THE DISTRIBUTION PROBLEM IN MANAGEMENT ACCOUNTING WITH GENETIC ALGORITHM AND FUZZY SETS

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1. ABSTRACT

Many different optimisation problems can be formulated on networks: For instance, we might be interested in finding the shortest path from one node to another, finding the cheapest way to connect all the nodes, or finding the best way to move objects through a network. The movement of objects through a network is called a flow.

In this paper we pretend to show a new approach to a classical network optimisation problem: the transportation problem. Firstly, allowing the vehicles that can establish routes, and, also, permitting to manage with imprecise or vague information. We propose to use the Fuzzy Sets Theory [9]; [4]; [10], with the aim of being able to handle the uncertainty, which is characteristic of decision-making processes in distribution problems.

Moreover, to optimise the distribution network we suggest to use a Genetic Algorithm (GA) [3]; [1]; [7]. The main reason for this is that GAs are heuristic optimisation methods which don't impose restrictions to the posing of the problem. In this study, the algorithm is characterised by the use of a Fuzzy Fitness Function that allows the evaluation of imprecise information.

Keywords: transportation and distribution problems, shortest route problem, fuzzy sets, genetic algorithms.

2. FUZZY DISTRIBUTION MODEL

The information necessary is related to:

- *Destination or client demands ($\tilde{C}$).* We consider a more realist representation using fuzzy sets.

- *Sources number and its supply capacity (F).* We suppose that each source has a supply capacity, normally known in a strict way.

- *Vehicle Capacity (V).* We consider that in each source the company has a vehicle used to distribute the commodities. Therefore, we must take into account the vehicle capacity, because the number of travels will depend of it.

- *Transporting costs of the empty vehicles (cv)* Sometimes, the vehicles must return to their sources due to they have distribute all their capacities. Thus, we must take into account what is the shipping cost when the vehicle is empty. This, normally depends of the distance (routed kilometres).

- *Increasing cost for each unit of commodities shipped ($\Delta c\tilde{v}$).* When the vehicles distribute the commodities the transporting cost increase with each unit shipped. As well, this increasing cost depends of the distance travelled. Moreover, this cost could be represented using fuzzy sets.

- *Distance among sources and destinations (D).* The distribution is made from the sources to the destinations or clients. So, it is necessary to know the distance among them.

- *Distance among destinations (D').* The vehicles can establish routes from the source to the several destinations that this source supplies, so it is necessary to know the distances among the destinations.

According with this approach, we are trying to reach the real distribution problem. Thereafter, we need some tool capable to solve this complex problem or able to give a reasonable solution. In this paper we propose to use a Genetic Algorithm with a Fuzzy Fitness Function. Also, although we describe to use fuzzy sets, we suggest representing all the fuzzy information as Trapezoidal Fuzzy Numbers (TFN) [5].

3. A GENETIC ALGORITHM FOR FUZZY DISTRIBUTION PROBLEMS

Some authors have applied Genetic Algorithms to transportation problems, in many cases solving the problem reducing their complexity or using precise knowledge for the variables.

In our approach we suggest a fuzzy representation of the information on held and a less restrictive model. Due to this, the GA must determine the quantities supplied from each source to each destinations and the routes travelled for the vehicles. For both objectives the criteria is the same: minimising the total distribution costs.

To solve the fuzzy distribution problem we propose to use a GA with the following components:

3.1. Genetic Representation

The solutions to the problems are a set of distribution quantities to be shipped from each source to each destination and the routes followed by the vehicles of each source. To codify this solutions we propose to use two matrixes: one that contains the quantities distributed from each origin to each destination, and, other, that represents the path followed for the several vehicles.

The codification of quantities matrix, for an example of five sources (S_i) that must supply to four destinations (D_i) could be:

Table 1: Codification of quantities matrix

S_1^1	S1	S2	S3	S4	S5	**Total**
D1	15	0	0	70	0	**85**
D2	0	40	0	0	0	**40**
D3	25	10	40	0	50	**125**
D4	0	0	60	0	5	**65**
Total	**40**	**50**	**100**	**70**	**55**	**315**

that indicates the quantities supplied from each source to each destination.

