CHARACTERISATION OF IMPLICATION OPERATORS IN FUZZY RULE BASED SYSTEMS FROM BASIC PROPERTIES

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Abstract

In this contribution we plan to characterise fuzzy implication operators based on three basic properties and the defuzzification methods suitable to be used in combination with them. We want to characterise the proper use of implication operators in Fuzzy Rule Based Systems for control problems.

Keywords: Fuzzy rule based systems, implication operators

1 INTRODUCTION

The design of Fuzzy Rule Based Systems (FRBSs) for control mainly involves two operations: the derivation of the Knowledge Base and the selection of the fuzzy inference and defuzzification processes that the FRBSs will use to perform the fuzzy reasoning.

Different studies have been carried out regarding to fuzzy inference and defuzzification mechanisms, both analytical [2] and experimental ones as [1, 3, 4, 9]. The current study is focused on this topic with the main objective of obtaining a characterisation of the fuzzy implication operators in the design of FRBSs which is appropriate from the engineering point of view.

In [4] we looked for the basic properties for the robustness of fuzzy implication operators in the sense of good average behaviour in any application and in combination with different defuzzification methods. Robust implication operators may be considered to be those that verify the properties shown below:

a) $I(h,0)=0,\ \forall h\in[0,1]$
b) $I(h,1)>0,\ \forall h\in(0,1)$ and $I(1,1)=1$
c) $I(0,y)=0,\ \forall y\in[0,1]$

On the one hand, these results support Mendel's affirmations [8] in the sense that he considered the minimum and the algebraic product t-norms as "*engineering implications*". These operators verify the three basic properties needed to have a robust behaviour. Talking about operators that verify the property $I(h,0) > 0$ (unlimited support), Mendel adds that *"this does not make much sense from an engineering perspective"* and *"violates engineering common sense"*.

On the other hand, Dubois and Prade [6] express their disagreement with Mendel assumptions saying that *"he apparently does not notice that there indeed exist genuine implications which do not have this behaviour, such as Gödel implication, or Goguen implication"*. From a different point of view, they postulate that *"the proper use of implication-based fuzzy rules is often misunderstood in fuzzy control"*.

We plan to characterise fuzzy implication operators based on the three said basic properties and the defuzzification methods suitable to be used in combination with them. *We want to characterise the proper use of implication operators in FRBSs for control problems.*

Properties **a)** and **c)** have different origins, but they are not independent. They have a common point: $h=0$. With the aim to have independent properties in this study, from now on, we are going to consider property **a)** as the union of properties *a1)* and *a2):*

a1) $I(h,0)=0,\ \forall h\in(0,1]$
a2) $I(0,0)=0$

This division is due to the fact that both properties, **a1)** and **a2)**, refer to different aspects. Property **a1)** $I(h,0)=0,\ \forall h\in(0,1]$ is related to the unlimited support.

On the other hand, property **a2)** is more related to the meaning of property **c)**, that is, to the non inference of consequent fuzzy sets when the matching between the inputs and the rule antecedent is 0.

As regards property **b)** *I(h,1)>0, ∀ h∈(0,1) and I(1,1)=1*, implication operators that verify it keep the information on the modal values of the fuzzy set when the matching degree is not 0.

Finally, implication operators that verify property **c)** *I(0,y)=0, ∀ y∈[0,1]* infer fuzzy sets that do not supply information when matching is 0. Rules for which the antecedents do not match the system input will not supply anything to the defuzzification interface.

We shall study all the possible cases resulting from the combinations in the verification of the basic properties. Given that **a1)** ∪ **a2)** = **a)** and that **c)** ⇒ **a2)**, we find that **a1)** ∪ **c)** ⇒ **a)** ∪ **c)**. Therefore, Table 1 shows the possible combinations or classes that could appear.

