Type-2 Fuzzy Trust Aggregation

John T. Rickard[1] Sam Hamilton[2] Wendy Hamilton[3]

1 Distributed Infinity, Inc., 4637 Shoshone Drive,
Larkspur, CO 80118, USA
2, 3 Distributed Infinity, Inc., 4033 Nabal Drive
La Mesa, CA 91941, USA
Email: {trickard, shamilton, whamilton@distributedinfinity.com}

Abstract— *Trust is a fundamental concept that is critical in human decision processes in almost all domains, but of particular relevance in the domain of computer security. In many computer systems deployed today, trust is only modeled indirectly through a serious of formal rules and regulations describing when privileges are to be granted or revoked. Recently, research has been conducted into the area of fuzzy trust modeling in order to allow a more intelligent tradeoff analysis by the computer security software. In this paper, we extend this work to support Type-2 fuzzy reasoning. The Type-2 input membership functions can be derived from multiple human experts' judgments. This allows us to incorporate the inherent imprecision in trust judgments in a mathematically principled manner.*

Keywords— Computer security, fuzzy systems, trust modeling, Type-2 fuzzy logic.

1 Introduction

In computer security, trust is described as the belief in the competence of an entity to act dependably, reliably and securely within a specific context, and distrust is described as the belief in the incompetence of an entity to act dependably, securely and reliably within a specific context [1]. In many computer systems deployed today, trust is only modeled indirectly through a serious of formal rules and regulations describing when privileges are to be granted or revoked. Manual human judgment is invoked whenever the system falls into an ambiguous state. For example, if a user tries to log on with an incorrect password too many times, many systems will revoke privileges for that user account and rely on the system administrator for the judgment of when it is appropriate to reissue those privileges.

This approach has significant weaknesses in that its strong reliance on a human in the decision process requires the Boolean security rule set to make inappropriately crisp statements, which either overly restrict users (reducing productivity) or are not restrictive enough (reducing security). Recently, research has been conducted into augmenting policy rules with Type-1 fuzzy trust modeling to help address this issue by softening the hard and fast set of computer security policy rules to allow a more intelligent tradeoff analysis [2-7].

The notion of trust relationships is traditionally modeled in a fuzzy sense, i.e., "entity A trusts entity B to a degree $x \in [0,1]$," where x is usually determined by the historical results of entity $A's$ interactions with entity B. Trust chains may also be involved in this determination, e.g., entity A can consult entity C as to its trust in entity B and factor that information into its decision. A discrete set K of contexts may also enter into this description, resulting in an ordered pair (x, y) describing trust, indicating that entity A trusts entity B to a degree $x \in [0,1]$ in a particular context $y \in K$.

The contribution of each context factor may be measured independently using two numbers, a *confidence* $c \in [-1,1]$ and a *plausibility* $p \in [0,1]$. The confidence c describes entity $A's$ belief in a particular outcome either *occurring* or *not occurring* (we use negative values to indicate the latter) in an interaction with entity B in a given context, and the plausibility p describes the degree to which entity A considers her confidence assessment to be reliable. For example, entity A might believe with high confidence (e.g., 0.8) in a particular outcome's occurrence, but if this belief is based upon little or no historical evidence, then her plausibility might be low (say, 0.1). Upon the accumulation of further direct evidence as a result of her interactions with entity B and/or consultation with other entities, she may simultaneously refine her confidence estimate and increase her plausibility in this refined confidence. Thus we treat these as independent variables.

Hamilton and Hamilton [8] developed an algebra for combining such assessments, based upon *singleton values* of input confidences and plausibilities, and producing a corresponding singleton aggregate confidence and plausibility. However, it is well known that human assessments of situations are pervasively imprecise, and thus humans are more reliable at estimating *intervals* of confidence or plausibility rather than point values. Likewise, the results of computer simulations are better described by an *interval* of outcomes than by a single number. Furthermore, it is often desirable to solicit such intervals from multiple experts, or to perform multiple simulations. In this case, the problem remains as to how to incorporate this multiplicity of interval inputs into the trust system. Multiple interval inputs can be combined into interval Type-2 input membership functions using the techniques described in [17].

In this paper, we extend the results of [8] initially to individual interval inputs of confidence and plausibility, to compute corresponding interval output membership functions for their aggregation. Then, employing α-cuts, we further extend these results to accommodate Type-1 and

ISBN: 978-989-95079-6-8

interval Type-2 fuzzy inputs. This allows us to propagate imprecision in our knowledge of the inputs to the corresponding imprecision in our knowledge of the output in a mathematically principled fashion.

