Type–II Fuzzy Possibilistic C-Mean Clustering

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Abstract**—***Fuzzy clustering is well known as a robust and efficient way to reduce computation cost to obtain the better results. In the literature, many robust fuzzy clustering models have been presented such as Fuzzy C-Mean (FCM) and Possibilistic C-Mean (PCM), where these methods are Type-I Fuzzy clustering. Type-II Fuzzy sets, on the other hand, can provide better performance than Type-I Fuzzy sets, especially when many uncertainties are presented in real data. The focus of this paper is to design a new Type-II Fuzzy clustering method based on Krishnapuram and Keller PCM. The proposed method is capable to cluster Type-II fuzzy data and can obtain the better number of clusters (c) and degree of fuzziness (m) by using Type-II Kwon validity index. In the proposed method, two kind of distance measurements, Euclidean and Mahalanobis are examined. The results show that the proposed model, which uses Mahalanobis distance based on Gustafson and Kessel approach is more accurate and can efficiently handle uncertainties.*

Keywords— *Type-II Fuzzy Logic; Possibilistic C-Mean (PCM); Mahalanobis Distance; Cluster Validity Index;*

1. Introduction

Clustering is an important method in data mining, decision-making, image segmentation, pattern classification, and etc. Fuzzy clustering can obtain not only the belonging status of objects but also how much the objects belong to the clusters. In the last 30 years, many fuzzy clustering models for crisp data have been presented such as Fuzzy K-Means and Fuzzy C-Mean (FCM) [1]. FCM is a popular clustering method, but its memberships do not always correspond well to the degrees of belonging and it may be inaccurate in a noisy environment [2]. To relieve these weaknesses, Krishnapuram and Keller presented a Possibilistic C-Mean (PCM) approach [3]. In addition, in real data, there exist many uncertainties and vaguenesses, which Type-I Fuzzy sets are not able to directly model as their membership functions are crisp. On the other hand, as Type-II membership functions are fuzzy, they are able to model uncertainties more appropriately [2, 4]. Therefore, Type-II Fuzzy Logic Systems have the potential to provide better performance than Type-I [5].

In the case of combining Type-II Fuzzy logic with clustering methods, the data can be clustered more appropriately and more accurately. The focus of this paper is to design a new Type-II Fuzzy clustering method based on Krishnapuram and Keller PCM. The proposed method is capable to cluster Type-II fuzzy data and can obtain the better number of clusters (*c*) and degree of fuzziness (*m*) by using Type-II Kwon validity index.

The rest of this paper is organized as follows: The clustering methods are reviewed in Section 2. Section 3 presents the historical review of Type-II Fuzzy Logic. Section 4 is dedicated to the proposed method and Section 5 presents the experimental results. Finally, conclusions are presented in Section 6.

2. Clustering Methods

The general philosophy of clustering is to divide the initial set into homogenous groups [6] and to reduce the data [1]. Clustering is useful in several exploratory decision-making, machine learning, data mining, image segmentation, and pattern classification [7]. In literature, most of the clustering methods can be classified into two types: Crisp clustering and Fuzzy clustering. Crisp clustering assigns each data to a single cluster and ignores the possibility that these data may also belongs to other clusters [8]. However, as the boundaries between clusters could not be defined precisely, some of the data could belong to more than one cluster with different positive degrees of memberships [6]. This clustering method considers each cluster as a fuzzy set and the membership function measures the degree of belonging of each feature in a cluster. So, each feature may be assigned to multiple clusters with some degree of belonging [8]. Two important applied models of fuzzy clustering, Fuzzy C-Means, and Possibilistic C-Means are described as follows:

Fuzzy C-Means (FCM): Fuzzy C-Means clustering model can be defined as follows [9]:

$$\min\left\{J(x,\mu,c)=\sum_{i=1}^{c}\sum_{j=1}^{N}\mu_{ij}^{m}\,d_{ij}^{2}\right\} \quad (1)$$

$$\text{ST:}\begin{cases}0<\sum_{j=1}^{N}\mu_{ij}<N \quad \forall i\in(1,2,\dots,c) & (2)\\ \sum_{i=1}^{c}\mu_{ij}=1 \qquad \forall j\in(1,2,\dots,N) & (3)\end{cases}$$

where, μ_{ij} is the degree of belonging of the j^{th} data to the i^{th} cluster, d_{ij} is the distance between the j^{th} data and the i^{th} cluster center, m is the degree of fuzziness, c is the number of clusters, and N is the number of the data.

