

# A LANGUAGE FOR EXPRESSING EXPERT KNOWLEDGE USING FUZZY TEMPORAL RULES

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## Summary

In this paper we undertake an initial approach to the formal definition of a grammar that describes what we call Fuzzy Temporal Propositions (FTP), or propositions with explicitly expressed information of a temporal type. The grammar contemplates the double perspective of the problem: it provides a syntactic description of the propositions that it models, and a procedure (semantic criteria) for evaluating the Degree Of Fullfillment (DOF) of these propositions. In this paper we principally deal with aspects of the language for the representation of FTPs.

**Keywords:** Fuzzy Temporal Knowledge, Fuzzy Temporal Reasoning.

## 1 INTRODUCTION

Up until the present applications based on fuzzy rules have constituted the most significant industrial and commercial success of what has been labelled fuzzy technology. Diagnostic, classification and/or pattern recognition systems and, to an even greater degree, fuzzy controllers are clear examples in which fuzzy rules have had and currently have unquestionable specific importance.

Nevertheless, in spite of the enormous success that the fuzzy set theory has had in these environments, it is also patently clear that the structure of the fuzzy rules that make them up does not appear to have evolved in a significant manner since the appearance of the first proposals. Fuzzy Knowledge Bases (FKBs) were already described in these proposals as being formed by fuzzy rules that are constructed as the conjunction/disjunction of basic propositions of the type “X is A” (e.g., “Temperature is High”, “Error is Low”), and grouped into a very regular and parallel structure, which has clearly differentiated state and control variables.

In some proposals ([2, 10]) more generic or flexible structures of the FKB have been suggested, by means

of chaining between rules, for example. Nevertheless, not too many variations have taken place in the basic structure of the propositions making up the rules. Syntactically, these propositions always bind a linguistic variable with a linguistic value.

A large number of applications ([5, 7, 8, 9]) show portions of knowledge that do not correspond to this very simple scheme of propositions. Thus, for example, in monitoring and/or control environments we may find ourselves with expressions such as “Temperature has been very low for a few seconds” or “Vomiting started later than 15 min after the beginning of the radiation exposure” ([7]) or “Sometime within the last hour Temperature 1 was much greater than Temperature 2”. In all these examples the time variable is introduced explicitly, playing a central role in the meaning of the proposition, be it as a temporal reference of the events (“for a few seconds”, “for the last hour”), or as a relation between the occurrence of events (“15 minutes after”). The structure of these propositions does not fit into the conventional atemporal scheme “X is A”, at least in an intuitive and direct manner, due to which it becomes necessary to increase the flexibility and possibilities of the fuzzy proposition model. This is the context of this paper, in which we undertake an initial approach to the formal definition of a grammar that describes what we call Fuzzy Temporal Propositions (FTP), or propositions with explicitly expressed information of a temporal type. The grammar contemplates the double perspective of the problem: it provides a syntactic description of the propositions that it models, and a procedure (semantic criteria) for evaluating the Degree Of Fullfillment (DOF) of these propositions.

For reasons of brevity, we will attempt to condense the most relevant characteristics of the grammar, paying special attention to graphically illustrating its use by means of examples in which the DOF of some significant propositions is calculated. Some of the aspects related to the model of FTPs, mainly concerning the semantics associated with part of the propositions here formalised, can be found in [4]. On the contrary, in this paper we deal with aspects of the language for the representation of FTPs.

## 2 FROM FUZZY PROPOSITIONS TO FUZZY TEMPORAL PROPOSITIONS: A GRAMMAR FOR FTPs

The grammar essentially consists of two well differentiated blocks or parts. In the first of these blocks, the general structure of the FTPs is defined, by syntactic rules which combine temporal and spatial entities:

- RS1:  $\langle \text{Propositional Clause} \rangle ::= \{ \langle \text{Proposition} \rangle \mid (\langle \text{Propositional Clause} \rangle [\langle \text{Temporal Reference} \rangle]) \}$   
 RS2:  $\langle \text{Proposition} \rangle ::= \{ (\langle \text{Variable} \rangle \langle \text{Spatial Value} \rangle [\langle \text{Temporal Reference} \rangle]) \mid (\langle \text{Variable} \rangle [\langle \text{Spatial Value} \rangle] [\langle \text{Temporal Reference} \rangle] \langle \text{Spatial Relation} \rangle \langle \text{Variable} \rangle [\langle \text{Spatial Value} \rangle] [\langle \text{Temporal Reference} \rangle]) \}$   
 RS3:  $\langle \text{Spatial Value} \rangle ::= \text{IS Any spatial value}$   
 RS4:  $\langle \text{Spatial Relation} \rangle ::= ([\langle \text{Quantifier} \rangle] \langle \text{Spatial Comparator} \rangle)$   
 RS5:  $\langle \text{Quantifier} \rangle ::= \{ (\text{VERY})\{\text{MUCH} \mid \text{LITTLE}\} \mid \text{QUITE} \mid \dots \}$   
 RS6:  $\langle \text{Spatial Comparator} \rangle ::= \{ \text{IS GREATER THAN} \mid \text{IS LOWER THAN} \mid \text{IS SIMILAR TO} \}$

