Linguistic Approach for interpolative reasoning

Nédra Mellouli, Bernadette Bouchon-Meunier
Université Pierre et Marie Curie
LIP6, mailbox 169, 4 place Jussieu
75252 Paris cedex 05, France

Abstract

We propose a new method to use an incomplete rule base with imprecize descriptions of variables. We extend classical interpolative reasoning to this case, under the assumption of graduality in variations of the variables, by using a linguistic approach.

Keywords: Fuzzy set theory, Analogical reasoning, Interpolation, Sparse rules.

1 Introduction

The process of interpolation is the common solution used to make decision under limited and incomplete knowledge. Dealing with numerical values involved in production rules, interpolation between the premises of the rules is the most classical way to associate a satisfiying solution to an observed new value.

In the case of fuzzy sparse rules, where values are linguistic, adopting interpolation to construct a linguistic result value for an observed linguistic value, is too complex. Many works exist in this topic. Some of them propose an extension of classical interpolation[1][2] based on the proportional distances between observation and premises. These approachs take into account only the distance comparison between fuzzy sets and neglects the information provided by the comparison between the shapes of fuzzy sets.

In this paper we propose a new interpolative reasoning approach for fuzzy sparse rules taking into account gradual knowledge and providing interpretable conclusions when the data are linguistic. Through this approach, we intend to take into account the semantic dimension involved in the comparison of two extreme situations. In the context of fuzzy sets theory, we have to distinctly undertake comparisons of uncertainty, precision and locations between observation and premises of the rules. We propose degrees of focusing and dispersion to take the uncertainty and precision aspects into account. We base our approach on an analogical scheme[5] in such a way that the conclusion is compared to consequents of rules, in a way analogous to the comparison of the observation to the antecedents of these rules.

2 Interpolative reasoning

Let us consider a rule base R_1, R_2..., R_n of the following form: (R_i) - if x is A_i then y is B_i, where x and y are two variables defined respectively on two finite intervals X and Y of $\mathbf{R}$.

A_i and B_i are fuzzy sets of X and Y representing respectively linguistic descriptions of x and y. We suppose that y has a gradual behavior with regard to x, which implies that the fuzzy sets A_i and B_i are ordered.

Let us suppose given the following observation: (O) - x is A,

such that A is located between two of the fuzzy sets $A_i,\ i = 1, n$ with respect to the considered order.

For the sake of simplicity, we assume that these fuzzy sets are A_1 and A_2.

The question is: which description of y can we deduce, under the hypothesis of graduality of y with respect to x ? We assume that this description will be represented by a fuzzy set B of $\mathbf{Y}$. The purpose of the approach we introduce in this paper, is the linguistic construction of the characterization B of y using an analogical sheme [5] and taking into account the graduality of the knowledge.

2.1 Interpolation of B

The comparison of B with B_1 and B_2 is analogous to the comparison of the description A with A_1 and A_2.

The comparison of the description A with A_1 and A_2 is done with regard to two basic aspects: the location of A between A_1 and A_2 and the shape of A related to the shape of A_1 and the shape of A_2. Our algorithm of construction of the characterization B of y is composed by three steps. The first step consists in defining the location of B in the context of B_1 and B_2 by interpolation of the location of A in the context of A_1 and A_2. In the second and the third steps we use the concept of *modifier* to take into account the difference between A and A_1 and A andA_2 .

At each step of this algorithm, a particular measure of comparison is used. We have at our disposal three types of comparison, the *location*, the *focusing* and the *dispersion*.

In the first step, we determine the *location* of B such that B is located between B_1 and B_2 in a way analogous to the way A is located between A_1 and A_2. The location of a fuzzy set is defined here by the middle of its kernel.

Then, the comparison of the shapes of fuzzy sets is split into two kinds of comparisons: the comparison of their kernels (*the focusing*) and the comparison of their supports (*dispersion*). Each kind of comparison leads us to the determination of a modifier related to the corresponding component of the fuzzy set.