On the other hand, the codification of vehicles routed could be:

Table 2: Codification of vehicles matrix

S_1^2	S1	S2	S3	S4	S5
D1	1	2	1	4	3
D2	4	3	2	3	2
D3	2	1	3	2	1
D4	3	4	4	1	4

that indicates the position of each destination in the vehicle routes.

Moreover, in order to generate usefulness solutions, the company must decide the covering level of each client fuzzy demand. The TFN can be identified as possibility distributions. So, we establish in the work the risk of not cover demand of each client. According to this, the quantity matrix represents available solutions for the decision.

3.2. Fuzzy Fitness Function

The fitness function must assign more value to good solutions to the distribution problem. To get this, we propose to calculate the total distribution cost that the two matrixes suppose. The steps are as follows:

First, each vehicle must depart from its source full of commodities. The first destination established for the routes matrix determine the cost of this first travel. Once the first destination is satisfied (with one or more travels) the second destination is attempted. If the vehicle is empty during the route it return empty to the source to full its capacity. As well, when all the destinations are satisfied then the vehicle go back to its origin, completing the route. The sum of all these transporting costs would be a part of the solution. To obtain the total cost we must add the cost of the other distribution routes contained in the matrixes.

Due to the fact that some of the information can be represented by fuzzy sets, we suggest to use the operations designed for them [2]. Moreover, in the cases that the operations could not produce a TFN, we have approximated the result as one of this [5], assuming a little error.

To set up a hierarchy among the solutions, the proposal is to use the fuzzy distance [6], based on Hamming Distance. Doing it, we calculate the distance from the origin (singleton 0) to each solution fuzzy cost, which is defined as follows:

$$d(\tilde{A},\tilde{B}) = \int_{\alpha=0}^{1} \left(\left| A_\alpha^1 - B_\alpha^1 \right| + \left| A_\alpha^2 - B_\alpha^2 \right| \right) d\alpha \qquad (1)$$

where $[A_\alpha^1, A_\alpha^2]$ is confidence interval of $\tilde{A}$ at the signification level α.

The more accurate solutions will have a lower distance. Thus for obtaining the fitness value we propose calculate the inverse of the distance of each solution.

3.2.1. Selection process

We propose to use Roulette Wheel Ranking [3] to select the "parents" of the next generation.

3.2.2. Crossover operator

Traditional crossovers cannot be used, because the solutions are two matrixes that have to accomplish some constraints. Due to this we propose to different crossovers for each matrix, as follows:

♦ **Distribution quantities matrixes**. To combine the information of these matrixes obtained from two "parents" we have used the crossover proposed for Vignaux and Michalewicz to Transportation Problem [8]:

- Continuing with the aforementioned example of five sources that must supply to four destinations. Lets imagine that after selection process we have two quantities matrixes to be crossed like these:

Tables 3 and 4: Matrixes to be crossed

S_1^1	S1	S2	S3	S4	S5	**Total**
D1	15	0	0	70	0	**85**
D2	0	40	0	0	0	**40**
D3	25	10	40	0	50	**125**
D4	0	0	60	0	5	**65**
Total	**40**	**50**	**100**	**70**	**55**	**315**

S_2^1	S1	S2	S3	S4	S5	**Total**
D1	0	0	50	0	35	**85**
D2	0	40	0	0	0	**40**
D3	5	0	50	70	0	**125**
D4	35	10	0	0	20	**65**
Total	**40**	**50**	**100**	**70**	**55**	**315**

- Using these "parents", we obtain two temporal matrixes: one containing the integer average of both "parents" (*MED*), and another that include an 1 in those cells that have been rounded to obtain the average, and a 0 in the remaining (*RED*).

- The *RED* matrix have some interesting properties. One of then indicates that the number of *ones* of each row or column is a par one. In other words, the source capacity values and the destination demand values are par integers. Thus, we can divide the *RED* matrix into two matrixes (*RED1* and *RED2*):

$$RED = RED1 + RED2, \qquad (2)$$

being equal the supplying capacity of *RED1* and *RED2* for each source, and, also, the demands of *RED1* y *RED2* for each destination. The offspring are obtained as follows:

$$\begin{aligned} S'_1 &= MED + RED1 \\ S'_2 &= MED + RED2 \end{aligned} \qquad (3)$$

Then, for the example, the "sons" could be:

Tables 5 and 6: "Sons"