In the other block, following the line of [1], the concept of *fuzzy temporal reference* is described, which is the temporal context in which the proposition or parts of the same are to be evaluated (only the rewriting rules that are essential for the construction of the accompanying examples are shown):

- RT1:  $\langle \text{Temporal Reference} \rangle ::= \{ (\{ \langle \text{Interval - Instant Relation} \rangle \mid \langle \text{Instant - Instant Relation} \rangle \} \langle \text{Instant} \rangle) \mid (\langle \text{Persistence} \rangle [\{ \langle \text{Instant - Interval Relation} \rangle \mid \langle \text{Interval - Interval Relation} \rangle \} \langle \text{Interval} \rangle]) \}$   
 RT2:  $\langle \text{Persistence} \rangle ::= \{ \text{IN} \mid \text{THROUGHOUT} \}$   
 RT3:  $\langle \text{Interval - Instant Relation} \rangle ::= ([\langle \text{Interval - Instant Relation} \rangle] \{ \langle \text{Time Distance} \rangle \mid [\text{APPROXIMATELY}] \{ \text{UNTIL} \mid \text{FOLLOWS} \} \mid \text{INCLUDES} \})$   
 RT4:  $\langle \text{Time Distance} \rangle ::= ([\langle \text{Time Extent} \rangle] \{ \text{AFTER} \mid \text{BEFORE} \})$   
 RT5:  $\langle \text{Time Extent} \rangle ::= ([\langle \text{Expansion Operator} \rangle] \langle \text{Absolute Time Extent} \rangle)$   
 RT6:  $\langle \text{Expansion Operator} \rangle ::= \{ \text{AT LEAST} \mid \text{AT MOST} \mid ([\langle \text{Quantifier} \rangle] \{ \text{MORE THAN} \mid \text{LESS THAN} \}) \}$   
 RT7:  $\langle \text{Instant} \rangle ::= \{ \langle \text{Direct Instant} \rangle \mid \langle \text{Occurrence of an Event} \rangle \}$   
 RT8:  $\langle \text{Occurrence of an event} \rangle ::= \text{Any explicit event}$

Basically, a fuzzy temporal reference can be described in an absolute manner (e.g. “at 20.00”), in a manner relative to the current moment (“ten minutes ago”) or in a manner relative to the occurrence of an event (“a little bit after an increase in pressure”, “between

30 min and 2 hours after the beginning of irradiation”- [7]). From a quantitative and qualitative point of view, it may be an instant or a temporal interval. We understand fuzzy instant  $T$  as being a normalized and unimodal possibility distribution  $\mu_T$  defined over the time axis  $\tau$ , such that for a time point  $\tau_0 \in \tau$ ,  $\mu_T(\tau_0)$  represents the possibility of  $T$  being precisely  $\tau_0$ . On the other hand, a fuzzy interval is defined based on its initial  $T_S$  and final  $T_E$  instants by means of a possibility distribution defined over  $\tau$  that comprises the time points that are possible after  $T_S$  and before  $T_E$ .

The concept of *persistence* is related to the use of fuzzy intervals as temporal references in propositions, and responds to the intuitive idea that a proposition in which it is said, for example, “Temperature was high throughout the last 30 minutes”, does not have an identical meaning to a proposition that indicates “Temperature was high in (at any moment within) the last 30 minutes”. In the former, the occurrence of the non-temporal part “Temperature High” is required for all the time points in the time interval, while in the latter, the proposition is true if this occurs for any time point in the interval. In the grammar, the prepositions “throughout” and “in” characterise the situations of total persistence and absence of persistence, respectively, giving rise to different semantic rules which realise the accurate evaluation of the meaning in each case. By way of illustration, and in a totally qualitative manner, Figure 1 shows the differences in meaning between these two situations for a simple example.

