Opposite Fuzzy Sets with Applications in Image Processing

Hamid R. Tizhoosh
Pattern Analysis and Machine Intelligence Laboratory
Systems Design Engineering, University of Waterloo
200 University Avenue West, Waterloo, Ontario, N2L 3G1, Canada
Email: tizhoosh@uwaterloo.ca

Abstract— *Diverse forms of the concept of opposition are already existent in philosophy, linguistics, psychology and physics. The interplay between entities and opposite entities is apparently fundamental for balance maintenance in almost a universal manner. However, it seems that we have failed to incorporate oppositional thinking in engineering, mathematics and computer science. Especially, the set theory in general, and fuzzy set theory in particular, do not offer a formal framework to incorporate opposition in inference engines. Considering sets along with their opposites can establish a new computing scheme with a wide range of applications. In this work, preliminary definitions for opposite fuzzy sets will be established. The underlaying idea of opposition-based computing is the simultaneous consideration of guess and opposite guess, and estimate and opposite estimate, in order to accelerate learning, search and optimization. To demonstrate the applicability and usefulness of opposite fuzzy sets, a new image segmentation algorithm will be proposed as well.*

Keywords— Fuzzy sets, opposition, opposite fuzzy sets, antonym, antonymy, complement

1 Motivation

Wherever we observe the physical world and the human society, some notions of opposition exist. The intensity of *oppositeness* and the interplay between entities and opposite entities may vary greatly by closer observation. However, the antipodality, duality, complementarity, antithetics, polarity, in one word, opposition is, in different forms and at various intensity levels, present almost everywhere.

Strikingly, the human communication, expressed verbally and in written form, is without opposition unimaginable. How often we hear "on the one hand" followed by "and on the other hand" when humans weigh opposite alternatives, or how frequently we make use of antonyms: cold vs. hot, short vs. tall, true vs. false etc., or how impressively we make a point by using sayings such as "actions speak louder than words" (activity vs. passivity), "All for one and one for all" (union of opposites), "the pen is mightier than the sword" (spirit versus might). In spite of the virtual omnipresence of opposition, there is no formal framework in logic and set theory to integrate oppositional concepts. This preliminary work aims at providing elementary definitions and thoughts in this very direction.

2 What is Opposition?

Opposition is related to entities, objects or their abstractions of the same nature that are completely different in some manner. For instance, *cold* and *hot* both describe a certain temperature perception (are of the same kind), however, are completely different since they are located at opposite spots of temperature scale. Transition from one entity to its opposite is understandably accompanied with rapid and fundamental changes. Social revolutions, for instance, mainly aim at attaining opposite circumstances, dictatorship vs. democracy, by initiating sudden transitions. On the other hand, in many cases the balance due to interplay between entities and opposite entities is more dominant than rapid transition from one state to its opposite. These may be taken as initial thoughts for the purpose to improve algorithms – by smart switching between estimates and opposite estimates. However, in many other cases, this is the coexistence, the synergy of things and opposite things, and not their antagonism, that creates changes and enhances the system state.

In natural language, opposition can be detected at different levels [1]. For instance, directional opposition (north-south, up-down, left-right), adjectival opposition (ugly-handsome, long-short, high-low), and prefix opposition (thesis vs. antithesis, revolution vs. counter-revolution, direction vs. opposite direction) are all different levels of linguistic opposition. Further, one can distinguish complements (mutually exclusive properties: dead-alive, true-false), antonyms (two corresponding points or ranges of a scale: long-short, hot-cold), directional converses (two directions along an axis: east-west, up-down), and relational converses (the relative positions of two entities on opposite sides: above-below, teacher-pupil) [1]. The human communication without utilizing linguistic opposition is unimaginable.

Many examples also exist in the philosophy which deal with opposition. The concept of **yin** and **yang** originates in ancient Chinese philosophy and metaphysics, which describes two primal *opposing* but complementary forces found in all things in the universe. Also in the dualism delivered by Avesta, the book of Zoroastrians, the everlasting fight between *Ahura* (God=good) and *Ahriman* (evil) defines the worldlook. The **Table of Opposites** of Pythagoras, delivered by Aristotle, has his own list of opposites: finite-infinite, odd-even, one-many, right -left, rest-motion, straight-crooked, light-darkness, good-evil, and square-oblong. Many philosophers have focused on *one vs. many* as one of the most fundamental questions in philosophy.

