# ON RELATIONSHIPS BETWEEN GOALS FOR AGGREGATION IN DECISION MAKING

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## Summary

A model of interactions between goals based on fuzzy relations is presented. In contrast to other approaches in multiple attribute decision making the interactive structure of goals for each decision situation is defined explicitly. The modeling of the interactive structure between goals provides for a better understanding of the decision situation and for better well-balanced aggregation.

**Keywords:** decision making, interacting goals, relationships between goals.

## 1 INTRODUCTION

When human decision makers deal with (even simple) decision situations, they intuitively deal with the interactive structure of the decision goals and reasons about relationships between goals. For instance, let us consider a decision maker who intends to earn money (goal 1) and to have fun (goal 2) simultaneously. If the only way to earn money is to work, then at least two situations are possible:

- Situation 1: The decision maker does not like to work. Therefore, while working the decision maker will not have fun. The alternative *working* has a positive impact on goal 1 but a negative impact on goal 2.
- Situation 2: The decision maker likes to work. Therefore, while working the decision maker will have fun. The alternative *working* has a positive impact on both goal 1 and goal 2.

In the first situation the goal "earn money" *affects negatively* the goal "have fun". In contrast to that, in the second situation the goal "earn money" *affects positively* the goal "have fun".

This simple example shows that there is a substantial need for a detailed reasoning on relationships between goals when dealing with decision making.

If we subsume for each goal $g$ the alternatives with a positive impact on $g$ in the fuzzy set $S_g$ and the alternatives with negative impact on the goal $g$ in the fuzzy set $D_g$, we observe the following characteristics of the two situations of the example drafted above:

- Situation 1:
- $S_{\text{earn money}} = \{\text{working}\} = D_{\text{have fun}}$
- Situation 2:
- $S_{\text{earn money}} = \{\text{working}\} = S_{\text{have fun}}$

Motivated by this observation, the relationships between two goals are defined based on a fuzzy inclusion and a fuzzy non-inclusion between the two fuzzy sets $S$ and $D$ of the corresponding goals.

## 2 BASIC DEFINITIONS

Before we define relationships between goals, which indicate how goals interact we introduce the notion of the positive impact set and the negative impact set of a goal.

**Def. 1)** Let $A$ be a non-empty and finite set of potential alternatives, $G$ a non-empty and finite set of goals, $A \cap G = \emptyset,\ a \in A,\ g \in G,\ \delta \in (0,1]$. For each goal $g$ we define the two fuzzy sets $S_g$ and $D_g$ each from $A$ into [0, 1] by:

1. *positive impact function of the goal g*

$$S_g(a) := \begin{cases} \delta, & a \text{ affects positively } g \text{ with degree } \delta \\ 0, & \text{else} \end{cases} \quad (1)$$

2. *negative impact function of the goal g*

$$D_g(a) := \begin{cases} \delta, & a \text{ affects negatively } g \text{ with degree } \delta \\ 0, & \text{else} \end{cases} \qquad (2)$$

The *fuzzy non-inclusion* **N** is defined as:

**Def. 2)** Let $S_g$ and $D_g$ be defined as in Def. 1). $S_g$ is called the *positive impact set of g* and $D_g$ the *negative impact set of g*.

The set $S_g$ contains alternatives with a positive impact on the goal $g$ and $\delta$ is the degree of the positive impact. The set $D_g$ contains alternatives with a negative impact on the goal $g$ and $\delta$ is the degree of the negative impact.

**Def. 3)** Let $A$ be a finite non-empty set of alternatives. Let $\mathcal{P}(A)$ be the set of all fuzzy subsets of $A$. Let $X, Y \in \mathcal{P}(A)$, $x$ and $y$ the membership functions of $X$ and $Y$ respectively. Let I be a *fuzzy inclusion*,

$$\mathrm{I}: \mathcal{P}(A) \times \mathcal{P}(A) \rightarrow [0,1], \qquad (3)$$

N a *fuzzy non-inclusion* defined as:

$$\mathrm{N}: \mathcal{P}(A) \times \mathcal{P}(A) \rightarrow [0,1] \qquad (4)$$

$$\mathrm{N}(X,Y) := 1 - \mathrm{I}(X,Y) \qquad (5)$$

The inclusions indicate the existence of interaction between two goals. The higher the degree of inclusion between the positive impact sets of two goals, the more cooperative the interaction between them. The higher the degree of inclusion between the positive impact set of one goal and the negative impact set of the second, the more competitive the interaction. The non-inclusions are evaluated in a similar way. The higher the degree of non-inclusion between the positive impact sets of two goals, the less cooperative the interaction between them. The higher the degree of non-inclusion between the positive impact set of one goal and the negative impact set of the second, the less competitive the relationship.

