A SELECTION METHOD BASED ON THE 2-TUPLE LINGUISTIC REPRESENTATION MODEL FOR DECISION-MAKING PROBLEMS WITH MULTI-GRANULARITY LINGUISTIC INFORMATION

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Abstract

The use of linguistic information in decision-making problems implies in most cases the need for using fusion processes to obtain aggregated values that summarize the input information. An important limitation of the fuzzy linguistic approach appears when fusion processes are applied to problems in which the linguistic information is assessed in term sets with different granularity of uncertainty, this type of information is denoted as multi-granularity linguistic information. In this contribution, taking as the base the 2-tuple fuzzy linguistic representation model and its computational technique, we shall present a method for easily dealing with multi-granularity linguistic information in fusion processes.

Keywords: Linguistic variables, fusion processes, granularity of uncertainty, multi-granularity linguistic information, decision-making.

1 INTRODUCTION

In some occasions,we can find decision-making problems that present several sources of information to qualify a phenomenon. When these phenomena present quantitative aspects they can be assessed by means of precise numerical values, however when the aspects presented by the phenomena are qualitative, then it may be difficult to qualify them using precise values. So, the use of the fuzzy linguistic approach [8] has shown itself as a good choice to model these phenomena, due to the fact that it represents qualitative aspects with qualitative terms by means of linguistic variables. The use of the fuzzy linguistic approach implies computing with words (CWW). In the specialized literature, three different linguistic computational techniques that provide linguistic operators for CWW can be found [1, 2, 5].

An important aspect when the fuzzy linguistic approach is used, is to determine the "granularity of uncertainty", i.e., the cardinality of the linguistic term set. Depending on the uncertainty degree held by a source of information qualifying a phenomenon, the term set will have more or less terms. Then, in those problems with several sources of information each one could express its knowledge by means of linguistic term sets with a different granularity of uncertainty from the other ones. We shall denote this type of information as multi-granularity linguistic information.

In decision-making problems with multi-granularity linguistic information the fuzzy linguistic approach together with the linguistic computational techniques introduced in [1, 2] present an important limitation because in these computational methods, neither a standard normalization process nor fusion operators are defined for this type of information. Therefore, it is highly complex to solve this type of problems using these methods and the results obtained are expressed in domains far removed from the initial ones [3, 4].

The aim of this contribution is to present an easy selection model for decision-making problems with multi-granularity linguistic information using as a base the 2-tuple fuzzy linguistic representation model [5, 6], together with the multi-granularity linguistic information fusion ideas presented in [3, 4]. So, we shall present a selection model that obtains the solution set of alternatives according to the following two steps:

1. *A fusion process for multi-granularity linguistic information based on the 2-tuple representation,* for obtaining collective values for each alternative.
2. *A selection process,* for obtaining the solution set.

2 PRELIMINARIES

Here, we present the scheme of a multi-expert decision-making (MEDM) problem with multi-granularity linguistic information and different methods for CWW.

2.1 MEDM PROBLEM

An MEDM problem can be defined as follows. Let $X = \{x_1, x_2, \ldots, x_n\}$ $(n \geq 2)$ be a finite set of alternatives to be qualified according to a finite set of experts $P = \{p_1, p_2, \ldots, p_m\}$ ($m \geq 2$). Each expert p_i provides a linguistic performance value μ^{ij} for each alternative x_j. Given that we shall deal with multi-granularity linguistic term sets in decision-making problems, we assume that each expert p_i may use a different linguistic term set S_i to express the performance values. The linguistic term sets $\{S_i, \forall i\}$ may have a different granularity and/or semantics. Therefore, for each expert p_i, the performance profile of the alternatives is defined as a linguistic fuzzy choice subset defined over X and assessed linguistically on S_i:

$$p_i \longrightarrow (\mu^{i1}, \ldots, \mu^{in})$$
$$\mu^{ij} \in S_i \quad S_i = \{s_0^i, \ldots, s_{g_i}^i\} \quad i \in \{1, \ldots, m\}$$

where $g_i + 1$ is the cardinality of S_i.

2.2 LINGUISTIC COMPUTATIONAL METHODS

The linguistic variables are used in processes of CWW that imply their fusion, aggregation, comparison, etc. To perform these computations there are three models in the literature. (i) The model based on the Extension Principle, (ii) the symbolic one and (iii) the model based on the 2-tuple fuzzy linguistic representation. Here we briefly review the first two models and we shall describe in depth the last one.

