Interval-valued Fuzzy propositions. An application to the L-Fuzzy contexts with absent values

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Abstract— *In order to obtain the information from an L-Fuzzy context, the complete relation between the objects and the attributes is needed. However, the contexts that model many situations have absent values.*
To solve this problem, at the beginning of the paper we remind the interval-valued linguistic variable definition and, later, we propose an extension of the fuzzy propositions to the interval-valued case.
In the second part, we apply these ideas to the problem of replacement of the absent values in an interval-valued L-Fuzzy Context. The use of interval-valued fuzzy propositions simplifies the process.

Keywords— Interval-valued fuzzy propositions, Interval-valued linguistic variables, Interval-valued L-Fuzzy Contexts.

1 Introduction

The first extension of the Formal Concept Analysis to the fuzzy case, the L-Fuzzy Concept Theory, is due to A. Burusco and R. Fuentes-González and was published in 1994 [1].

Later, to extract knowledge from a more general table, we defined the interval-valued L-Fuzzy context as an extension to the interval-valued case of the L-Fuzzy context defined in the L-Fuzzy Concept Theory [2]:

An *interval-valued L-Fuzzy context* is a tuple $(\mathcal{J}[L], X, Y, R)$, with X and Y two finite sets (of *objects* and *attributes*), R an interval-valued L-Fuzzy relation between X and Y and $\mathcal{J}[L]$ the set of the closed intervals of a lattice L with the usual order.

To replace the absent values, we have already developed a theory based on implications between attributes [3] that has given good results when has been applied to the Technique [4]. In these papers, in order to replace the absent values, we have used some labels that allow to obtain implications with high values of support and confidence.The good behavior of these labels, suggested us the application of the linguistic variables to our work [5].

In this work, we are going to define fuzzy propositions for the interval-valued case (they do not exist in the Literature of the subject) and later we will try to use them to replace the absent values in contexts instead of using implications between attributes. We will finally analyze the advantages of the method.

2 Linguistic variables defined in the set of closed intervals of $[0, 1]$

Let $(V, T(V), U, G, M)$ be a linguistic variable (Zadeh [6]) defined in the set $U = [0, 1]$, whose values or labels $T(V)$ are associated with the generalized trapezoidal fuzzy numbers defined by Yao and Lin [7]. We have extended the linguistic variable definition to the interval-valued case [5]:

Definition 1 *Taking as a departure point a linguistic variable V, we define an* interval-valued linguistic variable *associated with V as the linguistic variable* **V** *defined in the set $\mathcal{J}[U]$ of the closed intervals of U, characterized by the tuple $(\mathbf{V}, T(V), \mathcal{J}[U], G, M)$, where the compatibility function with each label $t \in T(V)$ is $\mathbf{c}_t : \mathcal{J}[U] \to \mathcal{J}[0, 1]$.*
Notation. *We denote the compatibility of the value $[\alpha, \beta] \in \mathcal{J}[U]$ with the label t by:*

$$\mathbf{c}_t([\alpha,\beta]) = [\alpha,\beta]_t = \\ = \begin{cases} [\min\{\alpha_t, \beta_t\}, \max\{\alpha_t, \beta_t\}] & \text{if } \alpha \geq b \text{ or } \beta \leq a \\ [\min\{\alpha_t, \beta_t\}, 1] & \text{in other case} \end{cases}$$

where a and b are the values that define the generalized trapezoidal fuzzy number associated with the label $t \in T(V)$.

With this definition we will try to represent the interval in which x_t have its values when $x \in [\alpha, \beta]$.

The defined interval-valued linguistic variable **V** is an extension of the linguistic variable V to the set of closed intervals of U.

Some properties of these interval-valued linguistic variables have been proven in [5].

3 Interval-valued fuzzy propositions

Before studying the interval-valued fuzzy propositions, we are going to remind the main aspects about fuzzy propositions.

3.1 Fuzzy propositions

As can be seen in [8], the main difference between classical and fuzzy propositions is that the truth value of the second ones belongs to the interval [0,1]. Moreover, four types of fuzzy propositions are defined by Klir [8]:

Unconditional and unqualified propositions: $p : \Phi$ *is* A, with Φ a variable and A a predicate attributed to the variable and represented by a fuzzy set. In other words, p : 'Φ *is* A' *is true.* The truth value of the proposition $p_\Phi : \Phi = \phi$ *is* A, is obtained by $T_r(p_\phi) = A(\phi), \forall \phi \in \Phi$.

