DEFUZZIFICATION AND CHAINING OF RULES IN HIERARCHICAL FUZZY SYSTEMS

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Summary

In hierarchical fuzzy systems, there occurs a problem what should be chosen as an input for fuzzy inferences which are connected to the outputs of another ones. We suggest to prefer defuzzification method of maxima type for fuzzy reasoning systems and of center of gravity type for fuzzy control systems.

Keywords: Fuzzy inference, defuzzification, hierarchical systems.

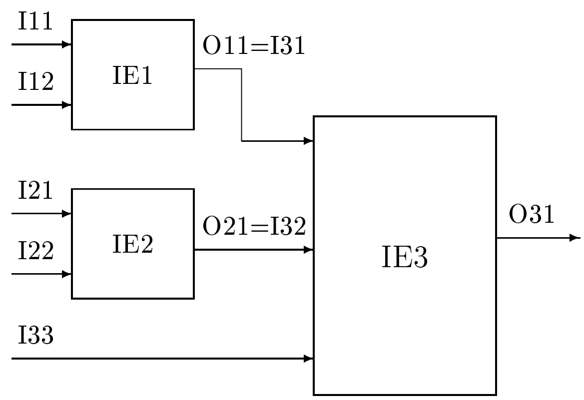

Figure 1: Example of hierarchical fuzzy system

1 INTRODUCTION

In this paper, we want to demonstrate some problems connected with the defuzzification and chaining of rules in situations when there are more fuzzy inference engines connected in a hierarchical way. As the inference mechanisms in such a complex fuzzy systems we use so called fuzzy logic inference proposed and developed by V. Novák (see e.g. [10]). This inference method is theoretically well founded and proved itself to be useful also in practical applications.

By hierarchical fuzzy system we understand several fuzzy inference engines connected in such a way that the output of one inference is input of another one (Figure 1). Here, there are three inference engines IE1, IE2 and IE3. IE1 and IE2 have two inputs, IE3 has three inputs, among them are outputs of IE1 and IE2. However, the structure of hierarchical fuzzy systems can be much more complicated.

Such systems may arise e.g. after a decomposition of some complicated fuzzy system with many antecedent variables. It is almost impossible to describe complicated system by means of one set of IF-THEN rules. Therefore, it is necessary to decompose it to smaller blocks (inference engines with maximally three or four inputs). In these situations, there arises the following problem. As a result of one inference step, we obtain a fuzzy set. There are several possibilities what we can choose as an input to the second inference, namely: we can use this fuzzy set directly as the input of the second inference engine and then we obtain inference with fuzzy inputs, which is computationally much more demanding. Beside that, it can also cause the outputs to be more and more "non-specific", i.e. output fuzzy sets tend to have the membership function close to the maximal membership degree on the whole universe of discourse after several inferences in such a hierarchical fuzzy system.

Therefore, it is necessary to search other possibilities how to transmit output of one inference to input of the other. We can defuzzify output fuzzy set and use as an input to the second step defuzzified value. In this case we have to take into account the loss of information caused by the defuzzification (instead of fuzzy set we use only one real number). We will discuss these possibilities in the subsequent sections.

As *linguistic description* we in the following understand the finite set of linguistic statements IF X is A_i THEN Y is B_i, where A_i, B_i are linguistic expressions of the form

$$[\langle \text{linguistic modifier} \rangle]\langle \text{atomic term} \rangle.$$

Here, ⟨linguistic modifier⟩ is an intensifying adverb with narrowing or extending effect (e.g. *very, more or less* etc.) and the negation *not*. The ⟨atomic term⟩ may be an adjective (we use *small, medium, big*), a fuzzy number or some special term (e.g. *undefined*). A typical example of the linguistic expression A is *very small*, *roughly big* etc.

2 INFERENCE WITH FUZZY INPUTS

The inference with fuzzy inputs has been studied in the previous papers [2, 3]. There were discussed problems with rapidly increasing computational complexity, which does not practically allow to use it for more than four antecedent variables. The fundamental formula which describes fuzzy logic inference mechanism is

$$B'y = \bigvee_{x_1 \in U_1, \dots, x_n \in U_n} ((A'_1 x_1 \wedge \cdots \wedge A'_n x_n) \otimes \otimes \bigwedge_{j=1}^{r} ((A_{1j} x_1 \wedge \cdots \wedge A_{nj} x_n) \rightarrow B_j y)), \quad (1)$$