In physics, *antiparticles* are subatomic particles having the same mass as one of the particles of ordinary matter but with opposite electric charge and magnetic moment. The positron (positive electron), hence, is the antiparticle of the negative electron (negatron). As another physical example, we define *electric polarization* as slight relative shift of positive and negative electric charge in opposite directions within an insulator, or dielectric, induced by an external electric field. Polarization occurs when an electric field distorts the negative cloud

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of electrons around positive atomic nuclei in a direction opposite the field[1]. And we could go back in time to the Newton's Mechanics with its principal of *action* and *reaction* which is a major example for discovering and exploiting oppositional behavior.

In mathematics we have several concepts employing some notions of opposition. For instance, the bisection method for solving equations makes use of *positive* vs. *negative* sign change in order to shrink the search interval. In probability theory, the probability of *contrary* situation is given by $1-p$ if the initial event occurs with probability p. In Monte Carlo simulation, *antithetic* random numbers are used to reduce the variance, and in category theory there is the concept of *duality*.

These examples show that opposition plays a central rule in many fields ranging from linguistics to physics. In spite of all examples, however, it should be mentioned that understanding and defining opposition may not be trivial in some cases.

3 Opposition-Based Computing

The concept of opposition-based computing (OBC) [2, 4] is based on a quite simple idea: Whenever we are looking for a solution, we should consider the opposite solution as well. This may appear too simple, however, since opposition is almost universal, it can be understood and implemented in many different ways such that the application fields and potential impacts are immense. The idea of opposition-based computing is not entirely new in machine intelligence. Numerous works have treated and employed oppositional concepts in different ways and under different labels [4].

Past research on oppositional concepts has not been conducted under a unified framework. As such, a universal view of opposition-based learning should regard and treat entities and their opposites in a much more general framework, particularly when for application in the context of machine intelligence schemes. Among others, we should not forget that opposition considers both qualitative and abstract entities, and not only numbers.

In a preliminary paper [2], OBC-extensions of neural nets (with opposite weights), genetic algorithms (with anti-chromosomes) and reinforcement agents (with opposite actions) were provided, with experimental results indicating modest improvements in learning/searching speed. More detailed reports, on improvement of the learning behavior of evolutionary algorithms, reinforcement learning, ant colonies and neural nets have been reported as well [3, 5, 6, 7, 9, 10]. Among other benefits of incorporating of OBC into existing learn and search algorithms, total computational time has decreased by 1.6 times for differential evolution (DE) by incorporating anti-chromosomes. (This becomes even more significant when we recall that DE is generally considered a fast algorithm to begin with).

OBC may be regarded as a new philosophy for integration of a-priori knowledge (interplay between things and opposite things during search and learning procedures). However, this is a very special type of a-priori knowledge. Among other potential benefits, working with opposite entities is not accompanied with uncertainties, if the initial entities and their evaluation/fitness are known. In contrast, any work based on similarity and dissimilarity is always to a certain degree uncertain.

[1] Encyclopdia Britannica

The main question, however, is how we should find/define the opposites for every given problem. This work will attempt to find some answers within fuzzy set theory.

4 Oppositional Ideas in Fuzzy Systems

Opposite fuzzy sets have been a part of fuzzy systems from the beginning. Fuzzy inferencing is without the concept of opposition virtually impossible. Linguistic terms are generally considered as pairs such as "short-tall", "cold-warm", "positive-negative" etc., something that is so natural and customary that we have overlooked its significance and failed to develop a more systematic paradigm to exploit its true potential.

Some explicit notions of employing opposition can be found in previous works published on fuzzy sets and their applications. One may immediately say that the concept of "negation" is the natural translation of opposition. A set A, describing a set of objects, is negated, $\neg A$, to quantify "not A". However, this is a general misconception. Negation cannot represent opposition, for it is too general to illustrate the oppositeness of attributes. For instance, "not very young" encompasses the entire spectrum of "young" to "very old", whereas the opposite of "very young" is clearly "very old". Most likely Roland Yager was the first one who used the term "opposition" in relation to fuzzy sets [11]. In literature on fuzzy systems, the opposition has been scarcely studied, and that only with respect to *antonyms* and their significance from a pure linguistic perspective [12, 13]. As we will see, antonyms are what we call in this paper "type-I opposite fuzzy sets".

5 Opposite Fuzzy Sets

In this section we attempt to establish a generic framework for opposite fuzzy sets. For all following definitions we consider a general class of membership functions (MFs) $\mu(x) = f(x; \mathbf{a}, \delta_1, \delta_2)$ where the vector **a** denotes the points with $\mu(a_i) = 1$, and δ_1 and δ_2 are the function widths to the left and right, respectively (Fig. 1).