1. *The linguistic computational methods based on the Extension Principle [2].* These methods use the extended arithmetic, based on the Extension Principle, on the membership functions associated to the linguistic terms to make linguistic computations.

2. *The linguistic computational symbolic models [1].* These methods do not use the membership functions of the labels to perform the computations, but they use the order index and properties of such linguistic assessments to make direct computations on labels.

3. *The 2-tuple Fuzzy Linguistic Representation Model [5, 6].* It is based on symbolic methods and takes as the base of its representation the concept of Symbolic Translation.

Definition 1. *The Symbolic Translation of a linguistic term $s_i \in S = \{s_0, ..., s_g\}$ is a numerical value assessed in $[-.5, .5)$ that supports the "difference of information" between a counting of information β assessed in $[0, g]$ obtained after a symbolic aggregation operation (acting on the order index of the labels) and the closest value in $\{0, ..., g\}$ that indicates the index of the closest linguistic term in S (s_i).*

From this concept we develop a linguistic representation model which represents the linguistic information by means of 2-tuples, (r_i, α_i), $r_i \in S$ and $\alpha_i \in [-.5, .5)$. r_i represents the linguistic label center of the information and α_i is the Symbolic Translation.

This linguistic representation model defines a set of functions to make transformations among linguistic terms, 2-tuples and numerical values.

Definition 2. *Let $s_i \in S$ be a linguistic term, then its equivalent 2-tuple representation is obtained by means of the function θ as:*

$$\theta : S \longrightarrow (S \ x \, [-0.5, 0.5))$$
$$\theta(s_i) = (s_i, 0)/s_i \in S$$

Definition 3. *Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value supporting the result of a symbolic operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:*

$$\Delta : [0, g] \longrightarrow Sx[-0.5, 0.5)$$

$$\Delta(\beta) = \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-.5, .5) \end{cases}$$

where round is the usual round *operation, s_i has the closest index label to "β" and "α" is the value of the symbolic translation.*

Definition 4.*Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and (s_i, α) be a linguistic 2-tuple. There is always a function Δ^{-1}, such that, from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g]$.*

$$\Delta^{-1} : Sx[-.5, .5) \longrightarrow [0, g]$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$$

Together with the fuzzy linguistic 2-tuple representation model a wide range of 2-tuple aggregation operators were developed in [5], which are neccesary for the development of our fusion method in order to combine the information.

3 A FUSION PROCESS FOR MULTI-GRANULARITY LINGUISTIC INFORMATION BASED ON THE 2-TUPLE REPRESENTATION MODEL FOR MEDM PROBLEMS

We want to obtain for each alternative, x_j, a collective value expressed by means of a linguistic 2-tuple. To do so, we present the following fusion process:

1. **Making the information uniform** (Normalization process). In this step the multi-granularity linguistic input information is unified into "fuzzy sets" in a Basic Linguistic Term Set (BLTS).

2. **Transforming fuzzy sets into 2-tuples**. Here we shall transform the fuzzy sets in the BLTS into 2-tuples based on the symbolic translation assessed in the BLTS.

3. **Fusion of 2-tuples**. Once the performance values, μ^{ij}, are expressed by 2-tuples assessed in the BLTS, we shall apply a 2-tuple fusion operator to obtain collective performance values.

4. **Backward step**. The 2-tuples obtained by the fusion method are assessed in the BLTS, it can be distant from the expression domains used by the sources of information. Therefore, it may be interesting to offer the option to make an approach of the collective values to the initial domains for a better comprehensiveness. This step is not neccesary, simply convenient.

Subsequently, we shall present in depth each step.

3.1 MAKING THE INFORMATION UNIFORM

With a view to manage the information we must make it uniform, i.e., the multi-granularity linguistic information must be transformed into a unified linguistic term set, called BLTS and denoted as S_T. Before defining a transformation function into this BLTS, S_T, we have to decide how to choose S_T. We consider that S_T must be a linguistic term set which allows us to maintain the uncertainty degree associated to each expert and the ability of discrimination to express the performance values. With this goal in mind, we look for a BLTS with the maximum granularity. We take into consideration two possibilities:

1. When there is only one term set with the maximum granularity, then, it is chosen as S_T.

2. If we have two or more linguistic term sets with maximum granularity then, S_T is chosen depending on the semantics of these linguistic term sets, finding two possible situations to establish S_T:

(a) All the linguistic term sets have the same semantics, then S_T is any one of them.