2 Trust aggregation

Suppose that we have two assessments of confidence and plausibility relating to our trust in a particular context factor, which we denote by A_{c_1,p_1} and A_{c_2,p_2}, respectively, where c_1 and c_2 denote the confidences, and p_1 and p_2 the plausibilities, of the corresponding factor. Suppose further that we wish to perform an aggregation of these values, denoted by the operator "$\wedge$", to yield a sensible aggregate confidence and plausibility based upon these two inputs. Thus we wish to describe $A_{c_3,p_3} = A_{c_1,p_1} \wedge A_{c_2,p_2}$ in terms of its aggregate confidence c_3 and plausibility p_3.

Hamilton and Hamilton [8] propose the following algebraic computations for c_3 and p_3:

$$c_3 = \frac{c_1 p_1^\alpha + c_2 p_2^\alpha}{p_1^\alpha + p_2^\alpha} \tag{1}$$

$$p_3 = \frac{(p_1 + p_2 - p_1 p_2)\left(2 - |c_1 - c_2|\right)^\beta + (p_1 + p_2 - 2p_1 p_2)|c_1 - c_2|^\beta}{\left(2 - |c_1 - c_2|\right)^\beta + |c_1 - c_2|^\beta} \tag{2}$$

The parameters $\alpha > 0$ and $\beta > 0$ govern the degree of influence of the input plausibilities p_1 and p_2 upon the aggregate confidence c_3, and the degree of influence of the *differences* in the two input confidences c_1 and c_2 upon the aggregate plausibility p_3, respectively. Typical values for these parameters might be $\alpha = 2$ and $\beta = 1/4$. It is straightforward to show that when the domains of the inputs are $c_1, c_2 \in [-1,1]$ and $p_1, p_2 \in [0,1]$, the resulting ranges for the aggregate confidence and plausibility are likewise $c_3 \in [-1,1]$ and $p_3 \in [0,1]$.

From (1), we see that the aggregate confidence c_3 is a simple convex combination of the input confidences, with weights proportional to the α-power of the plausibilities. The aggregate plausibility p_3 is a more complicated convex combination of two functions of the input plausibilities. At one extreme, when the input confidences are in perfect agreement, the aggregate plausibility is given by $p_3 = p_1 + p_2 - p_1 p_2 \; (c_1 = c_2)$. At the other extreme where the confidences are diametrically opposite and of unity magnitude, the aggregate plausibility is given by $p_3 = p_1 + p_2 - 2p_1 p_2, \; (c_1 = \pm 1, \; c_2 = -c_1)$.

To illustrate further the behaviors of (1) and (2), consider the following examples, where $\alpha = 2$ and $\beta = 1/4$:

$$A_{0.8,0} \wedge A_{0.3,0.5} = A_{0.3,0.5} \tag{3}$$

$$A_{-0.8,0.9} \wedge A_{0.8,0.9} = A_{0,0.5} \tag{4}$$

$$A_{0.8,0.3} \wedge A_{0.7,0.4} = A_{0.74,0.54} \tag{5}$$

In the first case, an assessment with zero plausibility adds no new information. In the second case, strongly opposing confidences reduce aggregate plausibility. In the final case, similar confidences reinforce aggregate plausibility.

A single fuzzy scalar value of our degree of trust $t\left(A_{c,p}\right)$ can be computed as the following convex combination involving confidence and plausibility:

$$t\left(A_{c,p}\right) = \frac{0.5(1-p)^\gamma + \frac{(1+c)}{2} p^\gamma}{(1-p)^\gamma + p^\gamma} \tag{6}$$

where $\gamma > 0$ controls the degree of influence of the plausibility upon the trust. A typical value might be $\gamma = 2$. Note that if $p = 0$, then $t\left(A_{c,p}\right) = 0.5$, i.e., we have a maximally ambiguous degree of trust (i.e., the "coin toss" situation), whereas if $p = 1$, then $t\left(A_{c,p}\right) = \frac{1+c}{2}$, (i.e., trust is equivalent to a rescaling of the confidence c to lie in the interval [0,1].) In the extremes of this latter case, if $c = -1$, we have zero trust in the given outcome, while if $c = 1$, we have complete trust in the given outcome.

3 Interval inputs

Suppose now that, rather than the point values of the inputs described above, we instead have interval values. Interval inputs might naturally arise from soliciting them from a human expert, or as the results of a computer simulation that includes the uncertainty of the simulations results. Specifically, let $c_1 \in [c_{1\ell}, c_{1r}]$, $c_2 \in [c_{2\ell}, c_{2r}]$, $p_1 \in [p_{1\ell}, p_{1r}]$ and $p_2 \in [p_{2\ell}, p_{2r}]$, where the subscripts ℓ and r refer to the left- and right-hand endpoints of the intervals, respectively. Bear in mind that the c intervals are subsets of $[-1,1]$, and the p intervals are subsets of $[0,1]$.