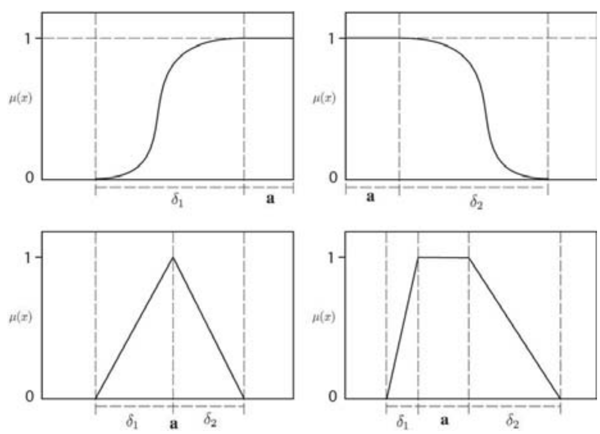

Figure 1: General MFs $\mu(x) = f(x; \mathbf{a}, \delta_1, \delta_2)$

Let X be the universe of discourse and $x \in X$ the elements, objects to be classified.

Definition (Fuzzy Set) - A fuzzy set $A \subset X$ with membership function $\mu_A(x)$ is defined as

$$A = \{(x, \mu_A(x)) | x \in X, \mu(x) \in [0,1]\} \tag{1}$$

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The membership function is given as $\mu_A(x) = f(x; \mathbf{a}, \delta)$ where $\mu_A(\mathbf{a}) = 1 \quad \forall a_i \in \mathbf{a}$ and δ is the somatic parameter that changes the shape of the membership function.

Definition (Opposite Fuzzy Set) - Given a fuzzy set $A \subset X$, the opposite fuzzy set $\breve{A} \subset X$ with membership function $\mu_{\breve{A}}(x)$ is defined as

$$\breve{A} = \{(x, \mu_{\breve{A}}(x)) | x \in X, \mu_{\breve{A}}(x) \in [0, 1]\} \quad (2)$$

where $\mu_{\breve{A}}(x) = f(x; \breve{\mathbf{a}}, \breve{\delta})$.

The vector $\mathbf{a} = [a_1, a_2, \dots]$ and its opposite vector $\breve{\mathbf{a}} = [\breve{a}_1, \breve{a}_2, \dots]$ represent the points on the universe of discourse with $\mu(a_i) = \mu(\breve{a}_i) = 1$; $\breve{\delta}$ will cause the opposite shape modification compared to original δ.

Example - If a Gaussian, bell curve, function is employed to represent $\mu_A(x) = f(x; \mathbf{a}, \delta)$, then $a = c$ and $\delta = \sigma$. The opposite membership function $\mu_{\breve{A}}(x) = f(x; \breve{\mathbf{a}}, \breve{\delta})$ will be defined with $\breve{a} = \breve{c}$ and $\breve{\delta} = \breve{\sigma}$

Since both elements of **a** and δ are real numbers, the definition of opposite fuzzy sets comes down to definition of the opposite of any arbitrary real number a.

Opposite numbers are generally defined for the set of irrational number $\mathbb{Z}$ on the interval $(-\infty, +\infty)$. For each $x \in \mathbb{Z}$ the opposite number $\breve{x}$ is defined by $\breve{x} = -x$. This definition, however, lacks generality. Following, we introduce more comprehensive definitions of opposite numbers [4].

Definition (Type-I Opposite Points[2]) - Let $P = (a_1, a_2, \cdots, a_n)$ be a point in an n-dimensional space, where $a_i \in [X^i_{min}, X^i_{max}] \in \mathbb{R}$. The type-I opposite point $\breve{P} = (\breve{a}_1, \breve{a}_2, \cdots, \breve{a}_n)$ is then completely defined where

$$\breve{a}_i = X^i_{max} + X^i_{min} - a_i. \quad (3)$$

For $n = 1$ following special cases exist:

$$\begin{array}{lllll} \breve{a} & = & -a & \text{for} & X_{max} = -X_{min}, \\ \breve{a} & = & 1 - a & \text{for} & X_{min} = 0 \;\; \& \;\; X_{max} = 1, \\ \breve{a} & = & a & \text{for} & a = \frac{X_{max} + X_{min}}{2}. \end{array} \quad (4)$$

Definition (Type I Opposite Fuzzy Sets) - The set $\breve{A}$ with membership function $\mu_{\breve{A}}(x) = f(x; \breve{a}, \breve{\delta})$ is type I opposite of the set A with membership function $\mu_A(x) = f(x; a, \delta)$ if $\breve{a}$ and $\breve{\delta}$ are type I opposites of a and δ, respectively.