Although FCM is a very good clustering method, it has some disadvantages: the obtained solution may not be a desirable solution and the FCM performance might be inadequate, specially when the data set is contaminated by noise. In addition, the membership values show degrees of

ISBN: 978-989-95079-6-8

probabilities of sharing [10] and not only depend on the distance of that point to the cluster center, but also on the distance from the other cluster centers [11]. In addition, when the norm used in the FCM method is different to the Euclidean, introducing restrictions is necessary, e.g., Gustafson and Kessel [12] and Windham limit the volume of the groups using fuzzy covariance and scatter matrices, respectively [13].

Possibilistic C-Mean (PCM): To improve the FCM weaknesses, Krishnapuram and Keller created a possibilistic approach, which uses a possibilistic type of membership function to describe the degree of belonging. It is desirable that the memberships for representative feature points be as high as possible and unrepresentative points have low membership. The objective function, which satisfies the requirements, is formulated as follows [3]:

$$\min\left\{J_m(x,\mu,c)=\sum_{i=1}^{c}\sum_{j=1}^{N}\mu_{ij}^{m}d_{ij}^{2}+\sum_{i=1}^{c}\eta_i\sum_{j=1}^{N}(1-\mu_{ij})^m\right\} \quad (4)$$

where, d_{ij} is the distance between the j^{th} data and the i^{th} cluster center, μ_{ij} is the degree of belonging of the j^{th} data to the i^{th} cluster, m is the degree of fuzziness, η_i is a suitable positive number, c is the number of the clusters, and N is number of the data. μ_{ij} can be obtained by using (5) [3]:

$$\mu_{ij}=\frac{1}{1+\left(\frac{d_{ij}^2}{\eta_i}\right)^{\frac{1}{m-1}}} \quad (5)$$

where, d_{ij} is the distance between the j^{th} data and the i^{th} cluster center, μ_{ij} is the degree of belonging of the j^{th} data to the i^{th} cluster, m is the degree of fuzziness, η_i is a suitable positive numbers. The value of η_i determines the distance at which the membership value of a point in a cluster becomes 0.5. In practice, (6) is used to obtained η_i values. The value of η_i can be fixed or changed in each iteration by changing the values of μ_{ij} and d_{ij}, but the care must be exercised, since it may lead to instabilities [3]:

$$\eta_i=\frac{\sum_{j=1}^{N}\mu_{ij}^{m}d_{ij}^{2}}{\sum_{j=1}^{N}\mu_{ij}^{m}} \quad (6)$$

The PCM is more robust in the presence of noise, in finding valid clusters, and in giving a robust estimate of the centers [14].

Updating the membership values depends on the distance measurements [11].The Euclidean and Mahalanobis distance are two common ones. The Euclidean distance works well when a data set is compact or isolated [7] and Mahalanobis distance takes into account the correlation in the data by using the inverse of the variance-covariance matrix of data set which could be defines as follows [15]:

$$\mathrm{D}=\sum_{\mathrm{i,j=1}}^{\mathrm{i,j=p}}A_{ij}\,(x_i-y_i)(x_i-y_j) \quad (7)$$

$$A_{ij}=\rho_{ij}\,\sigma_i\sigma_j \quad (8)$$

where, x_i and y_i are the mean values of two different sets of parameters, X and Y. σ_i^2 are the respective variances and ρ_{ij} is the coefficient of correlation between the i^{th} and j^{th} variates. Gustafson and Kessel proposed a new approach based on Mahalanobis distance, which enables the detection of ellipsoidal clusters. Their approach focused on the case where the matrix A is different for each cluster [12].