Table 1. Combination of properties

	Not verified							
Prop.	∅	b	b,c	c	a1,c	a1	a1,b	a1,b,c
Clas.	1	2	3	4	5	6	7	8

2 DEFUZZIFICATION METHODS

As regards the defuzzification methods we consider:

- The values of importance: matching, area and height.
- The characteristic values: Centre of Gravity (CG) and Maximum Value (MV).

The defuzzification methods taken into account are [3, 7]:

Mode A – FATI ***(First Aggregate Then Infer):*** The defuzzification interface performs the aggregation of the individual fuzzy sets inferred, B_i', to get the final output fuzzy set B'. The aggregation operator modelling the *also* connective may be selected as a t-norm (Mamdani´s Approximated Reasoning (MAR)) or a t-conorm (Formal Logic Reasoning (FLR)). Usually, the ones most used are the *Minimum* and *Maximum*, respectively. We consider the methods:

D_1: Middle of Maxima (MOM) of the individual fuzzy sets aggregated with *also* connective *Minimum*.
D_2: Centre of Gravity of the individual fuzzy sets aggregated with *also* connective *Minimum*.
D_3: Middle of Maxima (MOM) of the individual fuzzy sets aggregated with *also* connective *Maximum*.
D_4: Centre of Gravity of the individual fuzzy sets aggregated with *also* connective *Maximum*.

Mode B - FITA ***(First Infer Then Aggregate):*** It avoids the computation of the final fuzzy set B' by considering the contribution of each rule output individually, obtaining the final control action by taking a calculus (an average, a weighted sum or a selection of one of them) of a concrete characteristic value associated to each of them.

We will consider the following six defuzzification mechanisms working in Mode B – FITA for the experimental study we are going to develop:

D_5: Centre of Gravity weighted by the matching
D_6: Maximum Value weighted by the matching
D_7: Centre of Gravity of the fuzzy set with the largest matching
D_8: Maximum Value of the fuzzy set with largest matching
D_9: Average of the Maximum Values
D_{10}: Centre of Sums

3 RELATIONSHIP BETWEEN BASIC PROPERTIES AND DEFUZZIFICATION METHODS

This relation basically resides on the appropriate applicability of some defuzzification methods when a fuzzy implication operator does not verify one or more concrete basic properties. In this way, we could use implication operators that do not verify the three basic properties with a guarantee of good behaviour in engineering applications.

The study of the different implication operators classes introduced in Table 1 is shown in Table 2 in accordance with the following symbols:

X: it works fine,
⊗: it could work *(it has an acceptable behaviour in some cases, but not in all)*
- : it is out of order,
♦: we do not have enough information.

We are going to present a group of conclusions drawn from the observation of the study developed and reflected in Table 2.

1. The MV is a more general characteristic value than the CG because it is appropriate in most of the cases.

2. Mode B – FITA is more general than Mode A – FATI for the design of the defuzzification interface.

3. Defuzzification methods which work in Mode B – FITA and use the characteristic value MV and the value of importance matching, (e.g. the MV weighted by matching or the MV of the fuzzy set with largest matching), can overlook the non verification of any individual basic property or a couple of them.

Table 2. Correspondence between classes and defuzzification methods

Cl.	Does not / Verifies	CG	MOM / MV	h_i matching	s_i area	y_i height	Mode B	Mode A t-norm	Mode A t-conorm
1	∅ / a1,b,c	X	X	X	♦	♦	X	-	X
2	b / a1,c	♦	♦	⊗	♦	♦	⊗	♦	♦
3	b,c / a1	♦	♦	⊗	-	-	⊗	-	-
4	c / a1,b	X	X	X	-	-	X	-	-
5	a1,c / b	-	X	X	-	-	X	⊗	-
6	a1 / b,c	-	X	X	♦	♦	X	⊗	⊗
7	a1,b / c	-	♦	⊗	♦	♦	⊗	-	-
8	a1,b,c / ∅	-	♦	♦	-	-	♦	-	-

4. Mode A – FATI has two ways to aggregate the individual fuzzy sets inferred. Neither of them, MAR nor FLR, could be considered as more generic than the other one.