Definition (Type-I Super-Opposite Points) - Let $P = (a_1, a_2, \cdots, a_n)$ be a point in n-dimensional space with $a_i \in [X^i_{min}, X^i_{max}]$ and its opposite point $\breve{P} = (\breve{a}_1, \breve{a}_2, \cdots, \breve{a}_n)$. Then all points $\breve{P}^s$ are type-I super-opposite of P when $d(\breve{P}^s, P) > d(\breve{P}, P)$ where $d(\cdot, \cdot)$ denotes a metric such as the Euclidian distance.

For $n = 1$ the type-I super-opposite of $a \in [X_{min}, X_{max}]$ can be given as

$$\breve{a}^s \in \begin{cases} [X_{min}, \breve{a}) & \text{for} \quad a > (X_{min} + X_{max})/2 \\ [X_{min}, X_{max}] - \{a\} & \text{for} \quad a = (X_{min} + X_{max})/2 \\ (\breve{a}, X_{max}] & \text{for} \quad a < (X_{min} + X_{max})/2 \end{cases}$$

[2]The terms "type I" and "type II" opposition have been taken from previously published works on OBC [4] and should not be confused with type-1 and type-2 fuzzy sets. Type-I opposites reflect linear opposition whereas type-II opposites constitute nonlinear opposition.

In other words, for $a = \frac{X_{min} + X_{max}}{2}$ the entire interval except a becomes the super-opposite of a. This means that for $a \rightarrow \frac{X_{min} + X_{max}}{2}$ the opposition generally loses its benefit against random guesses, since any random guess will be a super-opposite of the initial guess.

Definition (Type I Super-Opposite Fuzzy Sets) - The set $\breve{A}^S$ with membership function $\mu_{\breve{A}^S}(x) = f(x; \breve{a}^S, \breve{\delta}^S)$ is type I super-opposite of the set A with membership function $\mu_A(x) = f(x; a, \delta)$ if $\breve{a}^S$ and $\breve{\delta}^S$ are type I super-opposites of a and δ, respectively.

Definition (Type-I Quasi-Opposite Points) - Let $P = (a_1, a_2, \cdots, a_n)$ be a point in n dimensional space with $a_i \in [X^i_{min} + X^i_{max}]$ and its opposite point $\breve{P} = (\breve{a}_1, \breve{a}_2, \cdots, \breve{a}_n)$. Then all points $\breve{P}^q$ are type-I quasi-opposite of P when $d(\breve{P}^q, \breve{P}) = d(\breve{P}^s, \breve{P})$ and $d(\breve{P}^q, P) < d(\breve{P}^s, P)$ for any super-opposite $\breve{P}^s$.

For $n = 1$ the type-I quasi-opposite of $a \in [X_{min}, X_{max}]$ can be given as

$$\breve{a}^q \in \begin{cases} (\breve{a}, 2\breve{a}] & \text{for} \quad a > (X_{min} + X_{max})/2 \\ [X_{min}, X_{max}] - \{a\} & \text{for} \quad a = (X_{min} + X_{max})/2 \\ [2\breve{a} - X_{max}, \breve{a}) & \text{for} \quad a < (X_{min} + X_{max})/2 \end{cases}$$

Definition (Type I Quasi-Opposite Fuzzy Sets) - The set $\breve{A}^q$ with membership function $\mu_{\breve{A}^q}(x) = f(x; \breve{a}^q, \breve{\delta}^q)$ is type I quasi-opposite of the set A with membership function $\mu_A(x) = f(x; a, \delta)$ if $\breve{a}^q$ and $\breve{\delta}^q$ are type I quasi-opposites of a and δ, respectively.

Figure 2 illustrates type-I super- and quasi-opposition for one dimensional cases. Figure 3 shows a fuzzy subset A, its opposite $\breve{A}$, quasi- and super-opposite.

