STOCK PRICE FORECASTING: AUTOREGRESSIVE MODELLING AND FUZZY NEURAL NETWOK

Dušan Marček
Department of Macro and Micro Economics, University of Žilina, Slovak Republic
e-mail: marcek@fria.utc.sk

Summary

This paper describes the basic notion of fuzzy linear regression models based on so called fuzzy parameter extension principle and presents the autoregressive model which uses the fuzzy parameters extension principle and fuzzy neural network priciple for estimating and predicting stock prices. The presented approach is supported and illustrated by practical application results.

Keywords: B-spline function, Artificial Neural Network (ANN), Fuzzy Autoregressive Model (FAR)

1 INTRODUCTION

Most models for the time series of stock prices have centered on autoregressive (AR) processes. Traditionaly, fundamantal Box-Jenkins analysis [2] have been the mainstream methodology used to develop time series models.

Next, we briefly describe the develop a classical AR model for stock price forecasting. A fuzzy regression model is then introduced. Following this description, an artificial fuzzy neural network based on B-spline member ship function is presented as an alternative to the stock prediction method based on AR models. Finnaly, we present our preliminary results and some further experiments that we performed.

2 AR MODELLING

We give an example that illustrates one kind of posible results. We will regard these results as the referential values for the approach of fuzzy autoregressive and ANN modelling.

To illustrate the Box-Jenkins methodology, consider the stock price time readings of a typical company (say VAHOSTAV company). We would like to develop a time series model for this process so that a predictor for the process output can be developed. The data was collected for the period January 2, 1997 to December 31, 1997 which provided a total of 163 observations (see Fig. 1). To build a forecast model the sample period for analysis $y_1, ..., y_{128}$ was defined, i.e. the period over which the forecasting model was developed and the ex post forecast period (validation data set), $y_{129}, ..., y_{163}$ as the time period from the first observation after the end of the sample period to the most recent observation. By using only the actual and forecast values within the ex post forecasting period only, the accuracy of the model can be calculated.

After some experimentation, we have identified two models for this series (see [4]): the first one (1) based on Box-Jenkins methodology and the second one (2) based on signal processing.

$$y_t = \xi + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t \qquad (1)$$

$$t = 1,2, \dots , N-2$$

$$y_t = -\sum_{k=1}^{7} a_k y_{t-k} + \varepsilon_t \qquad (2)$$

$$t = 1,2, \dots , \text{N - 7}$$

The final estimates of model parameters (1), (2) are obtained using OLS (Ordinary Last Square) and two adaptive filtering algorithms in signal processing [1]. The Gradient Lattice (GL) adaptive algorithm and Last Squares Latice (LSL) algorithm representing the parameter estimates of the predictors (1), (2) were used. In Table 1 the parameter estimates for model (2) and corresponding RMSE's are given. The Fig. 1 shows the GL prediction results and actual values for stock prices in both analysis and ex post porecast period.

3 FUZZY AUTOREGRESSIVE MODELLING

Next, we examine the application of fuzzy linear regression model [7], [8] to the stock price time readings used in (1) and (2).

Recall that the models in (1) and (2) fit to the stock prices were the AR(2) and AR(7) processes.

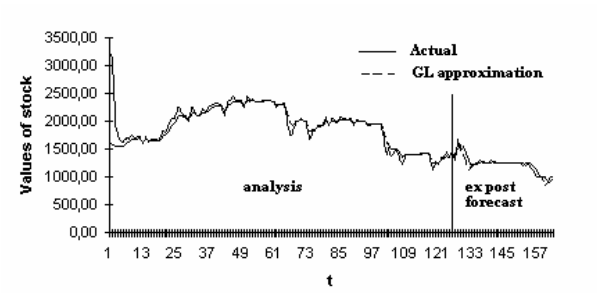

Figure 1: The data for VAHOSTAV stock prices (January 1997 - August 1997) and the values of the AR(7) model for VAHOSTAV stock prices estimated by GL algorithm

In the fuzzy regression model proposed by Tanaka et al. [8], the parameters are the fuzzy numbers. The regression function of such a fuzzy parameters can be modeled by the following equation

$$Y_t = A_0 * \varphi_0(x_{0t}) \oplus A_1 * \varphi_1(x_{1t}) \oplus, \ldots, \oplus A_k * \varphi_k(x_{kt}) = \mathbf{A}'\mathbf{x}_t \quad (3)$$

where $A_0, A_1, \ldots, A_k$ are fuzzy numbers, $\oplus$ and $*$ are fuzzy addition and fuzzy multiplication operators respectively, Y_t is fuzzy subset of y_t. This kind of fuzzy modelling is know as fuzzy parameter extension.