In the second step, we construct the kernel of B such that the comparison of the focusing of B with the focusing of B_1 and the focusing of B_2 is analogous to the comparison of the focusing of A with the focusing of A_1 and the focusing of A_2. As a rule of thumb, if A is more focused than A_1 then B will be more focused than B_1 respecting the hypothesis of graduality.

Let us note by mf_1 the modifier of focusing such that $mf_1(A_1) = A$, then by analogy and respecting the graduality, we construct a fuzzy description $B_1^{'}$ with the focusing satisfies $mf_1(B_1) = B_1^{'}$. We do the same with A and A_2 using another modifier mf_2 to construct a fuzzy description $B_2^{'}$ with the focusing satisfiying $mf_2(B_2) = B_2^{'}$.

Once we have constructed the location and the kernel, we use these informations to define the support of $B_1^{'}$ and $B_2^{'}$. In the final step, we decompose the construction of the support of B into the construction of its left dispersion (the elements which belong to the support and which are smaller than all elements of the kernel) and the right dispersion(the elements which belong to the support and which are greater than all elements of the kernel). we construct the left dispersion of B such that the comparison of the left dispersion of B with the left dispersion of B_1 and the left dispersion of B_2 is analogous to the comparison of the left dispersion of A with the left dispersion of A_1 and the left dispersion of A_2. We apply the same process to build the right dispersion. Thus, we construct B with respect to B_1 and B_2.

Let us note for example by md_1 the left dispersion modifier such that $md_1(A_1) = A$, then by analogy and respecting the graduality, we define the left dispersion of $B_1^{'}$ by $md_1(B_1) = B_1^{'}$. We do the same with A and A_2 using another modifier md_2 to define the left dispersion of $B_2^{'}$ by $md_2(B_2) = B_2^{'}$. The description B will be the aggregation of $B_1^{'}$ and $B_2^{'}$.

2.2 Linguistic aspects

The proposed approach, based on three kinds of criteria, can be described easily as a linguistic approach.

For a given characterization A of x, we look for the characterization B of y such that:

- B is located between B_1 and B_2 in a way analogous to the way A is located between A_1 and A_2.

- the focusing of B is modified related to the focusing of B_1 and the focusing of B_2 in a way analogous to the focusing of A is modified related to the focusing of A_1 and the focusing of A_2.

- the dispersion of B is modified related to the dispersion of B_1 and the dispersion of B_2 in a way analogous to the dispersion of A is modified related to the dispersion of A_1 and the dispersion of A_2.

Once we define the modification of A related to A_1 and to A_2, we use the obtained information to construct the characterization B of y. The hypothesis of graduality regarding the variations of x and y leads us to determine both the location, the focusing and the dispersion of B, with the further assumption that the variations of y between two extreme fuzzy values B_1 and B_2 are analogous to the variations of X between two extreme fuzzy values A_1 and A_2.

The result of using simultaneous modifiers concerning both focusing and dispersion to fuzzy sets A, A_1, A_2, B_1, B_2 can be interpreted as a global modification of the shape of A relatively to A_1 and A_2 and the shape of B related to B_1 and B_2. It can be for instance, described linguistically as "A is really more focused than A_1 and A_2, less dispersed than A_1 and A_2 ".

To model the concept of focusing modifiers and dispersion modifiers, we use respectively the degree of focusing and the degree of dispersion which we detail in the following. The location of fuzzy sets is not described in this paper but can be found in [6].

3 Comparison of shapes

To define the shape of fuzzy sets we propose to characterize the kernel and the support by introducing two concepts the focusing and the dispersion. These concepts are used throught different modifiers to compare between shapes of fuzzy sets.

Let **X** $= [binf(X), bsup(X)]$ and let **Y** $= [binf(Y), bsup(Y)]$. We note by $L(X) = bsup(X) - binf(X)$ and $L(Y) = bsup(Y) - binf(Y)$ the spreads of these intervals. Moreover for any fuzzy set F=(a,b,c,d) with membership function f_F, we note: $kernel(F) = \{x \in X\ /f_F(x) = 1\} = [b, c]$, $support(F) = \{x \in X\ /f_F(x) \neq 0\} = [a, d]$.