3.1 Confidence aggregation for interval inputs

From (1), c_3 is an interval weighted average [9], where the input variables take values continuously over interval ranges. Thus c_3 takes values continuously over an interval range, which we denote by $c_3 \in [c_{3\ell}, c_{3r}]$, where

$$c_{3\ell} = \min_{c_1,c_2,p_1,p_2} \frac{c_1 p_1^\alpha + c_2 p_2^\alpha}{p_1^\alpha + p_2^\alpha} = \min_{p_1,p_2} \frac{c_{1\ell} p_1^\alpha + c_{2\ell} p_2^\alpha}{p_1^\alpha + p_2^\alpha}, \tag{7}$$

$$c_{3r} = \max_{c_1,c_2,p_1,p_2} \frac{c_1 p_1^\alpha + c_2 p_2^\alpha}{p_1^\alpha + p_2^\alpha} = \max_{p_1,p_2} \frac{c_{1r} p_1^\alpha + c_{2r} p_2^\alpha}{p_1^\alpha + p_2^\alpha}. \tag{8}$$

It is well-known [10] how to solve this simple case of a two-element interval weighted average. We first re-index the triplets $(c_{i\ell}, p_{i\ell}, p_{ir})$ in order of increasing $c_{i\ell}$, i.e., to obtain $c_{1\ell} \le c_{2\ell}$. Then using these re-indexed triplets, $c_{3\ell}$ is given by

$$c_{3\ell} = \frac{c_{1\ell} p_{1r}^\alpha + c_{2\ell} p_{2\ell}^\alpha}{p_{1r}^\alpha + p_{2\ell}^\alpha}, \tag{9}$$

which is recognized as a closed-form two-element solution of the Karnik-Mendel (KM) algorithm [10], which is used more generally to find the endpoints of the output interval corresponding to an $n-$element interval weighted average. In similar fashion, after re-indexing the triplets $(c_{ir}, p_{i\ell}, p_{ir})$ in order of increasing c_{ir}, we have

ISBN: 978-989-95079-6-8

$$c_{3r} = \frac{c_{1r} p_{1\ell}^{\alpha} + c_{2r} p_{2r}^{\alpha}}{p_{1\ell}^{\alpha} + p_{2r}^{\alpha}}. \tag{10}$$

Thus we have a closed-form analytical solution for the output interval of $c_3 \in [c_{3\ell}, c_{3r}]$ in (1) when the inputs have interval values.

3.2 *Plausibility aggregation for interval inputs*

From (2), p_3 is a continuous function of its input values, so it too will take values over an interval $p_3 \in [p_{3\ell}, p_{3r}]$. By inspection of (2), we may re-write this equation as

$$p_3 = \lambda(p_1 + p_2 - p_1 p_2) + (1-\lambda)(p_1 + p_2 - 2p_1 p_2) \tag{11}$$

where $\lambda \in [0,1]$ is a function of c_1 and c_2 given by

$$\lambda(c_1, c_2) = \frac{\left(2 - |c_1 - c_2|\right)^{\beta}}{\left(2 - |c_1 - c_2|\right)^{\beta} + |c_1 - c_2|^{\beta}}. \tag{12}$$

Taking partial derivatives in (11) with respect to p_1 and p_2, we obtain

$$\frac{\partial p_3}{\partial p_1} = 1 - (2-\lambda) p_2 \begin{cases} > 0 \ \textit{for} \ p_2 < \dfrac{1}{2-\lambda} \\ < 0 \ \textit{for} \ p_2 > \dfrac{1}{2-\lambda} \end{cases}, \tag{13}$$

and

$$\frac{\partial p_3}{\partial p_2} = 1 - (2-\lambda) p_1 \begin{cases} > 0 \ \textit{for} \ p_1 < \dfrac{1}{2-\lambda} \\ < 0 \ \textit{for} \ p_1 > \dfrac{1}{2-\lambda} \end{cases}. \tag{14}$$

Given values of c_1 and c_2, and hence a given value of λ, we see that p_3 is monotonically increasing or decreasing as a function of p_1 and p_2 away from the saddle point $\left(\frac{1}{2-\lambda}, \frac{1}{2-\lambda}\right)$, which lies in the upper right-hand quadrant of the unit square since $\lambda \in [0,1]$. Thus the minimum (maximum) of (11) as a function of p_1 and p_2 always occurs at one of the four input interval extremal points $(p_{1\ell}, p_{2\ell})$, $(p_{1\ell}, p_{2r})$, $(p_{1r}, p_{2\ell})$ or (p_{1r}, p_{2r}).