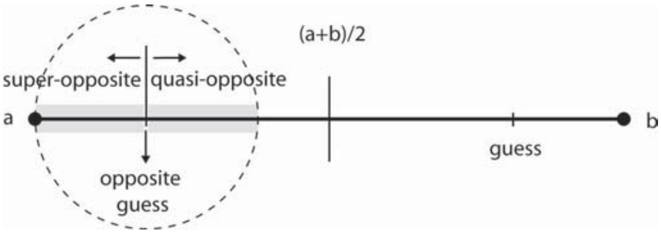

Figure 2: Definition of super- and quasi-opposition based on the distance of the opposite guess from the interval boundary.

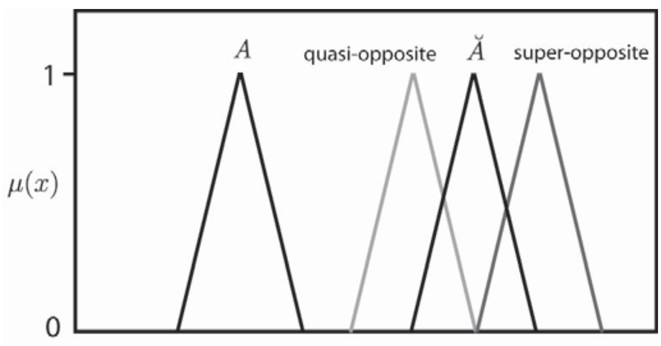

Figure 3: A fuzzy set and its opposites

Obviously type I opposition is a straightforward and a simple understanding of opposition. Generally, only linear,

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almost-linear and symmetric systems can benefit from type I opposition. In order to deal with complex and nonlinear $X - Y$-relationships we have to access higher levels of opposition.

Theorem - The entropy (fuzziness) of the fuzzy set A, denoted with $E(A)$ is equal to the entropy (fuzziness) of its type I opposite set $\breve{A}_I$: $E(A) = E(\breve{A}_I)$

Proof - since the membership function of $\breve{A}$ is just a reflection and translation of A, then $E(A) = E(\breve{A})$. □

Definition (Type-II Opposite Points) - Let $y = f(x_1, x_2, \cdots, x_n) \in \mathbb{R}$ be an arbitrary function with $y \in [y_{\min}, y_{\max}]$. For every point $P = (a_1, a_2, \cdots, a_n)$ the type-II opposite point $\breve{P} = (\breve{a}_1, \breve{a}_2, \cdots, \breve{a}_n)$ is defined by

$$\breve{a}_i = \{x| \quad \breve{y} = y_{\min} + y_{\max} - y\}. \tag{5}$$

This definition assumes that the function $f(x_1, x_2, \cdots, x_n)$ is not known but $y_{\min}$ and $y_{\max}$ are given or can be estimated. Alternatively, the temporal type-II opposite can be calculated according to

$$\breve{a}_i(t) = \left\{ x \;\middle|\; \breve{y}(t) = \min_{j=1,\ldots,t} y_j + \max_{j=1,\ldots,t} y_j - y(t) \right\}. \tag{6}$$

If only the evaluation function $h(X)$ is available, then we may define a temporal *degree of opposition* $\breve{\Phi}$ for any two variables a_1 and a_2:

$$\breve{\Phi}(a_1, a_2, t) = \frac{|h(a_1) - h(a_2)|}{\max\limits_{j=1,\ldots,t} h(a_j) - \min\limits_{j=1,\ldots,t} h(a_j)} \in [0, 1]. \tag{7}$$

The type-II opposite $\breve{a}_i$ of any variable a_i can then be given as

$$\breve{a}_i = a_j|_{\breve{\Phi}(a_i,a_j,t)=\max_k \breve{\Phi}(a_i,a_k,t)}. \tag{8}$$

Definition (Type II Opposite Fuzzy Sets) - The set $\breve{A}_{II}$ with membership function $\mu_{\breve{A}_{II}}(x) = f(x; \breve{a}, \breve{\delta})$ is type II opposite of the set A with membership function $\mu_A(x) = f(x; a, \delta)$ if $\breve{a}$ and $\breve{\delta}$ are type II opposites of a and δ, respectively.

Definition - Degree of Oppositeness for Type I Opposition For two real numbers $a, b \in X$ bounded in $[X_{min}, X_{max}]$, the degree of oppositeness can be given as

$$\breve{\varphi}_I(a, b) = \left(\frac{|a - b|}{X_{max} - X_{min}} \right)^{\beta} \tag{9}$$

where $\beta \in (0, 1]$ controls the opposition intensity.