Satisfying the underlying assumptions, such as shape and number, is another important issue in clustering methods, which could be obtained by validation indices. Xie & Beni's (XB) and Kwon are two common validity indices [1]. Xie and Beni defined a cluster validity, (9), which aims to quantify the ratio of the total variation within clusters and the separation of the clusters [1]:

$$XB(c)=\frac{\sum_{i=1}^{c}\sum_{j=1}^{N}(\mu_{ij})^m\left\|x_j-v_i\right\|^2}{N\min_{i\neq j}\left\|v_i-v_j\right\|^2} \quad (9)$$

where, μ_{ij} is the degree of belonging of the j^{th} data to the i^{th} cluster, v_j is the center of the j^{th} cluster, m is the degree of fuzziness, c is the number of clusters, and N is number of the data. The optimal number of clusters should minimize the value of the index [1]. However, in practice, when $c\rightarrow N \Rightarrow XB\rightarrow 0$ and it usually does not generate appropriate clusters. The $V_k(c)$, (10), was proposed by Kwon based on the improvement of the XB index [16];

$$V_k(c)=\frac{\sum_{i=1}^{c}\sum_{j=1}^{N}(\mu_{ij})^m\left\|x_j-v_i\right\|^2+\frac{1}{c}\sum_{i=1}^{c}\left\|v_i-\bar{v}\right\|^2}{\min_{i\neq j}\left\|v_i-v_j\right\|^2} \quad (10)$$

where, μ_{ij} is the degree of belonging of the j^{th} data to the i^{th} cluster, v_j is center of the j^{th} cluster, $\bar{v}$ is the mean of the cluster centers, m is the degree of fuzziness, c is the number of clusters, and N is number of the data. To assess the effectiveness of clustering method, the smaller the V_k, the better the performance [16].

All of the clustering methods and validation indices, mentioned above, are based on Type-I fuzzy set. However, in real world, there exists many uncertainties, which Type-I fuzzy could not model them. Type-II fuzzy set, on the other hand, is able to successfully model these uncertainties [4].

3. Type-II Fuzzy Clustering

The concept of a Type-II fuzzy set, was introduced by Zadeh as an extension of Type-I fuzzy set [17]. A Type-II fuzzy set is characterized by fuzzy membership function, i.e., the membership grade for each element of this set is a fuzzy set in interval [0,1]. Such sets can be used in situations where there are uncertainties about the membership values [18]. Type-II fuzzy logic is applied in many clustering methods e.g., [19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. There are essentially two types of Type-II fuzziness: Interval-Valued Type-II and generalized Type-II. Interval-Valued Type-II fuzzy is a special Type-II fuzzy, where the upper and lower bounds of membership are crisp and the spread of membership distribution is ignored with the assumption that membership values between upper and lower values are

ISBN: 978-989-95079-6-8

uniformly distributed or scattered with a membership value of 1 on the $\mu(\mu(x))$ axis (Figure 1.a). Generalized Type-II fuzzy identifies upper and lower membership values as well as the spread of membership values between these bounds either probabilistically or fuzzily. That is there is a probabilistic or possibilistic distribution of membership values that are between upper and lower bound of membership values in the $\mu(\mu(x))$ axis (Figure 1.b) [29].

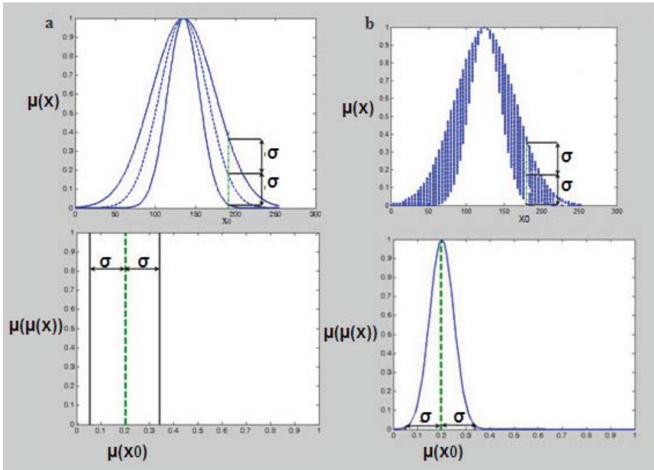

Figure 1:(a) Interval Valued Type-II (b) Generalized Type-II

4. Proposed Type-II PCM Method

Considering the growing application areas of Type-II fuzzy logic, designing a Type-II clustering method is essential. Several researchers designed a Type-II fuzzy clustering method based on FCM but FCM itself has some weaknesses, which make some of the developed methods ineffective in situations in which the data set is contaminated by noise, the norm used is different from the Euclidean, or the pixels on an input data are highly correlated. PCM could improves these weaknesses.