(b) There are some linguistic term sets with different semantics. Then, S_T is a term set with a larger number of terms than the number of terms that a person is able to discriminate (normally 11 or 13, see [7]). We define a BLTS with 15 terms simmetrically distributed.

Once S_T has been selected, we define a transformation function which transforms each linguistic value into a fuzzy set in S_T.

Definition 5 [3]. *Let $A = \{l_0, \ldots, l_p\}$ and $S_T = \{c_0, \ldots, c_g\}$ be two linguistic term sets, such that, $g \geq p$. Then, a multi-granularity transformation function, τ_{AS_T} is defined as,*

$$\tau_{AS_T} : A \longrightarrow F(S_T)$$
$$\tau_{AS_T}(l_i) = \{(c_k, \alpha_k^i) \; / k \in \{0, \ldots, g\}\}, \; \forall l_i \in A$$
$$\alpha_k^i = \max_y min\{\mu_{l_i}(y), \mu_{c_k}(y)\}$$

being $F(S_T)$ the set of fuzzy sets defined in S_T, $\mu_{l_i}(y)$ and $\mu_{c_k}(y)$ are the membership functions of the fuzzy sets associated to the terms l_i and c_k, respectively.

We shall denote each $\tau_{S_i S_T}(\mu^{ij})$ as r^{ij}, and represents each fuzzy set of performance, r^{ij}, by means of its respective membership degrees, i.e.,

$$r^{ij} = (\alpha_0^{ij}, \ldots, \alpha_g^{ij}).$$

3.2 TRANSFORMING FUZZY SETS INTO 2-TUPLES

So far, we have unified the multi-granular linguistic information into fuzzy sets in S_T. The fuzzy sets are complex to manage. Therefore, we shall use the 2-tuple fuzzy linguistic representation model to represent this information. To do so, we shall define the function χ that computes a value $\beta \in [0, g]$ that supports the information in the fuzzy set $\tau_{S_i S_T}(\mu^{ij})$.

Definition 6. *Let $\tau_{S_i S_T}(l_i) = \{(c_0, \alpha_0^i), \ldots, (c_g, \alpha_g^i)\}$ be a fuzzy set that represents a linguistic term $l_i \in S_i$ in S_T. We shall obtain a numerical value, that supports the information of the fuzzy set, assessed in the interval $[0, g]$ by means of the following function:*

$$\chi : F(S_T) \longrightarrow [0, g]$$
$$\chi(\tau_{S_i S_T}(l_i)) = \frac{\sum_{j=0}^{g} j\alpha_j^i}{\sum_{j=0}^{g} \alpha_j^i} = \beta$$

This value β is easy to transform into a linguistic 2-tuple using the function Δ (Definition 3). Therefore, we have unified the input information with linguistic 2-tuples assessed in S_T transforming the fuzzy sets, r^{ij}, by means of the functions χ and Δ:

$$\Delta(\chi(\tau_{S_i S_T}(\mu^{ij}))) = \Delta(\chi(r^{ij})) = (s_k, \alpha)^{ij}$$

with $s_k \in S_T$ *and* $\alpha \in [-.5, .5)$.

3.3 FUSION OF 2-TUPLES

Now the performance values, μ^{ij}, are modeled by means of linguistic 2-tuples assessed in S_T, $(s_k, \alpha)^{ij}$, and our objective is to aggregate this information to obtain collective values for each alternative x_j. In [5] a wide range of 2-tuple linguistic aggregation operators were presented, therefore, to aggregate the 2-tuples, $(s_k, \alpha)^{ij}$, we shall choose one of these operators and apply it for combining the 2-tuples, obtaining as result an aggregated linguistic 2-tuple assessed in S_T. Formally, it can be expressed as:

$$FO((s_k, \alpha)^{1j}, ..., (s_k, \alpha)^{nj})) = (s_k, \alpha)^j$$

where FO is any 2-tuple fusion operator, and $(s_k, \alpha)^j$ is the collective value for the alternative, x_j.