To continue this analysis, let us define four functions $f_{ij}(c_1, c_2)$, $i \in \{\ell, r\}$, $j \in \{\ell, r\}$ such that

$$f_{ij}(c_1, c_2) = \frac{(p_{1i} + p_{2j} - p_{1i} p_{2j})\left(2 - |c_1 - c_2|\right)^{\beta} + (p_{1i} + p_{2j} - 2p_{1i} p_{2j})|c_1 - c_2|^{\beta}}{\left(2 - |c_1 - c_2|\right)^{\beta} + |c_1 - c_2|^{\beta}} \tag{15}$$

It then remains to calculate

$$p_{3\ell} = \min_{i,j \in \{\ell, r\}} \left\{ \min_{c_1, c_2 \in [-1,1]} \frac{\pi_{ij}^{(1)} \left(2 - |c_1 - c_2|\right)^{\beta} + \pi_{ij}^{(2)} |c_1 - c_2|^{\beta}}{\left(2 - |c_1 - c_2|\right)^{\beta} + |c_1 - c_2|^{\beta}} \right\} \tag{16}$$

$$p_{3r} = \max_{i,j \in \{\ell, r\}} \left\{ \max_{c_1, c_2 \in [-1,1]} \frac{\pi_{ij}^{(1)} \left(2 - |c_1 - c_2|\right)^{\beta} + \pi_{ij}^{(2)} |c_1 - c_2|^{\beta}}{\left(2 - |c_1 - c_2|\right)^{\beta} + |c_1 - c_2|^{\beta}} \right\} \tag{17}$$

where

$$\pi_{ij}^{(1)} = p_{1i} + p_{2j} - p_{1i} p_{2j} \tag{18}$$

$$\pi_{ij}^{(2)} = p_{1i} + p_{2j} - 2p_{1i} p_{2j} \tag{19}$$

To this end, we employ the continuous change of variables $u = |c_1 - c_2|$, which reduces these equations to:

$$\begin{aligned} p_{3\ell} &= \min_{i,j \in \{\ell, r\}} \left\{ \min_{u \in [0,2]} \frac{\pi_{ij}^{(1)} (2-u)^{\beta} + \pi_{ij}^{(2)} u^{\beta}}{(2-u)^{\beta} + u^{\beta}} \right\} \\ &\triangleq \min_{i,j \in \{\ell, r\}} \left\{ \min_{u \in [0,2]} g_{ij}^{(\min)}(u) \right\} \end{aligned} \tag{20}$$

$$\begin{aligned} p_{3r} &= \max_{i,j \in \{\ell, r\}} \left\{ \max_{u \in [0,2]} \frac{\pi_{ij}^{(1)} (2-u)^{\beta} + \pi_{ij}^{(2)} u^{\beta}}{(2-u)^{\beta} + u^{\beta}} \right\} \\ &\triangleq \max_{i,j \in \{\ell, r\}} \left\{ \max_{u \in [0,2]} g_{ij}^{(\max)}(u) \right\} \end{aligned} \tag{21}$$

It is obvious by inspection of (18) and (19) that for any of the four combinations of $(i,j): i,j = 1,2$, $\pi_{ij}^{(1)} \geq \pi_{ij}^{(2)}$, so from (20) and (21) we observe that each of the functions $g_{ij}^{(\min)}(u)$ and $g_{ij}^{(\max)}(u)$ is a convex combination (whose coefficients are nonlinear functions of u over the interval $[0,2]$) of the values $\pi_{ij}^{(1)}$ and $\pi_{ij}^{(2)}$, having its maximum at $u = 0$ and its minimum at $u = 2$. We can show that the derivatives of these functions are uniformly non-positive over the interval $[0,2]$, and thus the functions $g_{ij}^{(\min)}(u)$ and $g_{ij}^{(\max)}(u)$ are uniformly non-increasing over any sub-interval of $[0,2]$. Therefore their minimum with respect to u in (20) over an interval $[u_\ell, u_r] \subseteq [0,2]$ will correspond to u_r, the right endpoint point of this interval, and their maximum with respect to u in (21) will correspond to u_ℓ, the left endpoint of this interval.

To determine the associated interval for u from the intervals $c_1 \in [c_{1\ell}, c_{1r}]$ and $c_2 \in [c_{2\ell}, c_{2r}]$, we must consider six cases of different interval overlap scenarios, depicted in Table 1.

Table 1: Interval overlap cases.

Case	**Interval Positions**	**Interval for** $u = \lvert c_1 - c_2 \rvert$
1	$c_{1r} < c_{2\ell}$	$u \in [c_{2\ell} - c_{1r}, c_{2r} - c_{1\ell}]$
2	$c_{1\ell} \leq c_{2\ell} \leq c_{1r} \leq c_{2r}$	$u \in [0, c_{2r} - c_{1\ell}]$
3	$c_{1\ell} \leq c_{2\ell} \leq c_{2r} \leq c_{1r}$	$u \in [0, \max(c_{1r} - c_{2\ell}, c_{2r} - c_{1\ell})]$
4	$c_{2\ell} \leq c_{1\ell} \leq c_{2r} \leq c_{1r}$	$u \in [0, c_{1r} - c_{2\ell}]$
5	$c_{2\ell} \leq c_{2r} \leq c_{1\ell} \leq c_{1r}$	$u \in [c_{1\ell} - c_{2r}, c_{1r} - c_{2\ell}]$
6	$c_{2\ell} \leq c_{1\ell} \leq c_{1r} \leq c_{2r}$	$u \in [0, \max(c_{2r} - c_{1\ell}, c_{1r} - c_{2\ell})]$