Unconditional and qualified propositions: The difference with the previous ones is that the proposition has a qualified truth value that modifies the truth value. They are

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characterized by the form p : 'Φ *is* A' *is* S, with S qualified truth value.
Examples of truth qualifiers are expressions as *true*, *very true*, *fairly true*, *false*, *very false* or *fairly false* (See Fig. 1).

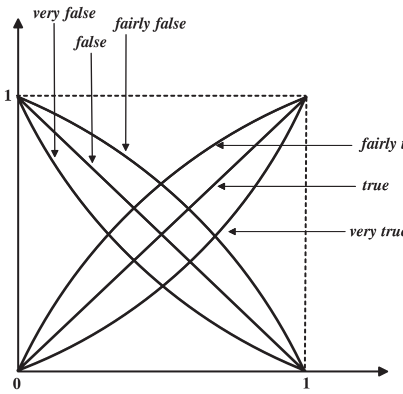

Figure 1: Some examples of truth qualifiers

Conditional and unqualified propositions: These propositions are expressed by p : *If* Φ *is* A*, then* Ψ *is* B, with Φ and Ψ variables and A and B predicates represented by fuzzy sets. That is, p : '*If* Φ *is* A*, then* Ψ *is* B' *is true*. The truth value of the proposition is calculated using an implication operator:

$$T_r(p_{\phi,\psi}) = I[A(\phi), B(\psi)]$$

Conditional and qualified propositions: p : '*If* Φ *is* A*, then* Ψ *is* B' *is* S, with S a qualified truth value. The truth value of the proposition is obtained by

$$T_{rS}(p_{\phi,\psi}) = S[T_r(p_{\phi,\psi})]$$

3.1.1 Fuzzy quantifiers

The two quantifiers of the predicate logic are *all* and *exists*. Fuzzy quantifiers are a tool for symbolizing quantified statements that minimizes the lost of information forced by the choice of quantifier.

The fuzzy propositions of any of the types introduced in the previous section may be quantified by a suitable fuzzy quantifier. In general, fuzzy quantifiers are fuzzy numbers which take part in the various propositional forms and affect the degrees of truth of specific fuzzy propositions. Each fuzzy quantifier expresses an approximate number of elements or an approximate proportion of elements in a given universal set that claims to satisfy a given property.

There are two types of fuzzy quantifiers: The absolute quantifiers expressed by fuzzy numbers defined on the set of real numbers or on the set of integers: *about a dozen*, *at most about 10*, *at least about 100...* And the relative quantifiers expressed by fuzzy numbers defined on [0,1]: *most*, *almost all*, *about half*, *about 20%*. These relative quantifiers are the most interesting for our study. So, we are going to see how the fuzzy propositions with these fuzzy quantifiers are.

Fuzzy propositions with quantifiers of the second kind have the general form ([8]):

p : *Among z's in* Z *such that* $\nu_1(z)$ *is* F_1 *there are* Q *z's in* Z *such that* $\nu_2(z)$ *is* F_2

Any propositions p can be expressed in a simplified form:

$$p' : QE'_1s \text{ are } E'_2s$$

where Q is the used quantifier, E_1 and E_2 are the fuzzy sets on Z defined by $E_1(z) = F_1(\nu_1(z))$, $E_2(z) = F_2(\nu_2(z)), \forall z \in Z$, with F_1 and F_2 fuzzy sets.

We may rewrite this proposition in the form:

$$p' : W \text{ is } Q$$

with W a variable that represents the degree of subsethood of E_2 in E_1, that is:

$$W = \frac{|E_1 \cap E_2|}{|E_1|}$$

where $|\quad|$ represents the cardinality of a set.

Using the standard fuzzy intersection, we obtain:

$$W = \frac{\sum_{z \in Z} \min\{F_1(\nu_1(z)), F_2(\nu_2(z))\}}{\sum_{z \in Z} F_1(\nu_1(z))}$$

for any E_1 and E_2.

Then, the truth value is $T_r(p) = T_r(p') = Q(W)$.

3.1.2 Linguistic hedges

Linguistic hedges are special linguistic terms by which other linguistic terms are modified: *Very*, *more or less*, *fairly* or *extremely*. They can be used for modifying fuzzy predicates, fuzzy truth values and fuzzy quantifiers.

Any linguistic hedge may be interpreted as an unary function h in [0,1]. For example, *Very* ($h(a) = a^2$), *fairly* ($h(a) = \sqrt{a}$). These functions h are also said to be modifiers or qualifiers.