Definition - Degree of Oppositeness for Type II Opposition Assuming the evaluation function $h(\cdot)$ is given, then for two real numbers $a, b \in X$, the degree of oppositeness can be given as

$$\breve{\varphi}_{II}(a, b) = \left(\frac{|h(a) - h(b)|}{h_{max} - h_{min}} \right)^{\beta} \tag{10}$$

where $\beta \in (0, 1]$ controls the opposition intensity. If the bounds of the evaluation function are not known, then a temporal type-II opposition may be calculated:

$$\breve{\varphi}^{(t)}_{II}(a, b) = \left(\frac{|h(a) - h(b)|}{h^{(t)}_{max} - h^{(t)}_{min}} \right)^{\beta} \tag{11}$$

Based on the introduced definitions, following observations can be made:

- Opposition, in general, can only be determined if the system parameters are bounded and *opposite* has a clear relationship with the finite interval, in which those parameters are defined.
- Type-I opposition delivers the real opposite (type-II) only for linear or quasi-linear functions. Type-I opposition can, however, be more easily calculated.
- Type-II opposition can, in absence of any information about the function $f(\cdot)$, only be calculated if an evaluation function $g(\cdot)$ (error, fitness, reward, cost etc.) is available.

Now that we have basic definitions available we can define the opposition-based computing.

Opposition-Based Computing (OBC) - For any system behaving according a (unknown) function $y = f(x_1, x_2, \cdots, x_n)$ with an evaluation function $h(x_1, x_2, \cdots, x_n)$ with higher/lower values being desirable, for any estimate vector $X = (x_1, x_2, \cdots, x_n)$ to estimate the input-output relationship we calculate its opposite $\breve{X} = (\breve{x}_1, \breve{x}_2, \cdots, \breve{x}_n)$ and consider $\max(h(X), h(\breve{X}))$. The learning/search will continue with X if $h(X) \gtreqless h(\breve{X})$ otherwise with $\breve{X}$.

The opposition can be calculated using the aforementioned definitions and based on the specific domain knowledge.

The definition of OBC is making two fundamental assumptions: 1) one of estimate and counter-estimate is always closer to the solution, and 2) considering the opposition is more beneficial than generating independent random solutions and taking the best among them. (this assumption has already been proved [8]).

The aforementioned definition of OBC is primarily meant to be used in the model-free context of machine intelligence, where an evaluation function such as error, cost, fitness, reward etc. is being used within multiple iterations/epochs/generations/episodes in order to find some optimal values. For pure fuzzy systems we may rely on the following definition.

Opposition-Based Fuzzy Inference Systems (OFIS) - An inference mechanism which systematically uses fuzzy sets along with their opposites to construct membership functions and rules and/or carry out the inference is an opposition-based fuzzy inference system.

6 Applications

As mentioned before, we have virtually been using opposite fuzzy sets since Zadeh's first paper. However, due to lack of necessary formalism and due to investigative studies on the nature of opposition in its diverse forms, we have not exploited their potentials.

Most likely, one major application for opposite fuzzy sets is generation of membership functions (MFs) for an inference system. From the beginning days of fuzzy sets, determination of membership functions has been a challenge. Oppositional

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relationships can be considered to generate membership functions. Table 1 provides a simple algorithm for this purpose. The underlaying idea is as follows: Assuming we are looking for 3 membership functions, and assuming we can fix the position of one of them namely the set A. Then if we automatically determine the opposite fuzzy set $\breve{A}$, the third (middle) membership function, which quantifies "everything else" can be given by $\neg(A \cup \breve{A})$ (see Figure 4).

Table 1: Oppositional generation of membership functions

1.	Determine n to generate $2n+1$ membership functions
2.	For $i = 1 : n$
3.	$\mu_{A_i} = f(x; a_i, \delta_i)$
4.	end
5.	For $i = n+2 : 2n+1$
6.	Determine $\breve{a}_i$ subject to the constraint $\breve{a}_i < \breve{a}_{i+1}$
7.	Determine $\breve{\delta}_i$
8.	$\mu_{\breve{A}_i} = f(x; \breve{a}_i, \breve{\delta}_i)$
9.	end
10.	Determine the middle MF based on $\neg(A \cup \breve{A})$