As noted above, in all cases, $[u_\ell, u_r] \subseteq [0,2]$, and due to the monotone non-increasing nature of the functions $g_{\min}(u)$ and $g_{\max}(u)$, we finally have

ISBN: 978-989-95079-6-8

$$p_{3\ell} = \min_{i,j\in\{\ell,r\}} \left\{ \min_{u\in[u_\ell,u_r]} g_{ij}^{(\min)}(u) \right\} = \min_{i,j\in\{\ell,r\}} \left\{ \frac{\pi_{ij}^{(1)} \left(2-u_r\right)^\beta + \pi_{ij}^{(2)} u_r^\beta}{\left(2-u_r\right)^\beta + u_r^\beta} \right\}, \tag{22}$$

$$p_{3r} = \max_{i,j\in\{\ell,r\}} \left\{ \max_{u\in[u_\ell,u_r]} g_{ij}^{(\max)}(u) \right\} = \max_{i,j\in\{\ell,r\}} \left\{ \frac{\pi_{ij}^{(1)} \left(2-u_\ell\right)^\beta + \pi_{ij}^{(2)} u_\ell^\beta}{\left(2-u_\ell\right)^\beta + u_\ell^\beta} \right\}. \tag{23}$$

Thus to find the interval boundaries for p_3, we simply must find the minimum (maximum) of the four function evaluations involved in (22) and (23).

3.3 *Interval calculations for trust*

As a final step in extending our results to interval values, we consider the trust function of equation (6) in the event that $c\in\left[c_\ell,c_r\right]$ and $p\in\left[p_\ell,p_r\right]$ are described by interval values. Since c appears only in the numerator, it is obvious by inspection that

$$\min t\left(A_{c,p}\right) = \min_{p\in[p_\ell,p_r]} \frac{0.5(1-p)^\gamma + \frac{\left(1+c_\ell\right)}{2} p^\gamma}{(1-p)^\gamma + p^\gamma} \tag{24}$$

$$\max t\left(A_{c,p}\right) = \max_{p\in[p_\ell,p_r]} \frac{0.5(1-p)^\gamma + \frac{\left(1+c_r\right)}{2} p^\gamma}{(1-p)^\gamma + p^\gamma} \tag{25}$$

Analogous to the functions $g_{ij}^{(\min)}(u)$ and $g_{ij}^{(\max)}(u)$ of the previous sections, we are here dealing with the minimum or maximum of a nonlinear convex combination of the values 0.5 and $(1+c)/2$ over the interval $p\in\left[p_\ell,p_r\right]$. We can show that the derivative of (6) with respect to p is monotonic, so that the minimum (maximum) of the above expressions occurs either at $p=p_\ell$ or $p=p_r$ depending upon the value of c (for $c<0$ the derivative is negative and for $c>0$ it is positive), with the result that $t\in\left[t_\ell,t_r\right]$, where

$$t_\ell = \min_{p\in[p_\ell,p_r]} t\left(A_{c,p}\right) = \begin{cases} \dfrac{0.5(1-p_r)^\gamma + \frac{\left(1+c_\ell\right)}{2} p_r^\gamma}{(1-p_r)^\gamma + p_r^\gamma}, & c_\ell < 0 \\ \dfrac{0.5(1-p_\ell)^\gamma + \frac{\left(1+c_\ell\right)}{2} p_\ell^\gamma}{(1-p_\ell)^\gamma + p_\ell^\gamma}, & c_\ell \ge 0 \end{cases} \tag{26}$$

$$t_r = \max_{p\in[p_\ell,p_r]} t\left(A_{c,p}\right) = \begin{cases} \dfrac{0.5(1-p_\ell)^\gamma + \frac{\left(1+c_r\right)}{2} p_\ell^\gamma}{(1-p_\ell)^\gamma + p_\ell^\gamma}, & c_r < 0 \\ \dfrac{0.5(1-p_r)^\gamma + \frac{\left(1+c_r\right)}{2} p_r^\gamma}{(1-p_r)^\gamma + p_r^\gamma}, & c_r \ge 0 \end{cases} \tag{27}$$

4 Type-1 fuzzy inputs

Suppose that instead of interval membership functions, the fuzzy membership functions for c_1, c_2, p_1 and p_2 in equations (1) and (2) are general convex Type-1 membership functions $\mu_{c_1}(x)$, $\mu_{c_2}(x)$, $\mu_{p_1}(x)$ and $\mu_{p_2}(x)$, respectively, which may not necessarily be normal (i.e., the maximum degree, or height, of the membership function is not necessarily unity). The approach we take in this case is similar to that used to analyze fuzzy weighted averages [9] via α-cuts of the input membership functions, as corrected in [10]. (Note that the term "α-cuts", as commonly used in the fuzzy systems literature, bears no relationship to the α exponent used in equation (1).)