3.2 Interval-valued fuzzy propositions

There are some situations where we work with interval-valued fuzzy variables and fuzzy propositions and we think that the truth value of an interval-valued fuzzy proposition must be also an interval in order to model in a more suitable way the represented data. An example of this situation can be seen in next section.

With this aim, we are going to study the interval-valued fuzzy propositions.

Then, we define an interval-valued fuzzy proposition with fuzzy quantifiers of the second kind as:

$$\mathbf{p}' : Q{E_1}'s \text{ are } {E_2}'s$$

where Q is the quantifier, and E_1,$E_2 \in \mathcal{J}[L]^X$ are interval-valued fuzzy sets.

As in the previous section, we can rewrite this proposition as:

$$\mathbf{p}' : W \text{ is } Q$$

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with W a variable that represents the idea of degree of subsethood of E_2 in E_1.

Using the standard fuzzy intersection, we can express this degree as:

$$W = \\ = \left[\frac{\sum_{x\in X} \min\{l_{E_1}(x), l_{E_2}(x)\}}{\sum_{x\in X} u_{E_1}(x)}, \frac{\sum_{x\in X} \min\{u_{E_1}(x), u_{E_2}(x)\}}{\sum_{x\in X} l_{E_1}(x)} \wedge 1 \right]$$

where $u_{E_1}, u_{E_2}, l_{E_1}$ y l_{E_2} are the upper and lower bounds of the interval-valued observations of E_1 and E_2.

Then, we define the truth value of the proposition $\mathbf{p}'$ as

$$\mathbf{T_r}(\mathbf{p}') = Q(W) = \\ = [\min\{Q(l_W), Q(u_W)\}, \max\{Q(l_W), Q(u_W)\}]$$

where l_W and u_W are the lower and upper bounds of W respectively.

4 Use of the linguistic variables and fuzzy propositions to replace the absent values in Contexts

4.1 Interval-valued L-Fuzzy Contexts with absent values

To extract knowledge from a table with incomplete and ambiguous information, we use the interval-valued L-Fuzzy context as an extension of the L-Fuzzy context given by an implication operator [1, 2] to the interval-valued case.

Definition 2 *Let $\mathcal{J}[L]$ be the set of the closed intervals of a lattice L with the usual order. An* interval-valued L-Fuzzy context *is a tuple $(\mathcal{J}[L], X, Y, R)$, with X and Y two finite sets (of* objects *and* attributes*) and $R \in \mathcal{J}[L]^{X\times Y}$ an interval-valued L-Fuzzy relation between X and Y.*

We are going to see an example of an interval-valued L-Fuzzy context to understand the problem.

Example 1 *In Table 1 we have collected the amounts of the different ingredients (I_i) of a certain food item produced under several trademarks (M_j). As can be seen, there are two unknown values. The table values have been normalized and belong to $\mathcal{J}[L]$, the set of intervals of the lattice $L = \{0, 0.1, 0.2, \dots 0.9, 1\}$.*
This is an example of an interval-valued L-Fuzzy context $(\mathcal{J}[L], X, Y, R)$, where the set of objects X is the set of trademarks or brands, the set of attributes Y is the set of ingredients and the relation R between both is represented in the table.

In the papers developed to study contexts with absent values [3, 4] we have analyzed the implications between attributes associated with labels to obtain good values to replace the absent ones.

In those papers, we represent these absent values by the interval [0,1] and analyze the implications between attributes with the intention of obtaining good values to reduce, as much as possible, the width of that interval.

The support and confidence definitions of an interval-valued implication are based on the idea of association rules [9].

Definition 3 *Let $(\mathcal{J}[L], X, Y, R)$ be an interval-valued L-Fuzzy context. Given the attribute sets $B, C \in \mathcal{J}[L]^Y$, we define the* support *of the implication $B \Rightarrow C$,* $\mathrm{supp}(B \Rightarrow C)$*, as the interval:*

$$\left[\frac{\sum_{x\in X} l_{(B\cup C)_2}(x)}{|X|}, \frac{\sum_{x\in X} u_{(B\cup C)_2}(x)}{|X|} \right]$$

where the membership function of the derived set B_2 is $B_2(x) = [l_{B_2}(x), u_{B_2}(x)]$ and represents the percentage of objects that share the attributes of B and C.