Note that the pair ($S_g$, $D_g$) represents the whole known impact of alternatives on the goal g. Dubois and Prade (1992) show that for ($S_g$, $D_g$) the so-called twofold fuzzy sets can be taken. Then $S_g$ is the set of alternatives which more or less certainly satisfy the goal g. $D_g$ is the fuzzy set of alternatives which are rather less possible, tolerable according to the decision maker.

## 3 RELATIONSHIPS BETWEEN GOALS

Based on the inclusion and non-inclusion defined above, basic types of interaction between goals are defined [6]. The interactions cover the whole spectrum from a very high confluence (analogy) between goals to a strict competition (trade-off). The independence of goals and the case of an unspecified dependence are also considered.

Let $S_{g_1}$, $D_1$, $S_{g_2}$ and $D_{g_2}$ be fuzzy sets given by the corresponding membership functions as defined in Def. 2). For simplicity we write $S_1$ instead of $S_{g_1}$ etc.. Let $g_1, g_2 \in G$ where $G$ is a set of goals. The types of interaction between two goals are defined as relations which are fuzzy subsets of $G \times G$. In [6] the full definitions are given. Here we consider the following five examples of relationships:

1. $g_1$ is independent of $g_2$: $\Leftrightarrow$

$$\textit{IS - INDEPENDENT - OF}(g_1,g_2) :=$$

$$\min(\mathrm{N}(S_1,S_2),\mathrm{N}(S_1,D_2),\mathrm{N}(S_2,D_1),\mathrm{N}(D_1,D_2)) \qquad (6)$$

2. $g_1$ cooperates with $g_2$: $\Leftrightarrow$

$$\textit{COOPERATES - WITH}(g_1,g_2) :=$$

$$\min(\mathrm{I}(S_1,S_2),\mathrm{N}(S_1,D_2),\mathrm{N}(S_2,D_1)) \qquad (7)$$

3 $g_1$ is analogous to $g_2$: $\Leftrightarrow$

$$\textit{IS - ANALOGOUS TO}(g_1,g_2) :=$$

$$\min(\mathrm{I}(S_1,S_2),\mathrm{N}(S_1,D_2),\mathrm{N}(S_2,D_1),\mathrm{I}(D_1,D_2)) \qquad (8)$$

4. $g_1$ competes with $g_2$: $\Leftrightarrow$

$$\textit{COMPETES - WITH}(g_1,g_2) :=$$

$$\min(\mathrm{N}(S_1,S_2),\mathrm{I}(S_1,D_2),\mathrm{I}(S_2,D_1)) \qquad (9)$$

5. $g_1$ is in trade-off to $g_2$: $\Leftrightarrow$

$$\textit{IS\_IN\_TRADE\_OFF}(g_1,g_2) :=$$

$$\min(\mathrm{N}(S_1,S_2),\mathrm{I}(S_1,D_2),\mathrm{I}(S_2,D_1),\mathrm{N}(D_1,D_2)) \qquad (10)$$

Please note that, there is a duality relation

assists $\leftrightarrow$ hinders, cooperates $\leftrightarrow$ competes,
analogous $\leftrightarrow$ trade off

which corresponds to the common sense understanding of the respective types of relationships between goals.
The relationships between goals are substantial for an adequate modeling of the decision making process because they reflect the way the goals depend on each other and describe the pros and cons of the decision alternatives, with respect to the goals. Together with information about goal priorities, the types of interaction between goals are the basic aggregation guidelines for the decision maker. For example, for cooperative goals a conjunctive aggregation is appropriate. If the goals are rather competitive, then an aggregation based on a exclusive disjunction is appropriate.

## 4 TYPES OF INTERACTION IMPLY WAY OF AGGREGATION

The observation, that cooperative goals imply conjunctive aggregation and conflicting goals rather lead to exclusive disjunctive aggregation, is easily to understand from the intuitive point of view.
Although a variety of aggregation operators has been developed [3], previously there were no systematic approaches how to choose the appropriate way of aggregation for a particular decision situation. The analysis of the decision situation based on the different types of interaction between goals presented in the previous subsection provides for a systematic method how to choose that way of aggregation which corresponds best to the requirements of the given particular decision situation.

The fact that the types of relationships between goals are defined as fuzzy relations based on both positive and negative impacts of alternatives on the goals provides for the information about the confluence and competition between the goals: The negative impact functions reflect the negative aspects of the decision alternatives with respect to each goal and, compared with other approaches, represent additional information which enables to distinguish the non-presence of confluence between two goals from an effective competition between them.

Figure 1 shows two different representative situations which can be distinguished appropriately only if besides the positive impact additionally the negative impact of decision alternatives on goals is represented.