The problem to find out fuzzy parameters gives the following linear programming solution [8]

$$\min\ s = c_0 + c_1 + \ \ldots\ + c_k$$

$$\text{subject to } c_j \geq 0$$

and (4)

$$(h-1)\mathbf{c}'|\mathbf{x}| - (y_t - \mathbf{x}'\alpha) \geq 0$$

$$(1-h)\mathbf{c}'|\mathbf{x}| - (y_t - \mathbf{x}'\alpha) \geq 0$$

for t = 1, 2, ..., N

where c_j , j = 0, 1, ... , k is the width or spread around the center of the fuzzy number, $\alpha = (\alpha_0, \alpha_1, \ \ldots\ , \alpha_k)$ denotes vector of center of the fuzzy numbers for model parameters, $\mathbf{x}' = (x_0, x_1, \ldots, x_k)'$ denotes vector of regressor variables in (3), *h* is an inposed threshold $h \in [0, 1]$ (see [8]). A choice of the *h* value influences the widths c_j of the fuzzy parameters. The *h* value expresses a measure of the fitting of the estimated fruzzy model (3) to the given data. The fuzzines of $\mathbf{c}' = (c_0, c_1, \ \ldots\ , c_k)$ of the parameters $\widetilde{A}_0$, $\widetilde{A}_1$, ... , $\widetilde{A}_k$ for the models (1) and (2) are given in Table 2. The forecast for future observation is generated sucessively throught the Eq. (3) by replacing the functions of the independent variables ($\varphi_j(x_{jt})$, *j* = 0, 1, ... , *k* by observations y_{t-j}. Then the forecasting function of the fuzzy AR process is

$$Y_{T+1}(T) = A_0 \oplus A_1 * y_T \oplus A_2 * y_{T-1} \oplus, \ldots, \oplus A_k * y_{T-k+1} \quad (5)$$

where $Y_{T+1}(T)$ is the forecast for period *T*+1 made at origin *T*. We observe that the forecasting procedure (5) produces forecast for one period ahead. As a new observation becomes available, we may set the new current period *T*+1 equal to *T* and compute the next forecast again according to (5).

4 B-SPLINE NEURAL NETWORK APPROACH

The concept of fuzzy neural network (FNN) can be approached from several different avenues. The one that we have used for stock price forecasts is based on the concept of the B-spline membership functions of the data described in [9]. This concept (often called as B-spline FNN) can be considered as a fuzzy system introduced by Kosko in [3].

The fuzzy system (see Fig. 2) associates output variables a_j, j = 1, 2, ... , p with input fuzzy sets $B_{j,k}$ (y_{t-j}).

The neuro-fuzzy networks compute the output variables a_j , weight them with the scalars y_{t-j} , and the fuzzy system sums them to produce the scalar output $\hat{y}_t$. The nonlinear output variable a_j in the fuzzy system according to Fig. 2 can be estimated as an inner product of B-spline functions $B_{j,k}$ (y_{t-j}) and weights $\omega_j(t)$. It is then passed through a

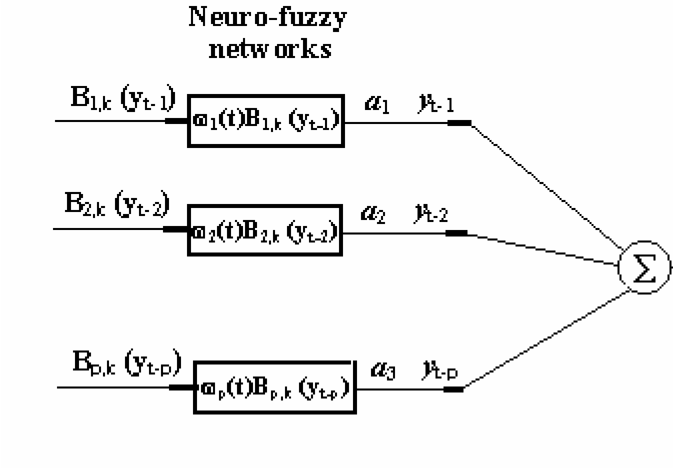

Figure 2: The fuzzy system for approximation of nonlinear function

$$a_j = f\left(\sum_{t=-j}^{n-j} \omega_j(t) B_{j,k}(y_{t-j})\right) = f(U_j) \qquad (6)$$

where k is order of the B-spline basis curve. In our case we used a hyperbolic tangent:

$$f(U_j) = \tanh(U_j) = \frac{\exp(gU_j) - \exp(-gU_j)}{\exp(gU_j) + \exp(-gU_j)}$$

where g is the parametr which controls the slope of the sigmoidal function.

Then, the fuzzy system in Fig. 2 can be represented as a neural network in the architecture of Fig. 3.