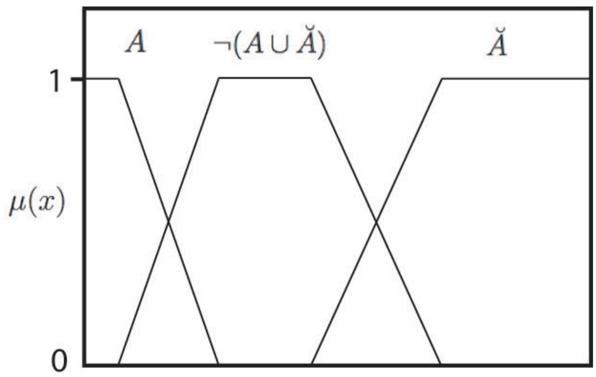

Figure 4: The set A, its opposite $\breve{A}$ and "everything else"

Processing of digital images is certainly an interesting application to verify the applicability of opposite fuzzy sets. The task of *image segmentation* is the most significant part in many vision-based application whereas segmentation is equivalent to classification of each pixel to different classes (segments). Here, based on proposed algorithm in Table 1 we can demonstrate how opposite fuzzy sets can help to segment a digital image.

Let us assume that we are interested in extracting only one object of interest in the image. The rest, regardless how many other objects, are irrelevant. Further assume we have the knowledge that the object of interest is *dark*. Then an oppositional fuzzy rule-based approach to object segmentation can be implemented as described in Table 2. As apparent from line 7 and 8 in this algorithm, we assume that the entropy of A and $\breve{A}$ are equal, which is, according to the theorem in section 5, only proved for type-I opposites. However, experimental results will show that this is a reasonable assumption, which still needs to be theoretically solidified.

Table 2: An OFIS for image segmentation

1.	Define the set $A = \{$ *dark pixels* $\}$ with $\mu_A(x)$
2.	Calculate the entropy of $A : E(A)$
3.	Initialize the opposite membership $\breve{\mu}_A(x)$ at $\breve{a} = L-1$
4.	For $i = L-1 : -1 : a+\delta$
5.	Define opposite fuzzy set $\breve{A}_i$ at $\breve{a} = i$ (Fig. 5)
6.	Calculate the entropy of $\breve{A}_i : E(\breve{A}_i)$
7.	Calculate the entropy difference $d_i = \lvert E(\breve{A}_i) - E(A) \rvert$
8.	Find the position j with $d_j = \min_i d_i$
9.	Define the final opposite fuzzy set at $\breve{a} = j$
10.	Define the middle membership function via $\neg(A \cup \breve{A})$
11.	(OPTION 1) Create a fuzzy segmentation map via $g' = \frac{\mu_A(g)+\mu_{\neg(A\cup\breve{A})}(g)\times(L-1)/2+\mu_{\breve{A}}(g)\times(L-1)}{\mu_A(g)+\mu_{\neg(A\cup\breve{A})}(g)+\mu_{\breve{A}}(g)}$
13.	(OPTION 2) Threshold the image with $T = (a+\breve{a})/2$
14.	$g' = 1$ if $g <= T$, and $g' = 0$ otherwise

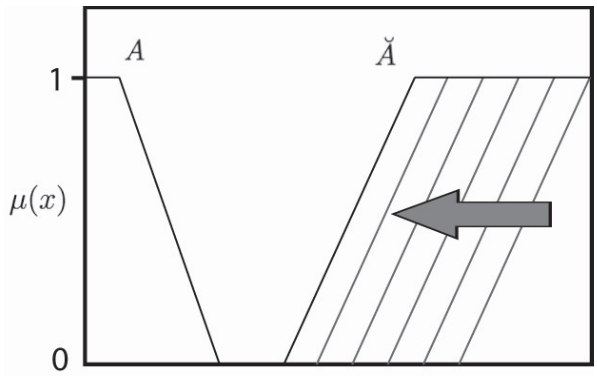

Figure 5: Moving an initial set toward A to find the opposite set $\breve{A}$

The results of this approach are presented in Figure 6. Needless to say that this simple algorithm should only demonstrate the usefulness of opposite fuzzy sets for a significant application. Most likely more sophisticated segmentation algorithms can be developed by incorporating oppositional thinking.

Breast ultrasound images have been used to verify the performance of this method (Fig. 6). The first two scan contain anechoic (dark) breast cysts. These are relatively easy to segment with almost any thresholding technique. The other two contain breast masses that are no cyst and should be examined on malignancy. These cases are challenging and cannot be easily segmented.

7 Conclusions

In this paper, we attempted to establish a formalism for opposite fuzzy sets and introduce some definitions. Preliminary results for a simple image segmentation method were provided as well. Opposition-based fuzzy inference systems (OFISs) exploit domain knowledge and seem to have real-world applications. Many questions, however, remain open with respect to the nature of opposition and how it can be embedded into existing fuzzy systems.