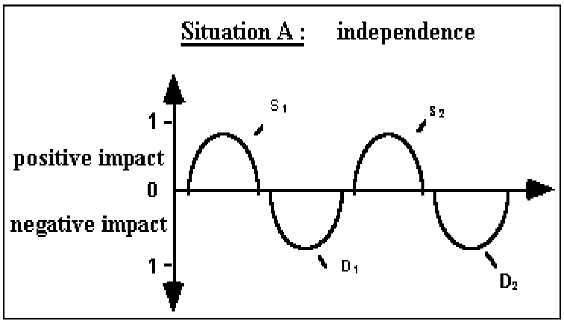


Figure 1 ( Part 1 ) :
Distinguishing independence and trade-off based on positive and negative impact functions of goals

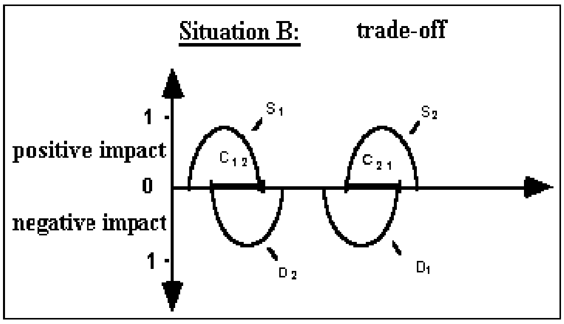


Figure 1 ( Part 2 ):
Distinguishing independence and trade-off based on positive and negative impact functions of goals

In case that the goals are represented only by the positive impact of alternatives on them, *situation A* and *situation B* could not be distinguished and a disjunctive aggregation would be the recommended in both cases.
However, in *situation B* a decision set $S_1 \cup S_2$ would not be appropriate because of the conflicts indicated by $C_{12}$ and $C_{21}$. In this situation the set $(S_1 \ / \ D_2) \cup (S_2 \ / \ D_1)$ could be recommended in case that the priorities of both goals are similar (where X / Y is defined as the difference between the sets X and Y, that means X / Y = X $\cap \overline{Y}$ , where X and Y are fuzzy sets). In case that one of the two goals, for instance goal 1, is significantly more important than the other one, the appropriate decision set would be $S_1$. The aggregation used in that case had not to be a disjunction, but an <u>exclusive</u> disjunction between $S_1$ and $S_2$ with emphasis on $S_1$.

This very important aspect, can easily be integrated into decision making models by analyzing the types of interaction between goals. The information about the interaction between goals in connection with goal priorities is used in order to define general decision rules which describe the way of aggregation. For conflicting goals, for instance, the following decision rule which deduces the appropriate decision set is given:

if ($g_1$ is in trade-off to $g_2$ ) and ($g_1$ is significantly more important then $g_2$ ) then $S_1$

Another rule for conflicting goals is the following:
if ($g_1$ is in trade-off to $g_2$ ) and ($g_1$ is insignificantly more important then $g_2$ ) then $S_1 / D_2$

Note that both rules use priority information which in case of a conflictive interaction between goals is substantial for a correct decision. Note also that the priority information can only by adequately used if the knowledge about whether or not the goals are conflictive is explicitly modeled.

## 5 RELATED APPROACHES, A BRIEF COMPARISON

Since fuzzy set theory has been suggested as a suitable conceptual framework of decision making [1], two directions in the field of fuzzy decision making can be observed. The first direction reflects the fuzzification of established approaches like linear programming or dynamic programming [11]. The second direction is based on the assumption that the process of decision making can be modelled by axiomatically specified aggregation operators [3]. None of the related approaches sufficiently addresses one of the most important aspects of decision making, namely the explicit and non-hard-wired modeling of the interaction between goals. The related approaches like Dubois and Prade [3], Biswal [2] either require a very restricted way of describing the goals or postulate that decision making shall be performed based on a few, very general mathematical characteristics of aggregation like commutativity or associativity. Other approaches are based on fixed (hard-wired) hierarchies of goals [10] or on the modelling of the decision situations as probabilistic or possibilistic graphs [8]. In contrast to that, human decision makers usually proceed in a different way. They concentrate on which goals are positively or negatively affected by which alternatives. Furthermore, they evaluate this information in order to infer how the goals interact with each other and ask for the actual priorities of the goals. In the sense that the decision making approach presented in this contribution explicitely refers to the interaction between goals, it significantly differs from other related approaches.

## 6 CONCLUSIONS

The importance of an appropriate analysis of the decision situation by modeling of types of relationships between goals in decision making has been discussed. Especially the explicit representation of both positive and negative impacts of decision alternatives on goals provides for the ability of adequate modelling of confluences and conflicts between goals and for an appropriate way of aggregation. The presented types of relationships between goals have been used in a novel decision making model successfully applied for real world decision making problems.

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