3.1 Degree of focusing

The main idea of introducing the focusing of fuzzy sets is, first to characterize the aspect of the kernel independently of the whole shape of the fuzzy set, second to compare kernels of fuzzy sets between them. In this direction there exist two principal concepts: the measure of *nonspecificity* defined by Klir [3] as a generalization of Hartley function to fuzzy set and possibility set theory and the Yager's measure of *specificity* based on $\alpha-$cuts [4]. The two main ideas of these concepts are the following: let Sp be a specificity measure taking values in $[0, 1]$, on one hand, $Sp(F)$ is maximum and equal to 1 if and only if F is a fuzzy set on X reduced to a unique element, and on the other hand, $Sp(F') \leq Sp(F)$ if and only if $F' \subseteq F$.

However we can note that there exist no fundamental definition of specificity, if we want to compare any fuzzy sets. Furthermore, it is impossible to compare two fuzzy sets F and F' such that $F' \nsubseteq F$ by means of nonspecificity or specificity. Thus, we introduce the concept of focusing to compare two fuzzy sets.

Definition 1 *For any fuzzy set* $F = (a, b, c, d)$ *of* X*, the focusing of* F *is defined as* $Foc(F) = c - b$

We remark that $Foc(F) = 0$ if $kernel(F)$ is a singleton and the corresponding fuzzy set is called *focal*. This case can occurs when we have enough information characterizing the variable. But if we need more uncertain information, the difference $c - b$ increases and the fuzzy set F becomes less focused.

If we want now to compare objectively the focusing of two fuzzy sets (for example: comparing the focusing weight of babies to the focusing weight of adults), we must first define a normalized function independent of both scale and position.

We propose to define a normalized degree of focusing function, the properties of which are studied in the following.

Definition 2 *The degree of focusing of a fuzzy set* F *of* X *is a mapping* DF*:[0,L(X)]*$\rightarrow$ *[0,1] such that: the degree of focusing* DF *is a positive and decreasing function on [0,1].*

$DF(0)$ *is equal to 1 and* $DF(L(X))$ *is equal to 0.*

Given a fuzzy set F of X, the *degree of focusing* of F is defined as $DF(Foc(F))$.

Through the degree of focusing, we obtain three families of possible functions satisfiying the last definition. The study of these functions leads us to define a threshold ρ between focused and spread fuzzy sets. Given a fuzzy set F, the meaning of ρ is the limit value of $Foc(F)$ for which F can be considered as focal.

The value of ρ allows us to discriminate between focal fuzzy sets and spread ones. Given a fuzzy set F of Y, we interprete the degree of focusing of F as follows:

- $DF(Foc(F)) = 0$, F is an *absolutely spread* fuzzy set,
- $DF(Foc(F)) = 1$, F is an *absolutely focal* fuzzy set,
- $DF(Foc(F)) < DF(\rho)$, F is a *spread* fuzzy set,
- for any $F \in X$ such that $DF(Foc(F)) \geq DF(\rho)$, F is a *focal* fuzzy set

Hence, we define three modifiers "really more focused", "more focused" and "spreader" enabling us to compare two fuzzy sets with regard to the focusing.

Let F_1,F_2 be two fuzzy sets of X, we note $x_1 = Foc(F_1)$ and $x_2 = Foc(F_2)$

- F_1 *is really more focused than* F_2 if and only if $DF(x_2) < DF(\rho) < DF(x_1)$
- F_1 is *more focused than* F_2 if and only if $DF(\rho) < DF(x_2) < DF(x_1)$
- F_1 *is spreader than* F_2 if and only if $DF(x_1) < DF(x_2) < DF(\rho)$

3.2 Degree of Dispersion

Our purpose for introducing the dispersion of fuzzy sets is to study the aspect of the support independently of the kernel. In this direction we can follow the measure of accuracy of fuzzy sets: a fuzzy set F_1 is more precise than F_2, when A and B have the same kernel, if $support(F_1) \subset support(F_2)$. The most precise fuzzy set associated to F_1 is its kernel $kernel(F_1)$. Consider now two fuzzy sets F_1 and F_2 having the same focusing degree but disjoint supports. Adopting this concept of accuracy to compare the precision between them is impossible. Our purpose is to define another concept of accuracy characterized by a 2-dimensional vector: the *left-dispersion* and the *right-dispersion.*

Let $F = (a, b, c, d)$, the dispersion vector $dis(F)$ is defined as the vector $(Ldis(F), Rdis(F))$, where $Ldis(F)$ is the left dispersion defined as $Ldis(F) = b - a$ and the right dispersion defined as $Rdis(F) = d - c$.