We recall that that an α-cut of a convex fuzzy membership function $\mu(x)$ is an interval on the independent variable axis defined by

$$\left[x(\alpha)_\ell, x(\alpha)_r\right] = \left\{x : \mu(x) \ge \alpha\right\}. \tag{28}$$

We may then represent $\mu(x)$ in terms of its α-cuts by [11]

$$\mu(x) = \sup_{\alpha\in[0,1]} \alpha I_{\mu_\alpha}(x), \tag{29}$$

where $I_{\mu_\alpha}(x)$ is the indicator function for the α-cuts, i.e.,

$$I_{\mu_\alpha}(x) = \begin{cases} 1, & x\in\left[x(\alpha)_\ell, x(\alpha)_r\right] \\ 0, & otherwise \end{cases}. \tag{30}$$

Note that the α-cuts of a fuzzy membership function exist for $0\le\alpha\le\alpha_{\max}\le 1$, where $\alpha_{\max}$ is the height of $\mu(x)$. Thus, given the α-cuts

$$\left[x(\alpha)_{c_1\ell}, x(\alpha)_{c_1 r}\right] = \left\{x : \mu_{c_1}(x) \ge \alpha\right\} \tag{31}$$

$$\left[x(\alpha)_{c_2\ell}, x(\alpha)_{c_2 r}\right] = \left\{x : \mu_{c_2}(x) \ge \alpha\right\} \tag{32}$$

$$\left[x(\alpha)_{p_1\ell}, x(\alpha)_{p_1 r}\right] = \left\{x : \mu_{p_1}(x) \ge \alpha\right\} \tag{33}$$

$$\left[x(\alpha)_{p_2\ell}, x(\alpha)_{p_2 r}\right] = \left\{x : \mu_{p_2}(x) \ge \alpha\right\} \tag{34}$$

we can calculate the corresponding α-cuts $\left[x(\alpha)_{c_3\ell}, x(\alpha)_{c_3 r}\right]$ and $\left[x(\alpha)_{p_3\ell}, x(\alpha)_{p_3 r}\right]$ of the fuzzy membership functions $\mu_{c_3}(x)$ and $\mu_{p_3}(x)$ for $0\le\alpha\le\min_i \alpha_{\max i}$ using (9)-(10) and (22)-(23), with the input α-cut intervals specified by (31)-(34). Then the Type-1 memberships for $\mu_{c_3}(x)$ and $\mu_{p_3}(x)$ are given by:

$$\mu_{c_3}(x) = \sup_{\forall\alpha\in[0,\alpha_{\max}]} \begin{cases} \alpha, & x(\alpha)_{c_3\ell} \le x \le x(\alpha)_{c_3 r} \\ 0 & elsewhere \end{cases}, \tag{35}$$

$$\mu_{p_3}(x) = \sup_{\forall\alpha\in[0,\alpha_{\max}]} \begin{cases} \alpha, & x(\alpha)_{p_3\ell} \le x \le x(\alpha)_{p_3 r} \\ 0 & elsewhere \end{cases}, \tag{36}$$

where $\alpha_{\max} = \min_i \alpha_{\max i}$. In a similar vein, we can compute the fuzzy membership function $\mu_t(x)$ of trust from the α-cut intervals for $c\in\left[c_\ell,c_r\right]$ and $p\in\left[p_\ell,p_r\right]$ using (26)-(27) to compute the corresponding α-cut intervals $\left[x(\alpha)_{t\ell}, x(\alpha)_{tr}\right]$ for t:

$$\mu_t(x) = \sup_{\forall\alpha\in[0,\alpha_{\max}]} \begin{cases} \alpha, & x(\alpha)_{t\ell} \le x \le x(\alpha)_{tr} \\ 0 & elsewhere \end{cases}. \tag{37}$$

These membership functions can be de-fuzzified to a representative scalar value by computing their centroids, where the centroid ς_μ of a fuzzy membership function $\mu(x)$ is defined by

ISBN: 978-989-95079-6-8

$$\varsigma_\mu = \frac{\int x\mu(x)dx}{\int \mu(x)dx} \quad (38)$$

In practice, the integrals in (38) are approximated by summations.