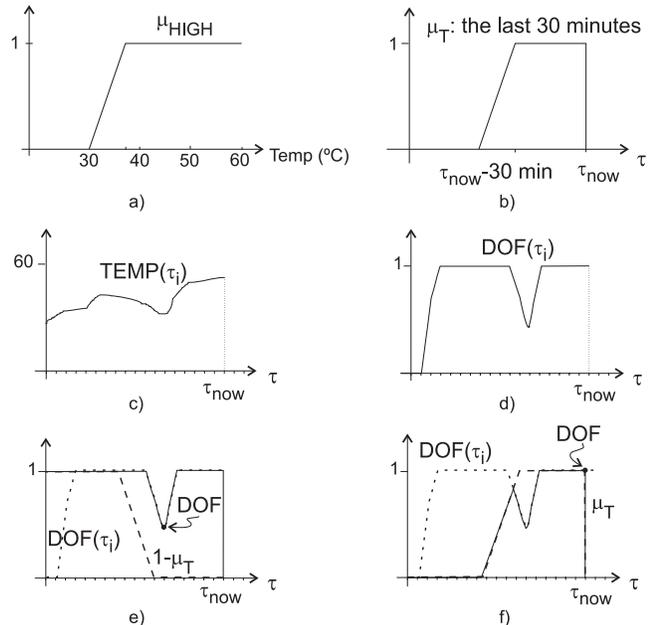


Figure 1: Calculation of DOF for proposition “Temperature was High throughout/in the last 30 minutes”.

We assume that membership function  $\mu_{HIGH}$  in Fig. 1a defines the linguistic value “High” for the linguistic variable “Temperature”, time membership function  $\mu_T$  in Fig. 1b, the time interval “the last 30 minutes”, and the recent history of Temperature observed values  $TEMP(\tau_i)$  is the one described in Fig. 1c. To obtain the DOF, in the first place, a *linguistic filtering* process (Fig. 1d) produces the history of the DOF of the non-temporal part of the proposition (“Temperature was high”):

$$DOF(\tau_i) = \mu_{HIGH}(TEMP(\tau_i)), \quad \forall \tau_i \in \tau \quad (1)$$

Simple maximum and minimum operations ([3]) between this DOF history and time interval  $\mu_T$  provide the DOF of the proposition. For the persistence situation (“Temperature was High throughout the last 30 minutes”) we have:

$$DOF = \bigwedge_{\tau_i \in \tau} DOF(\tau_i) \vee (1 - \mu_T(\tau_i)) = 0.4 \quad (2)$$

and for the non-persistence situation (“Temperature was High in the last 30 minutes”):

$$DOF = \bigvee_{\tau_i \in \tau} DOF(\tau_i) \wedge \mu_T(\tau_i) = 1 \quad (3)$$

Using these definitions, the weight of time points is proportional to their membership degree to  $\mu_T$ . Figures 1e and 1f show how this calculation process is performed.

The proposed grammar, in the simplest of cases, would allow propositions with the structure, already mentioned in prior examples, “X is A in T”, where T is the time reference and “X is A” is the atemporal (or, equivalently, “spatial”) part of the proposition. Obviously, more complicated cases than this one are contemplated. The propositions may be made up by relations between variables (“Temperature 1 is much higher than Temperature 2”) with different degrees of structural complexity (“Temperature 1 in the last few seconds was much higher than Temperature 2 throughout the last few minutes”). Evidently, the joint consideration of both grammars gives rise to much more generic structures, in which the temporal reference of the proposition and its spatial part are rewritten in a much more elaborate manner: “Pressure 1 was much higher than Pressure 2, a little bit after that Temperature 1 was much lower than Temperature 2”, “Temperature is low while Pressure is high”.

We now go on to describe, in a quantitative manner, an example (adapted from [7]) which shows the representative capabilities of the model.

Let us consider the following example: “Temperature was high at some instant between 15 min. and 1 h.

after the beginning of the irradiation”, which can be decomposed into a conjunction of two propositions: P1 (“Temperature was high at some instant at least 15 min. after the beginning of the irradiation”) AND P2 (“Temperature was high at some instant until 1 h. after the beginning of the irradiation”). These two propositions are modelled in the grammar according to the instantiation of the corresponding rewriting rules.

Thus, for their spatial part, the element “Variable” (rule RS2) is instantiated as TEMPERATURE, and “Spatial Value” as HIGH. For the temporal references, we have that, for P1, “Persistence” (rule RT2) is translated into IN (which means non-persistence), instantiation of RT3 to RT6 leads to AT LEAST 15 MINUTES AFTER, being 15 MINUTES an “Absolute Time Extent”; and then applying RT7, the event which occurs is BEGINNING OF THE IRRADIATION. For proposition P2, “Persistence” is IN as well, “Interval-Instant Relation” is UNTIL 1 HOUR AFTER, and the event for RT7 is the same BEGINNING OF THE IRRADIATION.