The confidence *of the implication,* $\mathrm{conf}(B \Rightarrow C)$*, is given by the interval:*

$$\left[\frac{\sum_{x\in X} l_{(B\cup C)_2}(x)}{\sum_{x\in X} u_{B_2}(x)}, \frac{\sum_{x\in X} u_{(B\cup C)_2}(x)}{\sum_{x\in X} l_{B_2}(x)} \wedge 1 \right]$$

and represents the percentage of objects that verify the implication, that is, the percentage of objects that having the attributes of B to a certain degree also have those of C to the same degree.

One kind of these implications that we have used are the implications associated with labels [3]. They are denoted by $y_{i_r} \Longrightarrow y_{j_s}$ where y_i and y_j are attributes of the context and r, s labels. In the calculation process, a function G_s (based on the distance between intervals) to measure the degree of similarity between the different labels and the attributes is needed. In addition, once the implications with highest support and confidence have been chosen, the absent values are replaced by intervals, in general, with less amplitude than [0,1].

In this work, we are going to approach all this process, in an easy way, using interval-valued linguistic variables and fuzzy propositions.

The implication between attributes $y_{i_r} \Longrightarrow y_{j_s}$ can be considered as a quantified fuzzy proposition since the expression:

$\mathbf{p}$: Most of those that have y_i(high, medium, low…)
also have y_j (high, medium, low…)

can be interpreted in the sense of section 3.2:

The quantifier Q is *most* and E_1,E_2 are interval-valued linguistic variables resulting of applying the linguistic variable (with values *high, medium, low…*) to the columns of the context. The quantifier Q will be applied to both of the bounds of the intervals. In this way, we can calculate the truth value of the proposition:

$$\mathbf{T_r}(\mathbf{p}) = Q(W) = \\ = [\min\{Q(l_W), Q(u_W)\}, \max\{Q(l_W), Q(u_W)\}]$$

4.2 Substitution of the absent values using the compatibility function of an interval-valued linguistic variable and the interval-valued fuzzy propositions.

In the study of contexts with absent values, we begin replacing these absent values by [0,1]. Next, we are going to define an interval-valued linguistic variable $(\mathbf{V}, T(V), \mathcal{J}[U], G, M)$, and use the interval-valued fuzzy propositions to estimate the

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Table 1: Amounts of the different ingredients under several trademarks.

R	$I1$	$I2$	$I3$	$I4$	$I5$	$I6$
$M1$	$[0.8, 1]$	$[0.1, 0.1]$	$[0.6, 0.6]$	$[0.2, 0.2]$	$[0.4, 0.4]$	$[0.4, 0.6]$
$M2$	$[0.8, 0.8]$		$[0.4, 0.6]$	$[0.4, 0.4]$	$[0.2, 0.2]$	$[0.6, 0.6]$
$M3$	[1,1]	[0,0]	$[0.2, 0.2]$	$[0.5, 0.5]$	$[0.4, 0.4]$	$[0.4, 0.4]$
$M4$	$[0.9, 0.9]$	$[0.2, 0.2]$	$[0, 0]$	$[0.4, 0.4]$	$[0.1, 0.2]$	
$M5$	$[0.8, 0.8]$	$[0, 0.2]$	$[0.4, 0.4]$	$[0.2, 0.2]$	$[0.2, 0.4]$	$[0.2, 0.2]$

absent values $R(x_i, y_j)$ of the interval-valued L-Fuzzy context by intervals with less amplitude than that of [0,1].

The compatibility function of the interval-valued linguistic variable will be used to establish the new columns in the table that measure the similarity between some attributes and the values or labels $t \in T(V)$ of the linguistic variable.

Example 2 *In the previous context, the two absent values ($R(M2, I2)$ and $R(M4, I6)$) have been replaced by the interval [0,1].*

We are going to define a linguistic variable ***V****, with labels* $T(V) = \{$high, medium, low$\}$.