where n is the number of antecedent variables, r is the number of rules in the linguistic description, $U_1, \dots, U_n$ are universes of discourse of antecedent variables, $A_{1j}, \dots, A_{nj}$, $A'_1, \dots, A'_n$, B_j are fuzzy sets which represent the meanings of linguistic expressions in the antecedent part of the linguistic description, observations and linguistic terms in the consequent part of linguistic description, respectively. Construction of these meanings is described in the paper [11]. The symbols $\otimes$ and $\rightarrow$ are the Łukasiewicz conjunction and implication, respectively. The computational complexity of fuzzy logic inference with fuzzy inputs can be described by

$$\mathcal{C} = \mathcal{O}(rP^{n+1}), \quad (2)$$

where P is precision of discretization (for the sake of simplicity, we assume that it is equal for all antecedent as well as succedent variables). Besides problems with computational complexity, there is also another problem: increasing fuzziness of outputs. When inputs do not match no rule fully, the result is fuzzy set which membership degrees are greater than zero for all $x \in X$. When such a set is used as a input for another inference step, the result will be now even more "fuzzy". Finally, it often occurs that as a result at the end of hierarchically chained inferences we obtain fuzzy set with all membership degrees close to one. Therefore, we can conclude that direct use of the output of one inference step as an input of the other is not advisable.

3 DEFUZZIFICATION

Defuzzification is a procedure which assigns a crisp value to a given fuzzy set. It is necessary to use it in situations when numerical output of fuzzy system is required, e.g. in fuzzy control. However, it is clear that there must occur some information loss. There are a lot of defuzzification techniques described in literature. In paper [13] there are provided certain criteria which defuzzification methods should obey. Among the most important are core selection (this is fulfilled if defuzzified value has always maximal membership degree), scale invariance wrt. various scale transformations, monotonicity and continuity (small variation in the membership function of a given fuzzy set should not result in a big change in the defuzzification value). Further, there is a classification of defuzzification methods proposed so far into several groups, namely: maxima methods, distribution methods and area methods. The prototypes of these groups are MOM (middle of maxima method), COG (center of gravity method) and COA (center of area method).

As is shown in [13], MOM method fulfills most of the defuzzification criteria, with the exception of continuity, and are therefore good candidates for fuzzy reasoning systems. Distribution and area methods are, on the other hand, continuous, which make them candidates for use in fuzzy control systems, where continuity is crucial.

For the use in hierarchical fuzzy systems with fuzzy logic inference mechanism it is necessary to modify the prototype distribution method COG. It can be, in its basic form, described by the following formula for fuzzy set defined on discretized universe $X = \{x_1, x_2, \dots, x_P\}$ as a mapping from $[0,1]^X$ to X

$$COG(A) = \frac{\sum_{i=1}^{P} x_i A x_i}{\sum_{i=1}^{P} A x_i}. \quad (3)$$

However, when we use fuzzy logic inference, then this method has serious drawback which can be explained as follows. When there is no rule which fully fires for a given input, then the result is a fuzzy set whose membership degrees are greater than zero for all $x \in X$. When we perform COG on such a set, the result will be shifted to the universe center. Therefore, simple COG is not suitable for fuzzy logic inference. Center of Gravity method can be principally used here, but it needs to be modified in order to make it insensitive to those parts of membership function which bear no relevant information. This modification is based on computation of gravity center from an strong α-cut ${}^{\alpha+}A$ [6] only of fuzzy set A, defined as ${}^{\alpha+}A = \{x \in X | \; Ax > \alpha\}$ where α is equal to the smallest membership degree of A. Formula (3) then becomes

$$MCOG(A) = \begin{cases} \frac{x_1+x_P}{2} & \min\limits_{x_i\in X} Ax_i = \max\limits_{x_i \in X} Ax_i \\ \frac{\sum_{x_i\in {}^{\alpha+}A} x_i Ax_i}{\sum_{x_i \in {}^{\alpha+}A} Ax_i} & \text{otherwise.}\end{cases} \tag{4}$$

This modified COG method preserves the following continuity property, which is weaker than that defined in [13]:

$$\begin{aligned} &\forall x_0 \in {}^{\alpha+}A \; \forall \epsilon > 0 \; \exists \Delta^* > 0 \; \forall \Delta \; (|\Delta| < \Delta^* \Rightarrow \\ &\Rightarrow |MCOG(\Delta_{x_0}A) - MCOG(A)| < \epsilon), \end{aligned} \tag{5}$$

provided that $\min\limits_{x_i\in X} Ax_i \neq \max\limits_{x_i\in X} Ax_i$. $\Delta_{x_0}Ax$ is defined as

$$\Delta_{x_0}Ax = \begin{cases} Ax + \Delta & x = x_0 \\ Ax & \text{otherwise.} \end{cases}$$