4 CHARACTERISATION OF IMPLICATION OPERATORS

We can analyse the behaviour of different fuzzy implication operators building different FRBSs designed by means of the combinations between these implication operators and 10 different choices for the defuzzification methods. We will run them over three different applications (the fuzzy modelling of three different surfaces) described in [4] and compute different performance degrees: an Adaptation Degree associated to the Medium Square Error (AD_SE) for every application and an Average Mean Adaptation Degree (AMAD) used to unify the results obtained in the three experiments (giving a global measure to compare the behaviour of the different implication operators in the three applications). All the measures are defined in [0,1] and the near value is to 1, the better behaviour is. The connective operator used in the antecedent is always the minimum t-norm.

In the following we show the analysis made on the T-norms, S-Implications and other implication operator.

4.1 T-NORMS

A function $T:[0,1]\times[0,1]\to[0,1]$ is a t-norm iff $\forall x,y,z \in [0,1]$ it verifies:

(1) Existence of a unit 1: $T(1,x)=x$.
(2) Monotonicity: If $x\leq y$ then $T(x,z) \leq T(y,z)$.
(3) Conmmutativity: $T(x,y)=T(y,x)$.
(4) Associativity: $T(x,Ty,z))=T(T(x,y),z)$.
(5) $T(0,x)=0$.

By definition, t-norms verify the three basic properties. On the one hand, **a1)** and **c)** are satisfied due to the verification of properties (5) and (3), whilst **b)** is satisfied by the verification of (1).

Therefore, we can conclude that, by their own definition, t-norms are robust implication operators and, then, they are suitable to be used in any application and in combination with a wide group of defuzzification methods composed by:

- Those which work in Mode B – FITA, with every value of importance and characteristic values

- Those which work in Mode A – FATI when a t-conorm is considered to model the *also* connective (FLR), but not when aggregating with a t-norm (MAR) because in this case there is a loss of information in the fuzzy inference process.

4.2. S-IMPLICATIONS

Corresponding to the definition of the implication in classical Boolean Logic: $A\to B=\neg A\vee B$, they present the form: $I(x,y)=S(N(a),b)$, with S being a t-conorm and N a negation function.

S-implications do not verify additional basic properties to the one verified by all the implication functions. That is, S-implications only verify the basic property **b)**, but not **a1)** and **c)**.

We know that in this situation the appropriate defuzzification methods are:

- Predominantly those which work in Mode B – FITA with the matching as a value of importance and the MV as a characteristic value.

- Those which work in Mode A – FATI aggregating with a t-norm (MAR) and defuzzifying with the MOM.

Table 3 shows the good behaviour of the S-implications

Diene : $\mathbf{I_1(x,y)} = \mathrm{Max}(1-x, y)$

$$\text{Dubois-Prade}: \mathbf{I_2(x,y)} = \begin{cases} 1-x, & \text{if } y=0 \\ y, & \text{if } x=1 \\ 1, & \text{otherwise} \end{cases}$$

Mizumoto : $\mathbf{I_3}$ **(x,y)** = 1-x + x·y

Lukasiewicz : $\mathbf{I_6}$ **(x,y)** = Min (1, 1-x+y)

in combination with the defuzzification methods MV weighted by matching (D_6) and MOM of the fuzzy set aggregated with *also* connective *Minimum* (D_1), opposite to bad average behaviour, AMAD, with other defuzzification methods.

Table 3: AMAD and AD_SE for S-implications with the best defuzzification methods (D_1 and D_6).

	AMAD	AD_SE_{YX}	AD_SE_{F1}	AD_SE_{F2}	D
I_1	0.62016	0.92301	0.95150	0.95679	1
I_1	0.62016	0.99642	0.99756	1.00000	6
I_2	0.65620	0.97633	0.96642	0.96365	1
I_2	0.65620	0.96170	0.99112	0.98935	6
I_3	0.63623	0.99550	0.97448	0.96910	1
I_3	0.63623	0.99642	0.99756	1.00000	6
I_6	0.64610	1.00000	0.97529	0.97022	1
I_6	0.64610	0.99141	0.99557	0.99742	6

4.3 OTHER IMPLICATION OPERATOR

We have analysed two important families of implication operators, but there are also implication operators that do not belong to any family.