The proposed method is the extension of Krishnapuram and Keller Possibilistic C-Mean (PCM). Here, the membership functions are Type-II Fuzzy, the distance is assumed to be Euclidean and Mahalanobis and Type-II Kwon validity index is used to find the optimal degree of fuzziness (*m*) and number of clusters (*c*). The proposed Type-II PCM model is as follows:

$$J_m(x,\tilde{\mu},c) = min\left[\sum_{i=1}^{c}\sum_{j=1}^{N}\tilde{\mu}_{ij}^{m} D_{ij} + \sum_{i=1}^{c}\eta_i\sum_{j=1}^{N}\left(1-\tilde{\mu}_{ij}\right)^{m}\right] \quad (11)$$

$$S.T:\begin{cases} 0<\sum_{j=1}^{N}\tilde{\mu}_{ij}<N & (12)\\ \tilde{\mu}_{ij}\in[0,1] \quad \forall i,j & (13)\\ \max\tilde{\mu}_{ij}>0 \quad \forall j & (14)\end{cases}$$

where, $\tilde{\mu}_{ij}$ is Type-II membership for the i^{th} data in the j^{th} cluster, D_{ij} is the Mahalanobis distance of the i^{th} data to the j^{th} cluster's center, η_i is positive numbers, c is the number of the clusters, and N is the number of input data. The first term make the distance to the cluster's center be as low as possible and the second term make the membership values in a cluster to be as large as possible. The membership values for data in each cluster must lie in the interval [0,1], and their sum are restricted to be smaller than the number of input data, as shown in (12), (13), and (14).

Minimizing $J_m(x,\tilde{\mu},c)$ with respect to $\tilde{\mu}_{ij}$ is equivalent to minimizing the individual objective function defined in (15) with respect to $\tilde{\mu}_{ij}$ (provided that the resulting solution lies in the interval [0,l]).

$$J_m^{ij}(x,\tilde{\mu},\mathrm{c}) = \tilde{\mu}_{ij}^{m} D_{ij} + \eta_i\left(1-\tilde{\mu}_{ij}\right)^{m} \quad (15)$$

Differentiating (15) with respect to $\tilde{\mu}_{ij}$ and setting it to 0, leads to (16) which satisfies (12), (13), and (14).

$$\tilde{\mu}_{ij} = \frac{1}{1+\left(\frac{D_{ij}}{\eta_i}\right)^{\frac{1}{m-1}}} \quad \forall \mathrm{i}=1,\ldots,\mathrm{c} \quad (16)$$

$\tilde{\mu}_{ij}$ is updated in each iteration and depends on the D_{ij} and η_i. As mentioned in [3], the value of η_i determines the distance at which the membership value of a point in a cluster becomes 0.5. In general, it is desirable that η_i relate to i^{th} cluster and be of the order of D_{ij} [3].

$$\eta_i = \frac{\sum_{j=1}^{N}\tilde{\mu}_{ij}^{m} D_{ij}}{\sum_{j=1}^{N}\tilde{\mu}_{ij}^{m}} \quad \forall \mathrm{i}=1,\ldots,\mathrm{c} \quad (17)$$

where, D_{ij} is the distance measure and number of clusters (*c*) and degree of fuzziness (*m*) are unknown. Since the parameter η_i is independent of the relative location of the clusters, the membership value $\tilde{\mu}_{ij}$ depends only on the distance of a point to the cluster centre. Hence, the membership of a point in a cluster is determined solely by how far a point is from the centre and is not coupled with its location with respect to other clusters [11].