To normalize the concept of dispersion, we must take into account the influence of the universe X and the focusing of the associated fuzzy set.

Definition 3 *The degree of dispersion of a fuzzy set F of X is defined as a mapping Ddis: $[0, L(X)] \times [0, L(X)] \to [0, 1] \times [0, 1]$, such that:*

- $Ddis(L(X), L(X)) = (0, 0)$*: the dispersion is maximum.*
- $Ddis(0, d - c) = (1, .)$*: the left dispersion ($Ddis(b - a, 0) = (., 1)$: the right dispersion) is minimum.*

The degree of dispersion of a fuzzy set F of X is defined as Ddis(F)=Ddis(Ldis(F), Rdis(F)).

A possible degree of dispersion is the following:

$Ddis(b-a, d-c) = (\frac{d-c}{L(X)-Foc(F)}, \frac{b-a}{L(X)-Foc(F)})$, that can be interpreted as the estimation of the precision related to typical values of a fuzzy set.

To adopt the dispersion degree for comparing two fuzzy sets, we must at first define the limit between dispersed and non dispersed fuzzy set in the same way as we have defined the threshold between spread and focal fuzzy sets. Denote by σ the limit value of dispersion.

By way of the degree of dispersion, we define three modifiers,"really less dispersed ", "less dispersed" and "more dispersed" which enable us to compare two fuzzy sets with regard with the dispersion. Let $F_i = (a_i, b_i, c_i, d_i)$, i=1,2, we note the left dispersion degrees: $x_1 = b_1 - a_1$ and $x_2 = b_2 - a_2$. We say that, with regard to the left dispersion,

- F_1 is really less dispersed than F_2 if and only if $Ddis(x_1, .) < Ddis(\sigma, .) < Ddis(x_2, .)$
- F_1 is less dispersed than F_2 if and only if $Ddis(\sigma, .) > Ddis(x_2, .) > Ddis(x_1, .)$
- F_1 is more dispersed than F_2 if and only if $Ddis(x_1, .) > Ddis(x_2, .) > Ddis(\sigma, .)$

We do the same thing with the right dispersion degree to define the right dispersion modifier.

4 Conclusion

We have proposed a linguistic construction of a linguistic observation using a sparse rule base. Our method is essentially based on the concept of graduality in the variations of the variable y respecting the variations of the variable x, and on an underlying process of analogy. The purpose of this paper is to show the power of a linguistic approach of interpolation to involve the semantic dimension of fuzzy descriptions.

5 References

[1] Koczy L.T, Hirota K., Approximate Reasoning by Linear Rule Interpolation and General Approximation. International Journal of Approximate Reasoning, 9, pp.197-225, 1993.

[2] Koczy L. T., Hirota K., Interpolative reasoning with insufficient evidence in sparse fuzzy rules bases, Inform. Science. 71, pp. 169-201, 1993.

[3] Klir G. J., Yuan B., Fuzzy sets and fuzzy logic: Theory and Applications.Published by Prentice Hall PTR, 1995.

[4] Yager R., Measuring tranquilllity and anxiety in decision making: an application of fuzzy sets, Intern.J. General systems 8, pp. 139-146, 1982.

[5] Bouchon-Meunier B., Delechamp J., Marsala C., Rifqi M., Several Forms of Fuzzy Analogical Reasoning, FUZZ-IEEE, pp. 45-50, 1997.

[6] Bouchon-meunier B., Delechamp J., Marsala C., Mellouli N., Rifqi M., Zerrouki L., Fuzzy interpolation in the case of sparse rules, Joint Conference EUROFUSE-SIC'99, Budapest, 1999.