Proof: Denote

$$1/\gamma = \sum\nolimits_{x_i \in {}^{\alpha+}A} Ax_i, \quad \beta = \sum\nolimits_{x_i\in {}^{\alpha+}A} x_i Ax_i$$

hence

$$MCOG(\Delta_{x_0}A) - MCOG(A) =$$
$$\gamma\left(\frac{x_0\Delta + \beta}{\Delta\gamma + 1} - \beta\right) = \frac{\Delta\gamma}{\Delta\gamma+1}\left(x_0 - \gamma\beta\right).$$

Further, right hand side of the implication (5) can be rewritten as

$$\left|\frac{\Delta\gamma}{\Delta\gamma+1}\left(x_0 - \gamma\beta\right)\right| < \varepsilon. \tag{6}$$

This inequality holds when $x_0 = MCOG(A) \, (= \gamma\beta)$ for arbitrary Δ. We obtain for $x_0 \neq MCOG(A)$ from (6)

$$|g(\Delta)| < \frac{\varepsilon}{|x_0 - \gamma\beta|}, \quad g(\Delta) = \frac{\Delta\gamma}{\Delta\gamma + 1}.$$

The Δ^* can be chosen such that

$$\forall \Delta \; |\Delta| < \Delta^* \quad \gamma = \text{const.}, \; \beta = \text{const.}$$

then the function $g(\Delta)$ is continuous in neighborhood of $\Delta = 0$ and $\forall x_0 \in {}^{\alpha+}A \; \forall \varepsilon > 0$ we always find Δ^* such that

$$\forall \Delta \quad |\Delta| < \Delta^* \quad \Longrightarrow \quad |g(\Delta)| < \frac{\varepsilon}{|x_0 - \gamma\beta|}.$$

□

The result of the middle of maxima defuzzification method is defined as middle element from the kernel of fuzzy set A, provided that number of elements of the kernel is odd. Otherwise it is the middle element from the kernel with excluded leftmost (or rightmost) element, depending on the choice of implementation [13].

4 EXAMPLE

As an example for testing the methods we described we modeled a following simple fuzzy system, which structure is shown in Figure 2:

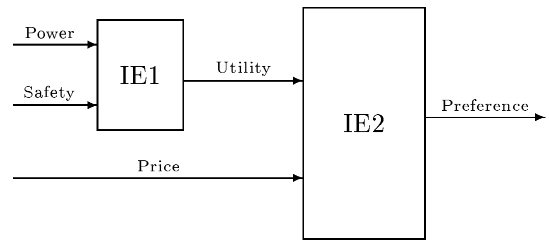

Figure 2: Simple fuzzy system

There are three input variables, namely *Price*, *Safety* and *Power*, output is a preference of a given car. Linguistic descriptions used in IE1 and IE2 are in Tables 1 and 2.

Table 1: Linguistic description 1

Rule	**Power**	**Safety**	**Utility**
1	Small	Small	Small
2	Small	Medium	Medium
3	Small	Big	Medium
4	Medium	Small	Small
5	Medium	Medium	Roughly Big
6	Medium	Big	More or Less Big
7	Big	Small	Very Small
8	Big	Medium	Medium
9	Big	Big	Big

Table 2: Linguistic description 2

Rule	**Price**	**Utility**	**Preference**
1	Small	Small	Medium
2	Small	Medium	Big
3	Small	Big	Quite Roughly Big
4	Medium	Small	Small
5	Medium	Medium	Roughly Medium
6	Medium	Big	Big
7	Big	Small	Very Small
8	Big	Medium	Small
9	Big	Big	Medium

Inference was performed by means of the formula (1). The universa of all variables have been discretized with the same precision equal to 101 points. The results support the preference of defuzzification of intermediate results. Here we present the difference in nonspecificity of overall results measured by energy-type measure of fuzziness Sq [9], defined by the following formula:

$$\mathrm{Sq}(A) = \sum_{i=1}^{P} (Ax_i)^2.$$

Table 3 depicts results of inference for *Power* = 0.4, medium *Safety* and *Price* which varies from 0 to 1 by 0.1 step (Power and Price are normalized to take values from [0 ,1]).

Table 3: Measure of fuzziness Sq of inference results

Price	Direct	MOM	MCOG
0.0	64.0	50.4	53.6
0.1	64.0	50.5	53.6
0.2	64.0	58.7	58.7
0.3	87.5	87.5	87.5
0.4	54.6	33.1	39.5
0.5	75.3	75.3	75.3
0.6	43.1	43.1	43.1
0.7	32.6	20.2	20.2
0.8	32.6	12.3	14.7
0.9	32.6	10.8	14.7
1.0	32.6	10.8	14.7

From these results it can be observed that defuzzification of intermediate results reduces vagueness of overall results, and is therefore a preferable method for transferring the output of one inference to the input of another one. We also recommend, in accordance with [13] to use modified center of gravity method in fuzzy control systems and middle of maxima method in other situations.

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