The clustering method needs a validation index to define the number of clusters (*c*) and degree of fuzziness (*m*), which are used in (15), (16), and (17). Therefore a Type-II Kwon Index based on Kwon index is designed, which is represented by (18):

$$\tilde{V}_k(c) = \frac{\sum_{i=1}^{c}\sum_{j=1}^{N}\tilde{\mu}_{ij}^{m}\left\|x_j-\tilde{v}_i\right\|^2+\frac{1}{c}\sum_{i=1}^{c}\|v_i-\bar{\tilde{v}}\|^2}{\min_{i\neq j}\left\|\tilde{v}_i-\tilde{v}_j\right\|^2} \quad (18)$$

where, $\tilde{\mu}_{ij}$ is Type-II possibilistic membership values for the i^{th} data in the j^{th} cluster, $\tilde{v}_i$ is the i^{th} center of cluster, $\bar{\tilde{v}}$ is the mean of centers, N is the number of input data, c is the number of the classes and m is the degree of fuzziness. The first term in the numerator denotes the compactness by the sum of square distances within clusters and the second term denotes the separation between clusters, while denominator denotes the minimum separation between clusters, so the smaller the $\tilde{V}_k(c)$, the better the performance.

In sum, the steps of the proposed clustering method are described bellow and are shown in Figure 2.

Step 1: Define the initial parameters including:

- Maximum iteration of the method (R)
- Number of the clusters (*c=2* is the initial value)

ISBN: 978-989-95079-6-8

- Degree of fuzziness (*m=1.5* is the initial value)
- Primary membership functions ($\tilde{\mu}_{ij}{}^{0}$) (Note that these membership functions are Type-II)

Step 2: Estimate η_i by using (17).
Step 3: Calculate the membership functions for each data in each cluster ($\tilde{\mu}_{ij}{}^{r}$) by using (16).
Step 4: If the difference between two membership functions for each data is bigger than the threshold, defined by user, ($|\tilde{\mu}_{ij}{}^{r} - \tilde{\mu}_{ij}{}^{r-1}| > \varepsilon$) go to step 4.1 and 4.2. On the other hand, go to step 4.3.

Step 4.1: $r = r + 1$
Step 4.2: Recalculate $\tilde{\mu}_{ij}{}^{r}$
Step 4.3: Compute the Kwon index ($\tilde{V}_k$).

Step 5: If the difference between two Kwon indexes for each membership functions is bigger than the threshold, ($|\tilde{V}_k{}^{r} -$ *Vkr−1>ε)*, go to step 5.1. On the other hand, go to step 5.2.

Step 5.1: Increase degree of fuzziness, m and run the method for another iteration.
Step 5.2: If the number of clusters are smaller than the number of data, ($c < N$), go to step 5.2.1. On the other hand, go to step 5.2.2.
Step 5.2.1: Run the method for another iteration
Step 5.2.2: Returns the value of *c, m*, and $\tilde{\mu}_{ij}{}^{r}$.

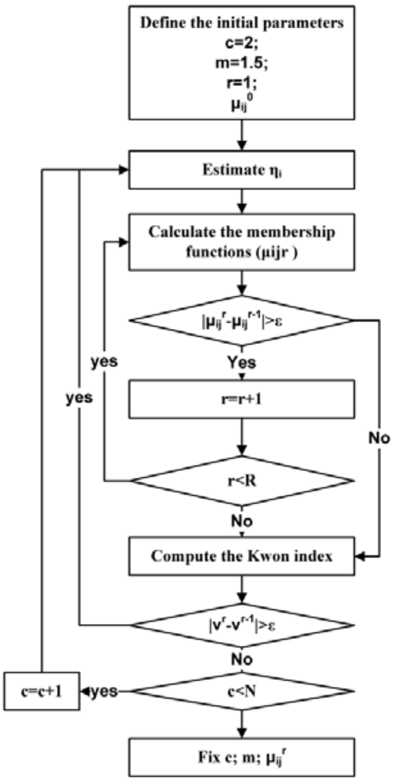

Figure 2: Type-II PCM Algorithm

5. Expand and Compare

In order to show the behavior of the proposed method, an image is used as input data. There may have been many uncertainties in images such as uncertainties caused by projecting a 3D object into a 2D image or digitizing analog pictures into digital images, and the uncertainty related to boundaries and non-homogeneous regions. Therefore, Type-II fuzzy logic can provide better performance than Type-I, this is shown by generating two models based on Type-I and Type-II Possibilistic C-Mean (PCM), each has two kind of distance measure, Euclidean and Mahalanobis. The Kwon Validity index is used to validate the results (*m* and *c*) as shown in Figure 2 and table 1, 2, 3 and 4. The results show that Type-II PCM using Mahalanobis distance can obtain better values for degree of fuzziness and number of clusters, which both are used for calculating the membership functions.