$S_1^{1\prime}$	S1	S2	S3	S4	S5	**Total**
D1	8	0	25	35	17	**85**
D2	0	40	0	0	0	**40**
D3	15	5	45	35	25	**125**
D4	17	5	30	0	13	**65**
Total	**40**	**50**	**100**	**70**	**55**	**315**

$S_2^{1\prime}$	S1	S2	S3	S4	S5	**Total**
D1	7	0	25	35	18	**85**
D2	0	40	0	0	0	**40**
D3	15	5	45	35	25	**125**
D4	18	5	30	0	12	**65**
Total	**40**	**50**	**100**	**70**	**55**	**315**

♦ **Vehicle routes matrixes**. As well, we must cross the route matrixes of both "parents". To this, we first select randomly one source and interchange between the "parents" the routes that the vehicle of this origin must follow.

3.3. Mutation operator

The intention of this operator is to introduce diversity into the solutions. Trying to this we must make differences among the two types of matrixes, as follows:

♦ **Distribution quantities mutation.** We propose to use a special mutation method designed to keep the solutions resulting as feasible ones. The steps are as follow:

- *Step 1*. We select one source (Source A) and one destination (Destination A) of the matrix. This destination must receive a non-zero quantity from the source selected. For the example, if we mutate the first matrix obtained after the crossing process, the source and destination could be:

Table 7: Selection of source A and destination A

S_1^1	S1	S2	S3	**S4**	S5	**Total**
D1	8	0	25	35	17	**85**
D2	0	40	0	0	0	**40**
D3	15	5	45	**35**	25	**125**
D4	17	5	30	0	13	**65**
Total	**40**	**50**	**100**	**70**	**55**	**315**

- *Step 2*. We generate a random number between 0 and the quantity assigned to this destination. For the example could be 20.

- *Step 3*. We search in the remaining sources, one quantity assigned to a different destination which amount is bigger that the random number generated before (Source B and Destination B). For the example could be:

Table 8: Selection of source B and destination B

S_1^1	S1	**S2**	S3	S4	S5	**Total**
D1	8	0	25	35	17	**85**
D2	0	**40**	0	0	0	**40**
D3	15	5	45	35	25	**125**
D4	17	5	30	0	13	**65**
Total	**40**	**50**	**100**	**70**	**55**	**315**

• *Step 4*. We discount the random number to the assigned amounts from the Source A to Destination A and from the Source B to Destination B. After that, we add the random number to the assigned amounts from Source A to Destination B and from Source B to Destination A. According to this, for the example, the mutated matrix could be:

Table 9: The mutated matrix

$S_1^{1\prime}$	S1	**S2**	S3	**S4**	S5	**Total**
D1	8	0	25	35	17	**85**
D2	0	**20**	**20**	0	0	**40**
D3	15	**25**	**25**	35	25	**125**
D4	17	5	30	0	13	**65**
Total	**40**	**50**	**100**	**70**	**55**	**315**

Finally, with the aforementioned process we can mutate the quantity matrixes maintaining them as feasible ones.

♦ **Vehicles routes mutation**. On the other hand, for mutating the route matrix we propose an *interchange* method. First, we select randomly one source. After that, we select two destinations and interchange their position in the route followed for the source vehicle.

3.4. Halt criteria for the best solution search

The proposal is for the algorithm to go through a number of generations specified by the user until the best solution is found. Moreover, in order not to lose good solutions, the characteristic termed *elitism* [3] has been introduced. This procedure consists of keeping the best individual from a population in successive generations unless and until some other individual succeeds in doing better in respect of suitability. In this way, the best solution is not lost until outclassed by a more suitable solution.

As explained, application of the model proposed here allows solving the distribution problem under uncertain environmental conditions.

4. CONCLUDING REMARKS

There are two main results obtained from this paper. One is the formulation of an approach to Distribution Problem that could be more adapted to real situations. The other is showing the GA capacity to solve complex problems with imprecise or vague information. In that way the work group has developed an example that deals with the distribution problem for a production furniture company. The solution has been found using the GA proposed.

However, trying to adjust properly the posing of the problem, as proposal of future work, could be interesting take into account the accomplishing of production and demand cadences, multiple products distribution, the intermediate warehouses, etc.

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