The semantics associated with these rewriting rules permits the calculation of the DOF for the propositions, according to the steps illustrated for the previous example in Fig. 1. Nevertheless, in this new example, the time intervals are not explicitly given, rather they refer to the occurrence of an event. Therefore, a first step must be included in order to obtain the explicit representation for the temporal references ( $\mu_{T1}$  and  $\mu_{T2}$ ). This process is shown in Fig. 2, where  $\tau_0$  is the time origin (time for the first observed value of temperature),  $\tau_I$  the instant when the beginning of the irradiation occurs, and  $\tau_{now}$  the current instant. A discretization step  $\Delta = 5min$  is assumed.

$\mu_{T1}$ : at least 15 min. after the beginning of the irradiation - - -  
 $\mu_{T2}$ : until 1 h. after the beginning of the irradiation . . . . .

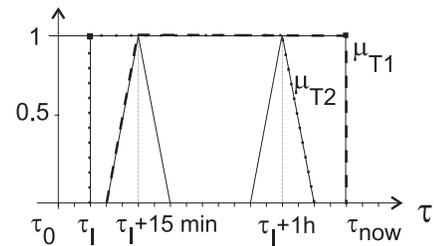


Figure 2: Calculation of the temporal references  $\mu_{T1}$  and  $\mu_{T2}$  for propositions P1 and P2 in the example.

We assume the history of temperature values is the one shown in Fig. 1c for the previous example:  $Temp(\tau_i) = \{27, 30, 35, 38, 38, 40, 45, 43, 42, 41, 39, 35, 33, 41, 46, 50, 51, 52, 53\}$ ,  $\tau_i \in [\tau_0, \tau_{now}]$

As only the time points belonging to the supports of  $\mu_{T1}$  and  $\mu_{T2}$  need to be taken into account, this set is reduced, for P1 and P2, to:

P1:  $\text{Temp}(\tau_i)=\{38,40,45,43,42,41,39,35,33,41,46,50,51,52,53\}$

P2:  $\text{Temp}(\tau_i)=\{35,38,38,40,45,43,42,41,39,35,33,41,46,50\}$

These values must be "linguistically filtered" with value "high", in order to obtain:

P1:  $\text{DOF}(\tau_i)=\{1,1,1,1,1,1,1,0.7,0.4,1,1,1,1,1,1\}$

P2:  $\text{DOF}(\tau_i)=\{0.7,1,1,1,1,1,1,1,1,0.7,0.4,1,1,1,1\}$

with the following temporal references (Fig. 2):

P1:  $\text{T1}(\tau_i)=\{0.5,1,1,1,1,1,1,1,1,1,1,1,1,1,1\}$

P2:  $\text{T2}(\tau_i)=\{1,1,1,1,1,1,1,1,1,1,1,1,0.5\}$

Since the propositions have a meaning of non-persistence, the DOF is calculated according to expression (3):

P1:  $\text{DOF}=\sqrt{(0.5,1,1,1,1,1,1,0.7,0.4,1,1,1,1,1,1)} = 1$

P2:  $\text{DOF}=\sqrt{(0.7,1,1,1,1,1,1,1,0.7,0.4,1,1,0.5)} = 1$

Finally, the global DOF for the initial proposition (conjunction of P1 and P2) is obtained, using a t-norm (for example, "minimum"):  $\text{DOF}=1 \wedge 1=1$ .

### 3 DISCUSSION

In this work we have tried to illustrate the descriptive potential of what we call Fuzzy Temporal Propositions by means of examples. The formal description of the grammar in which they are included has been introduced, including the semantic part, which allows the calculation of the DOF of the propositions that are syntactically correct.

The current state of the design of the grammar contemplates a large number of cases of interest, at the same time as completely encompassing totally atemporal propositions.

We are at present studying the incorporation of new potentialities to the model, such as the possibility of graduating spatial or temporal persistence, which, amongst other things, allows the modelling of propositions such as "X is A in part of T", or the inclusion of temporal and/or spatial relations of other types (for example, "the last time Temperature was high, Pressure was low"). One aspect of interest that we are approaching is the incorporation into the model of what we call Fuzzy Temporal Profiles ([6]), which permit the evaluation of tendencies in variables over time. The inclusion of these and the complete formalisation of the grammar that we outline here will enable us to clearly advance in increasing the expressive capacity of fuzzy propositions, which will permit the opening up of new application environments for fuzzy rule based systems.

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