Now, we take the label $t =$medium $\in T(V)$ *represented by Fig. 2,*

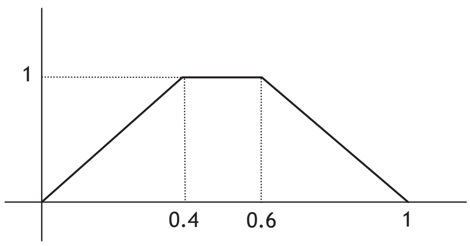

Figure 2: L-Fuzzy set associated with the label t

where the compatibility of the value $x \in X = [0, 1]$ *with the label* $t =$medium *has the following expression:*

$$x_t = \begin{cases} \dfrac{x}{0.4} & \text{if } x \leq 0.4 \\ 1 & \text{if } 0.4 \leq x \leq 0.6 \\ \dfrac{1-x}{1-0.6} & \text{if } 0.6 \leq x \end{cases}$$

and the interval-valued compatibility is defined, $\forall [\alpha, \beta] \in \mathcal{J}[X]$, *by:*

$$[\alpha, \beta]_t = \begin{cases} [\min\{\alpha_t, \beta_t\}, \max\{\alpha_t, \beta_t\}] & \text{if } \alpha \geq 0.6 \text{ or } \beta \leq 0.4 \\ [\min\{\alpha_t, \beta_t\}, 1] & \text{in other case} \end{cases}$$

From this point, we can extend the table with a new column $I6_{medium}$ *where the compatibility of the attribute* $I6$ *with the label* $t = medium$ *is valued, obtaining Table 2.*

Remark. Sometimes, the obtained values do not belong to the lattice $L = \{0, 0.1, 0.2, \ldots 0.9, 1\}$. Then, we will round the fractions to the nearest element of L.

One of the advantages of the use of linguistic variables to establish the compatibility between the values of the relation R and the labels is the maintenance of the unknowledge. It is possible to observe this property in the example where $R(M4, I6) = [0, 1] = \widehat{R}(M4, I6_{medium})$ and it is certain for any label $t \in T(V)$, as it was proven in [5].

On the other hand, the values here obtained are not very different from which turned out applying the techniques developed in [3] which can be seen in the Table 3. Nevertheless, we consider that the use of linguistic variables solves the problem in a simpler and more natural way.

Table 3: Values obtained from a previous method.

	$I6_{medium}$
$M1$	$[0.8, 1]$
$M2$	$[0.8, 1]$
$M3$	$[0.8, 1]$
$M4$	$[0.4, 1]$
$M5$	$[0.6, 0.8]$

Once extended the table, the interval-valued fuzzy propositions related to the new attribute $I6_{medium}$ can be calculated to find those that have highest values. In this case, this is the proposition $I1 \Rightarrow I6_{medium}$:

$\mathbf{p}$: *Most of those that have* $I1$ *also have* $I6_{medium}$

Then, $Q =$most, $E_1 = I1$, $E_2 = I6_{\text{medium}}$, where

$$W = \left[\frac{3.1}{4.5}, \frac{4.2}{4.3} \wedge 1\right] = [0.68, 0.97]$$

Now, if we take the quantifier Q defined by ([10]):

$$Q(x) = \begin{cases} 1.25x & \text{if } x \leq 0.8 \\ 1 & \text{in other case} \end{cases}$$

and we apply to both bounds of the interval, then we have:

$$Q([0.68, 0.97]) = [Q(0.68), Q(0.97)] = [0.85, 1]$$

that is the truth value of the proposition $\mathbf{p}$.

The obtained results are similar to those with implications between attributes but, in this case, the process is easier.

We can conclude that, in many cases (at least 85%), the ingredient $I1$ is associated with a medium amount of ingredient $I6$.

Given the high percentage of cases in which the implication is valid, we suppose that it also holds for brand $M4$. Therefore we expect that:

$$\widehat{R}(M4, I6_{medium}) \geq \widehat{R}(M4, I1) = [0.9, 0.9]$$

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Table 2: New relation $\widehat{R}$.

$\widehat{R}$	$I1$	$I2$	$I3$	$I4$	$I5$	$I6$	$I6_{medium}$
$M1$	$[0.8, 1]$	$[0.1, 0.1]$	$[0.6, 0.6]$	$[0.2, 0.2]$	$[0.4, 0.4]$	$[0.4, 0.6]$	$[1, 1]$
$M2$	$[0.8, 0.8]$	$[0, 1]$	$[0.4, 0.6]$	$[0.4, 0.4]$	$[0.2, 0.2]$	$[0.6, 0.6]$	$[1, 1]$
$M3$	[1,1]	[0,0]	$[0.2, 0.2]$	$[0.5, 0.5]$	$[0.4, 0.4]$	$[0.4, 0.4]$	$[1, 1]$
$M4$	$[0.9, 0.9]$	$[0.2, 0.2]$	$[0, 0]$	$[0.4, 0.4]$	$[0.1, 0.2]$	$[0, 1]$	$[0, 1]$
$M5$	$[0.8, 0.8]$	$[0, 0.2]$	$[0.4, 0.4]$	$[0.2, 0.2]$	$[0.2, 0.4]$	$[0.2, 0.2]$	$[0.5, 0.5]$