3.4 THE BACKWARD STEP

This is an "*optional step*" in the fusion process. Depending on the problem we are dealing with, it may be that the aggregated 2-tuple assessed in S_T is expressed in a expression domain distant from the domains, S_i, used by the information sources. In these situations, to offer the possibility of making an approach to the initial expression domains, for improving the comprehensiveness of the results, might be appropiate. To accomplish the backward step we shall present:

1. *A new representation for the linguistic information.* Using 2-tuples based on the "degree of membership", i.e., 2-tuples whose first component is a linguistic label and the second one indicates the degree of membership of the counting of information represented in the linguistic term. The following function δ transforms a 2-tuple based on the symbolic translation into two 2-tuples based on the degree of membership supporting the same counting of information.

Definition 7. *Let (s_k, α) be a linguistic 2-tuple based on the symbolic translation with $s_k \in S_T$ and $\alpha \in [-.5, .5)$ whose equivalent numerical value is $\Delta^{-1}((s_k, \alpha)) = \beta$ (Definition 4) with $\beta \in [0, g]$. The function δ computes two 2-tuples based on the degree of membership, from the initial 2-tuple, that support the same counting of information:*

$$\delta : [0, g] \longrightarrow \{S_T x[0,1]\} x \{S_T x[0,1]\}$$
$$\delta(\beta) = \{(s_h, 1-\gamma), (s_{h+1}, \gamma)\}$$

where

$$h = trunc(\beta) \quad and \quad \gamma = \beta - h$$

trunc is the usual trunc operation.

2.*A process that obtains a 2-tuple (s_k^i, α), with $s_k^i \in S_i$ and $\alpha \in [-.5, .5)$ based on the symbolic translation, from two 2-tuples based on the degree of membership assessed in a different domain from S_i.*

(a) A matching procedure using $\tau_{S_T S_i}$ is applied to s_h *and* s_{h+1} obtaining two fuzzy sets on S_i.

$$\tau_{S_T S_i}(s_h) = \{(s_0, \alpha_0^h), ..., (s_{g_i}, \alpha_{g_i}^h)\} \; with \quad r_h = (\alpha_0^h, ..., \alpha_{g_i}^h)$$
$$\tau_{S_T S_i}(s_{h+1}) = \{(s_0, \alpha_0^{h+1}), ..., (s_{g_i}, \alpha_{g_i}^{h+1})\}$$
$$with \;\; r_{h+1} = (\alpha_o^{h+1}, ..., \alpha_{g_i}^{h+1})$$

(b) The fuzzy sets are converted into numerical values assessed in $[0, g_i]$,

$$\chi(\tau_{S_T S_i}(s_h)) = \chi(r_h) = \beta_h$$
$$\chi(\tau_{S_T S_i}(s_{h+1})) = \chi(r_{h+1}) = \beta_{h+1}$$

(c) To achieve our objective we need to obtain a value β assessed in $[0, g_i]$ that supports the same information as $(s_h, 1-\gamma)$, (s_{h+1}, γ). We have β_h *and* $\beta_{h+1} \in [0, g_i]$, that represent the information supportted by s_h *and* s_{h+1}, now we make a linear combination using the degrees of membership of these labels in their respective 2-tuples to obtain the value that we are looking for:

$$(\beta_h * (1-\gamma)) + (\beta_{h+1} * \gamma) = \beta \in [0, g_i]$$

where β represents the same information as the two 2-tuples based on the degree of membership. Then $\Delta(\beta)$ obtains the linguistic 2-tuple based on the symbolic translation assessed in S_i that we were looking for:

$$\Delta(\beta) = (s_k^i, \alpha)$$

This process will be carried out $\forall \; S_i$.

4 SELECTION PROCESS

The objective of the decision process is to find a set of alternatives with the best ones. To do so, a selection process is applied to the collective preferences:

1. First, we select a choice degree.

2. The choice degree is applied to the collective values to rank them.

3. Finally, we select for the solution set the best alternatives according to the choice degree.

5 CONCLUDING REMARKS

In this paper we have presented a fusion method based on the 2-tuple fuzzy linguistic representation that allows us to easily deal with multi-granularity linguistic information in fusion processes.

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