5 Interval Type-2 fuzzy inputs

Now suppose that the fuzzy membership functions for c_1, c_2, p_1 and p_2 in equations (1) and (2) are interval Type-2 [13] fuzzy membership functions $\tilde{\mu}_{c_1}(x)$, $\tilde{\mu}_{c_2}(x)$, $\tilde{\mu}_{p_1}(x)$ and $\tilde{\mu}_{p_2}(x)$, respectively. Each of these functions exhibits a "footprint of uncertainty" (FOU) that represents the union of an uncountable number of Type-1 primary fuzzy membership functions lying within the FOU, each having a secondary membership equal to unity [14]. Thus we wish to propagate this imprecision in the input variables through to the corresponding imprecision in the output aggregation of two inputs.

In this case, we use an approach analogous to that of Wu and Mendel [15,16] for linguistic weighted averages, which relies upon the observation that an interval Type-2 fuzzy set is completely specified by the upper and lower convex membership functions (UMF and LMF) that bound its FOU. We denote the Type-2 membership functions for the input confidences and plausibilities by $\mu_{c_1}^{UMF}(x)$, $\mu_{c_1}^{LMF}(x)$, $\mu_{c_2}^{UMF}(x)$, $\mu_{c_2}^{LMF}(x)$, $\mu_{p_1}^{UMF}(x)$, $\mu_{p_1}^{LMF}(x)$, $\mu_{p_2}^{UMF}(x)$ and $\mu_{p_2}^{LMF}(x)$, respectively. To determine the corresponding interval Type-2 membership functions $\tilde{\mu}_{c_3}(x)$ and $\tilde{\mu}_{p_3}(x)$ for the aggregate confidence and plausibility, we employ the α-cut technique of the previous section applied separately to the UMF's and LMF's to compute the output UMF's and LMF's for confidence and plausibility, respectively. The area between these bounding functions represents the primary Type-2 fuzzy membership functions for these variables, with a secondary membership equal to unity over this FOU. In similar fashion, we can find the UMF and LMF for the trust Type-2 membership function $\tilde{\mu}_t(x)$.

Once any of the Type-2 memberships have been calculated by the above method, we have a very rich description of the imprecision of the corresponding fuzzy membership function. These Type-2 membership functions can be *type-reduced* to a Type-1 membership function by calculating the *centroid* of the Type-2 membership function via the Karnik-Mendel algorithm [12]. For interval Type-2 membership functions, the centroid membership function is itself an interval $[z_\ell, z_r]$. This is of course a reduced representation of the imprecision that is fully described by the Type-2 membership function. The interval Type-1 membership function can then be *defuzzified* by calculating the midpoint of this interval to obtain a scalar representation of the aggregate membership function.

6 Example

We now consider an example that approximates the first case depicted in equation (3), but incorporates more general interval Type-2 fuzzy membership functions to describe the distributions of confidence and plausibility. In all cases below, we use $\alpha = 2$ and $\beta = 1/4$.

Fig. 1 shows two Type-2 fuzzy membership functions for the confidence of two different observations, where the FOU of each membership is the area between the solid and dotted curves. In this example, both confidences have positive support intervals, centered upon the values 0.8 (with + shading) and 0.3 (with x shading) respectively, analogous to the point values of confidence in equation (3).

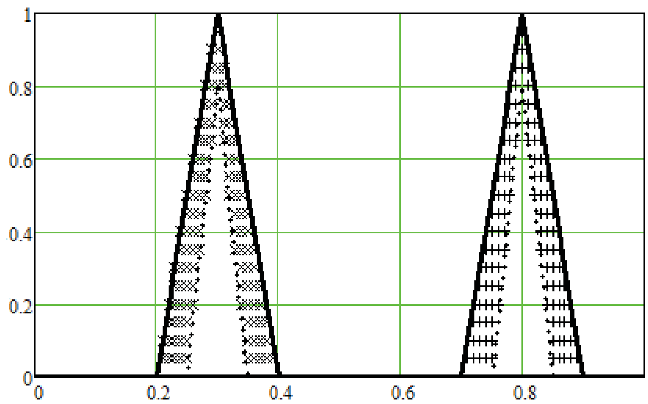

Figure 1 Interval Type-2 fuzzy membership functions for the confidence of two different observations.

In Fig. 2, we show the corresponding interval Type-2 membership functions of the plausibility of these two observations. Note the plausibility of the first observation (+ shading in Fig. 1) is concentrated near zero, while that of the second observation is centered on 0.5, but both cases exhibit a FOU of values.

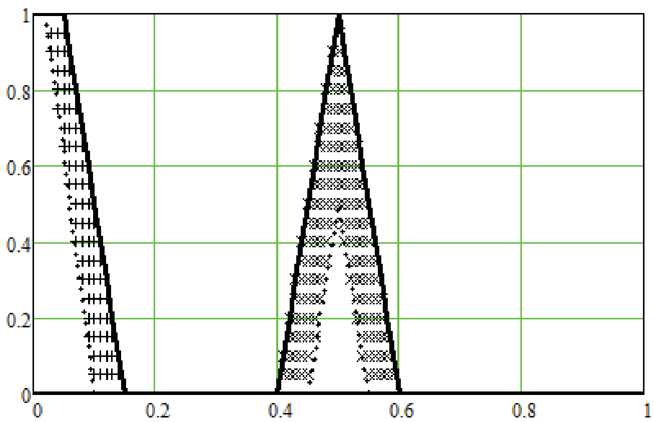

Figure 2 Interval Type-2 fuzzy membership functions for the plausibility of the observations whose confidence distributions are depicted in Fig. 1.