Table 1 shows the results of Kwon Validity index for Type-I PCM and Table 2 shows the results for Type-II PCM. In both of these tables Euclidean distance is used as the distance function. The elements of the tables are number of clusters (*c*) and degree of fuzziness (*m*) as input variables and the Kwon index values ($\tilde{V}_k$) as results, e.g., for *m*=2.7 and *c*=3, (2.7,3), the Kwon index for Type-I PCM is 4.5696 and is 750.34 for Type-II PCM. For *m*=3.3 and *c*=2, (3.3,2), the Kwon index could not be defined in both Type-I and Type-II PCM.

Table 3 shows the results of Kwon Validity index for Type-I PCM and Table 4 shows the results for Type-II PCM. In both of these tables Mahalanobis distance are used. The elements of the tables are number of clusters *(c)* and degree of fuzziness *(m)* as an input variables and the Kwon index values as results, i.e., for *m*=4.1 and *c*=3, (4.1,3), the Kwon index is 1.88 for Type-I PCM and is 33.686 for Type-II PCM.

ISBN: 978-989-95079-6-8

Table 1- Kwon Values for Type-I PCM with Euclidean Distance

c\m	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5
2	0.250	0.25	0.25	0.2500	0.2501	0.2509	0.2515	0.2662	0.2758	NaN	NaN
3	3.734	3.958	3.956	3.9877	4.0142	4.0762	4.5696	5.7768	6.889	NaN	NaN
4	5.95	5.944	6.05	6.3488	6.4236	2.464	987.99	7.0394	NaNi	NaN	NaN
5	5.351	5.367	5.474	5.7481	5.818	5.4528	991.14	11769	NaNi	NaN	NaN

Table 2- Kwon Values for Type-II PCM with Euclidean Distance

c\m	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5
2	57.82	38.36	14.53	14.436	30.184	6.2069	2.6636	1.5969	2.1884	NaN	NaN
3	514.7	113.3	92.05	37.547	484.97	201.14	750.34	392.91	10.381	NaN	NaN
4	9.17E+05	2.62E+05	8654.5	3.78E+05	1979.5	92.563	1854.1	331.16	NaN	NaN	NaN
5	1.01E+06	77060	6454	1534.8	386.57	130.98	9276.8	1240.2	NaN	NaN	NaN

Table 3- Kwon Values for Type-I PCM with Mahalanobis Distance

c\m	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.7	3.9	4.1	4.3	4.5	4.7	4.9	5.1
2	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.27	0.28	0.28	0.28	0.28	0.27	0.27	0.27	0.27	0.27	0.26	0.26
3	3.73	3.96	3.95	3.99	4.01	4.08	4.57	5.78	6.88	7.56	2.53	1.02	0.98	1.88	3.76	8.04	16.6	37.5	84.2
4	5.95	5.94	6.05	6.35	6.42	2.46	988	7.04	10.4	32.1	28.6	46.4	117	351	716	3080	5262	12599	28180
5	5.35	5.37	5.47	5.75	5.82	5.45	991	11769	587	282	56.5	49.3	121	356	716	3061	5235	12625	28521

Table 4- Kwon Values for Type-II PCM with Mahalanobis Distance

c\m	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.7	3.9	4.1	4.3	4.5	4.7	4.9	5.1
2	57.8	38.3	14.5	14.4	30.1	5.75	2.69	1.60	2.74	4.42	6.2	9.09	9.26	9.54	9.71	9.23	11.2	8.58	8.96
3	515	113	92.1	37.5	485	345	600	481	1043	29.5	11.1	9.24	13.9	33.7	76.2	232	399	853	2318
4	9.17E+05	2.62E+05	8654.5	3.78E+05	1979.5	92.563	1929.2	4171.5	22020	61.774	177.69	539.7	1322.5	4903.6	7743.6	50061	57076	1.41E+05	4.21E+05
5	1.01E+06	77060	6454.5	1534.8	386.57	130.98	9564.9	4806.6	23029	8855.7	27864	1.03E+05	6.50E+05	1.93E+06	1.26E+07	2.32E+07	4.90E+07	6.78E+07	1.11E+08