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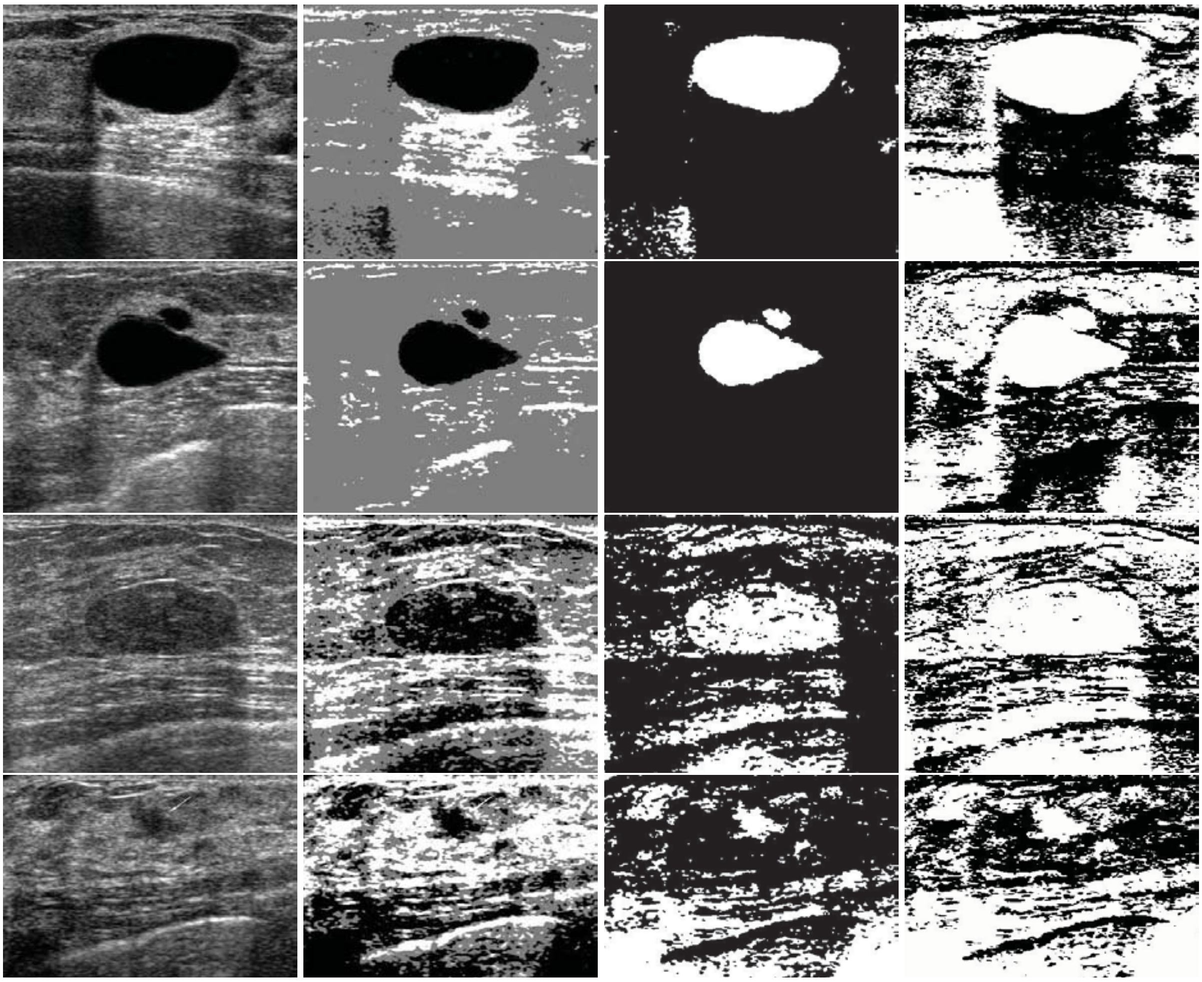

Figure 6: From left: first column: original breast ultrasound scans, second column: fuzzy segmentation with $A, \neg(A \cup \breve{A})$ and $\breve{A}$ (OPTION 1), third column: thresholding with $T = (a + \breve{a})/2$ (OPTION 2). The calculated values from top to bottom: $a = \{1, 1, 64, 62\}$ and $\breve{a} = \{143, 159, 127, 117\}$. The last column depicts the results by Otsu method for comparison [Source of images: Philips]

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