The learning algorithm is based on error signal. The neuro-fuzzy system modifies the weights in synaptic connections $(\omega_j(t))$ with respect to the desired fuzzy system output y_t. The error of the fuzzy system, i.e. the difference between the fuzzy system forecast $\hat{y}_t$ and the actual value, is analysed throught the RMSE. This measure is systematically minimised by adjusting the weights $\omega_j(t)$ according to the normalised Back-Propagation algorithm (see [6]).

5 RESULTS AND CONCLUSION

The statistical forecast accuracy of the FNN according to Fig. 3 depend on the formulation of the B-spline curve in (6) and the type of nonlinear transfer function. The approximation is better the higer the value of k. In all B-spline basis functions are cubic ones set. Periodically, during the training period, the RMSE of the FNN was measured not only on the training set but also on the validation (ex post forecast) set. The final FNN chosen for the stock price prediction is the one with the lowest error on the validation set. Note also, the training phase was finished after 2.10^4 epochs the best model being obtained after 5.10^3 epochs.

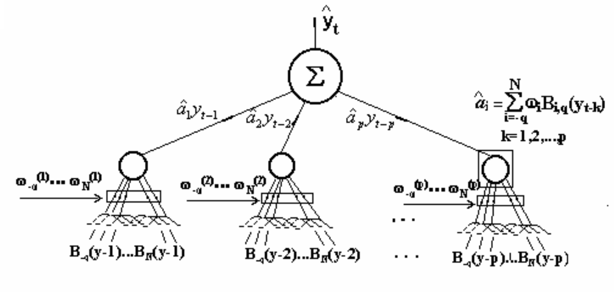

Figure 3: The neuro fuzzy system architecture

The RMSE's of our predictor models are shown in Table 3. From this table can be seen that the basic ANN architecture das not support the use ANN for daily frequencies. The initial results from using FNN architecture are clearly better.

Acknowledgement

This work was supported by Slovak grant foundation under the grant No. 1/6257/99. I thank M. Tkacin for the data sets and R. Ondrejáš for computational support.

References

[1] BAYHAN, G., M.: *Sales Forecasting Using Adaptive Signal Processing Algorithms*. Neural Network World 4-5/1997, Vol. 7.(579-589)

[2] BOX, G.,E., JENKINS, G.,M.:*Time Series Analysis, Forecasting and Control.* Holden-Day, San Francisco, CA 1976

[3] KOSKO, B.: Neural networks and fuzzy systems - a dynamical systems approach to machine inteligence. Prentice-Hall International, Inc. 1992

[4] MARČEK, D.: *Stock Price Prediction Using Autoregressive Models and Signal Processing Procedures.* Proceedings of the 16th Conference MME'98, Cheb 8.-10.9.1998 (114-121)

[5] MARČEK, D.: Neural Network Model for Stock Prices Forecasting. Neural Network World 3/1997 (347-352)

[6] POLYCARPOU M. M., IOANNOU P. A.: Learning and convergence analysis of neural-type structured networks. IEE Transactions on Neural Networks, 3, 1992

[7] SAVIC, D., A., PEDRICZ, W.: Evaluation of fuzzy regression models. Fuzzy Sets and Systems. 39 (1991) North-Holland (51-53)

[8] TANAKA. H., UEJIMA, S. and ASI, K.> Linear Regression Analysis with Fuzzy Model. IEEE Transactions on Systems, Man and Cybernetics, Vol. SMC - 12 . No. 6, 11-12/1982 (903-906)

[9] WU Z. Q., HARRIS Ch. J.: Indirect Adaptive Neurofuzzy Estimation of Nonlinear Time Series. Neural Network World 3/96 (407-416)

Table 1: OLS, GL and LSL estimates of AR models

Model	Order	Est.proc	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	$\hat{a}_4$	$\hat{a}_5$	$\hat{a}_6$	$\hat{a}_7$	RMSE *
(1)	2	OLS	1.113	-0.127				$\xi = 26.639$		67.758
(2)	7	GL	-0.7513	-0.1701	-0.0230	-0.0128	-0.0028	-0.0472	0.0084	68.540
(2)	7	LSL	-0.8941	-0.6672	0.7346	-0.2383	0.1805	-0.5692	0.4470	94.570

*ex post forecast period

Table 2: The fuzzines and modal values for model (3)

h=0.5 $\tilde{A}_k$ k:	0	1	2	3	4	5	6	7
Model FAR(2)								
Modal values(α)	26.639	1.113	-0.127					
Spread (**c**)	0	0	0.229008					
Model FAR(7)								
Modal values(α)	45.930	1.085	0.0861	-0.2531	0.0836	-0.0057	0.2081	-0.2281
Spread (**c**)	0	0	0	0	0.209587	0	0	0

Table 3: The RMSE characteristics

Model	RMSE*
AR(2)	67.7
Basic (non fuzzy) neural network (see [5])	67.2
Fuzzy system (see Fig. 2)	55.2

* Validation set