By comparing Type-I and Type-II PCM, the following conclusions can be obtained:

- In the same distance function cases, Kwon index is ascending for Type-I PCM, and its procedure is not clear for Type-II PCM case. Therefore, in Type-I PCM in all conditions the optimum *(m,c)* pairs are (1.5,2), (1.5,3), (1.5,4), (1.5,5), which may not be good results. However, in Type-II PCM the optimum *(m,c)* pairs are (2.9,2), (3.7,3), (3.3,4), (2.5,5), which seems to be good results. Figures 3 and 4 show the Kwon index results for *c=2* measured by Mahalanobis and Euclidean distance.

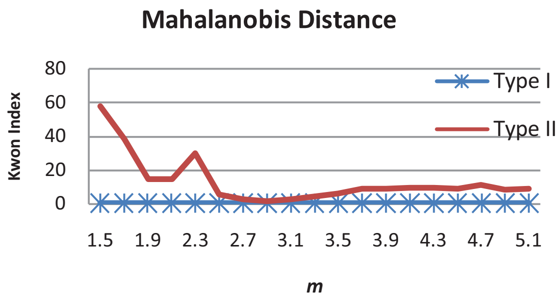

Figure 3: Kwon Index Result for c=2 and Mahalanobis Distance (based on Table 3 and 4)

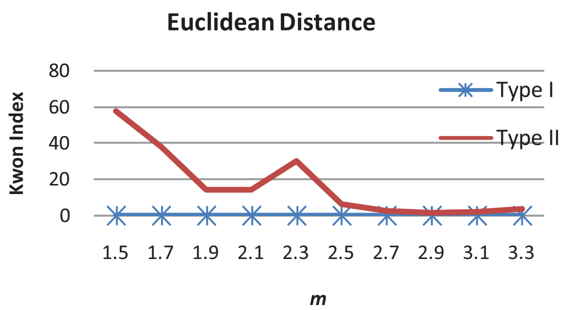

Figure 4- Kwon Index Result for c=2 and Euclidean Distance (based on Table 3 and 4)

- In the same type of fuzzy logic (Type-I or Type-II) case, for m<3.1 using two different distance functions did not show many differences for Kwon index results. However, for m>3.1, the Kwon index could be calculated by Mahalanobis but it is not defined for Euclidean distance, as shown in Figures 5.

ISBN: 978-989-95079-6-8

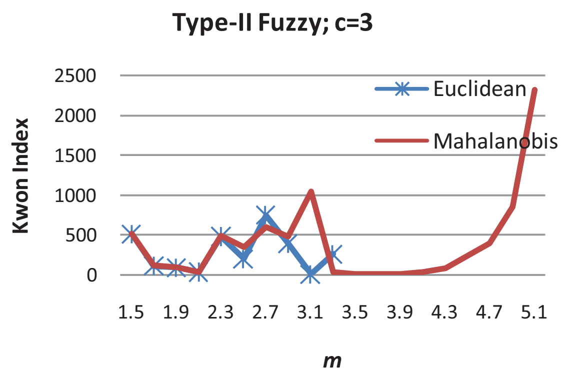

Figure 5: Kwon Index for c=3 and Different Distance Functions (based on Table 2 and 4)

6. Conclusions

This paper has presented a Type-II Possibilistic C-Mean (PCM) method for clustering purposes. The results of the proposed method are compared with Type-I PCM using an image as an input data and two kind of distance functions, Euclidean and Mahalanobis. The results show that Type-II PCM using Mahalanobis distance can provide better values for degree of fuzziness and number of clusters, which both are used in calculating the membership functions. Therefore the proposed clustering method is more accurate, can provide better performance and can handle uncertainties that exist in the data efficiently.

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