The way to operate in this case involves to determine what defuzzification methods can be used, analysing what basic properties are verified by the specific operator and then, determining the appropriate defuzzification method in view of the information collected in Table 2.

As an example of how to operate, we consider the Gaines Implication.

$$\mathbf{I_{35}(x,y)} = \begin{cases} 1, & \text{if } x \le y \\ 0, & \text{otherwise} \end{cases}$$

It verifies the basic properties **a1)** and **b)** but not **c)**. Hence, it belongs to class 4 and the defuzzification methods appropriate will be those which work in Mode B – FITA using the matching as a value of importance (especially those based on weighting) to compensate the action of the rules fired with matching 0. Both the CG and the MV will show good behaviour.

These defuzzification methods are, for example, the CG weighted by matching (D_5) and the MV weighted by the matching (D_6). Table 4 shows their values.

Table 4. Gaines Implication with the best defuzzification methods (D_5 and D_6).

	AMAD	AD_SE_{YX}	AD_SE_{F1}	AD_SE_{F2}	D
I_{35}	0.80471	0.99141	0.99557	0.99742	5,6

5 CONCLUDING REMARKS

We can characterise different families of implication operators taking as a base the basic properties and the defuzzification methods suitable to be used in combination with them to obtain FRBSs appropriate for engineering problems. An extended characterisation study can be found in [5].

Finally, we should point out that the implication operators that apparently *"does not make much sense from an engineering perspective"* and *"violates engineering common sense"* [8] can be considered for engineering applications if they are used in combination with an appropriate defuzzification method that guarantees a good behaviour. For some authors, this is a compensation for its lack of sense. For many others, this is to know how to interpret the meaning of the implication as was printed in [6], *"the proper use of implication-based fuzzy rules is often misunderstood in fuzzy control"*.

REFERENCES

[1] Z. Cao, D. Park and A. Kandel, Investigations on the applicability of fuzzy inference, *Fuzzy Sets and Systems* **49** (1992) 151-169.

[2] J.L. Castro, J.J. Castro and J.M. Zurita, The Influence of Implication Functions in Fuzzy Logic Controllers with Defuzzification First and Aggregation After, *Proc. 7th IFSA Congr*., Prague 1997, 237-242.

[3] O. Cordón, F. Herrera and A. Peregrín, Applicability of the Fuzzy Operators in the Design of Fuzzy Logic Controllers, *Fuzzy Sets and Systems* **86** (1997) 15-41.

[4] O. Cordón, F. Herrera and A. Peregrín, Searching for Basic Properties Obtaining Robust Implication Operators in Fuzzy Control, *Fuzzy Sets and Systems* (1999). To appear.

[5] O. Cordón, F. Herrera and A. Peregrín, Chareacterisation of Implication Operators in Fuzzy Control based on Basic Properties and Defuzzification Methods. Technical Report DECSAI-99108, Dept. of Computer Science and A.I., University of Granada, 1999.

[6] D. Dubois and H. Prade, What are fuzzy rules and how to use them, *Fuzzy Sets and Systems* **84** (1996) 169-185.

[7] M.R. Emami, J.B. Türksen and A.A. Goldenberg, Development of A Systematic Methodology of Fuzzy Logic Modelling, *IEEE Trans. on Fuzzy Systems* **6** (1998) 346-361.

[8] J.M. Mendel, Fuzzy Logic Systems for Engineering: A Tutorial, Proc. *IEEE* **83** (1995) 345-377.

[9] I. Rojas, O. Valenzuela, M. Anguita, A. Prieto, Analysis of the Operators Involved in the Definition of the Implication Functions and in the Fuzzy Inference Process, *Int. J. of Approx. Reasoning* **19** (1998) 367-389.