The election of the attributes and labels to obtain interval-valued fuzzy propositions with high truth values is very important because we will have then good values of substitution for the absent ones.
At this point, we have obtained the value $\widehat{R}(M4, I6_{medium}) = [0.9, 0.9]$. Now, using this value we want to get the best value to replace the initial absent value $R(M4, I6)$. To do this, denoting by $R(x_i, y_j) = [\alpha, \beta]$ the unknown value, we will use the following proposition associated with compatibility functions to obtain this value:

Proposition 1 *$\forall x \in [0,1]$ and $[\alpha, \beta] \in \mathcal{J}[0,1]$ it is verified that*

$$[\alpha, \beta]_t \subseteq [x, 1] \Leftrightarrow [\alpha, \beta] \subseteq [ax, 1 - x(1-b)]$$

where $a, b \in [0,1]$ are the values that appear in the fuzzy set associated with the label $t \in T(V)$.

Proof

$\Leftarrow$) Let us suppose that $[\alpha, \beta] \subseteq [ax, 1 - x(1-b)]$.
In [5], we have proven that:

If $[\alpha, \beta] \subseteq [ax, 1 - x(1-b)]$ holds,

then

$$[\alpha, \beta]_t \subseteq [ax, 1 - x(1-b)]_t$$

Moreover, for all $x \in [0,1]$, it is verified

$$\left.\begin{array}{l} ax \leq a \\ 1 - x(1-b) \geq b \end{array}\right\} \Rightarrow$$

Therefore,

$$\begin{aligned} [\alpha, \beta]_t &\subseteq [ax, 1 - x(1-b)]_t = \\ &= [\min\{(ax)_t, (1 - x(1-b))_t\}, 1] = \\ &= \left[\min\left\{\frac{ax}{a}, \frac{1 - (1 - x(1-b))}{1-b}\right\}, 1\right] = [x, 1] \end{aligned}$$

$\Rightarrow$) By reduction to the absurd, suppose that

$$[\alpha, \beta] \nsubseteq [ax, 1 - x(1-b)]$$

then we have $\alpha < ax$ or $\beta > 1 - x(1-b)$.

- If $\alpha < ax < a$, then:

$$\alpha_t = \frac{\alpha}{a} < \frac{ax}{a} = x \Rightarrow \min\{\alpha_t, \beta_t\} < x$$

- If $\beta > 1 - x(1-b) > b$, then:

$$\beta_t = \frac{1-\beta}{1-b} < \frac{1 - (1 - x(1-b))}{1-b} = x \Rightarrow$$
$$\Rightarrow \min\{\alpha_t, \beta_t\} < x$$

Therefore $[\alpha, \beta]_t \nsubseteq [x, 1]$.

Remark. Really, we are only interested in the implication

$$[\alpha, \beta]_t \subseteq [x, 1] \Rightarrow [\alpha, \beta] \subseteq [ax, 1 - x(1-b)]$$

To replace the absent value, we will take the widest interval verifying the condition, that is, $[ax, 1 - x(1-b)]$.

As $[a, b] \subseteq [ax, 1 - x(1-b)]$, we have to avoid to take labels with a wide amplitude of the interval $[a, b]$ because, in other case, the interval $[ax, 1 - x(1-b)]$ will be also wide.

Example 3 *Returning to the example, if we apply the previous proposition associated with the label t=medium (with the values $a = 0.4$ and $b = 0.6$), then we obtain*

$$R(M_4, I_6) = [0.36, 0.64]$$

that will be the interval with which we will replace the absent value (also it will be necessary here to round the results to the nearest element of L). The final result given in [3] for this same absent value $R(M_4, I_6)$ was $[0.5, 0.5]$.
In both cases we have a fuzzy value distant from 0 and 1.

Finally, the other unknown value, $R(M2, I2)$ can be replaced using interval-valued linguistic variables and fuzzy propositions by the interval [0,0.2]. In this case, the proposition with the highest value is

p : *Most of those that have $I1_{high}$ also have $I2_{low}$*

5 Conclusions and future work

The use of linguistic variables and fuzzy propositions in the field of contexts with absent values supposes a simplification of the process of replacement of these values. The old method, based on implication between attributes, needs the calculus of functions to measure the similarity between attributes and labels that makes difficult the process.
In future works we will analyze its reliability.

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