The mathematical apparatus described in previous sections for propagating the imprecision of these values when aggregating these two observations' confidences and plausibilities results in the aggregate confidence (+ shading) and plausibility (x shading) interval Type-2 membership functions shown in Fig. 3. The vertical dashed lines straddling the confidence FOU depict the centroid interval that results from type-reduction of the confidence membership function, while the vertical solid lines depict the midpoint (i.e., the de-fuzzified value) of the centroid intervals for both confidence and plausibility. These have

ISBN: 978-989-95079-6-8

values of 0.315 and 0.51, respectively, which roughly compares with the point wise values in equation (3), confirming that the aggregation of an observation with high confidence but very low plausibility results in essentially the same confidence and plausibility associated with the other observation. Note, however, that there is considerable additional information regarding both distributions that is lost in the type-reduction of a Type-2 membership function to a Type-1 interval membership function, and still more information is lost by representing the entire FOU by its de-fuzzified midpoint value.

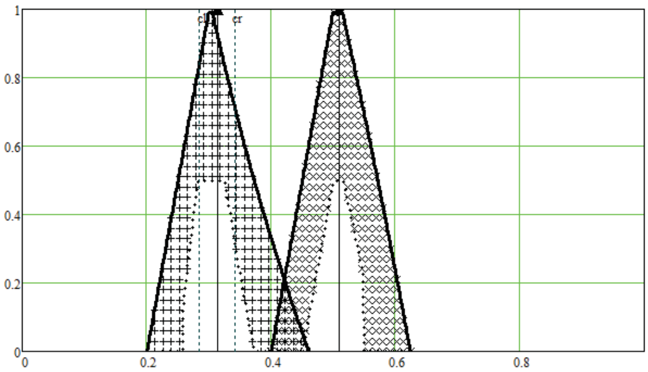

Figure 3 Interval Type-2 fuzzy membership functions for the aggregate confidence (+ shading) and plausibility (x shading) of the case depicted in Figures 4 and 5.

7 Conclusion

In this paper, we have generalized the trust aggregation algebra of Hamilton [8] for point values to admit interval, Type-1 and/or Type-2 input fuzzy membership values, and shown how these more complex inputs can be propagated in a mathematically principled manner to the corresponding output fuzzy membership functions.

References

[1] Grandison, T. and M. Sloman, "A survey of trust in internet applications," *IEEE Communications Surveys and Tutorials,* 3 pp. 2–16, 2000.

[2] Ray, I., S. Chakraborty, and I. Ray, "VTrust: a trust management system based on a vector model of trust," *Proceedings of International Conference on Information Systems Security*, Dec. 2005.

[3] Blaze, M., J. Feigenbaum and J. Lacy, "Decentralized trust management," *Proceedings of 17th IEEE Symposium on Security and Privacy*, Oakland, California, USA, pp. 164-173, IEEE Computer Society Press, 1996.

[4] Blaze, M., J. Feigenbaum and M. Strauss, "Compliance checking in the policymaker trust management system," *Proceedings of the 2nd Financial Crypto Conference*, vol. 1465 of *Lecture Notes in Computer Science*, pp. 254-274, Anguilla, Springer-Verlag, 1998.

[5] Jajodia, S., P. Samarati and V.S. Subramanian, "Logical language for expressing authorizations," *Proceedings IEEE Symposium on Security and Privacy*, pp. 31-42, May 1997.

[6] Damianou, N., N. Dulay, E. Lupu and M. Sloman. "The ponder policy specification language," *Policies for Distributed Systems and Networks*, vols. 1995, 2001.

[7] H. Hosmer, "Using fuzzy logic to represent security policies in the multipolicy paradigm," *ACM SIGSAC Review*, vol. 10, no. 4, pp. 12-21, Fall 1992.

[8] Hamilton, S. N. and Hamilton W. L. Hamilton, "Applying trust models to access control," submitted to *ACM Symposium on Access Control Models and Technologies*, June 2009.

[9] Wu, D. and J. M. Mendel, "Aggregation using the linguistic weighted average and interval Type-2 fuzzy sets," *IEEE Trans. Fuzzy Systems*, vol. 15, no. 6, pp. 1145-1161, Dec. 2007.

[10] Wu, D. and J. M. Mendel, "Corrections to "Aggregation using the linguistic weighted average and interval Type-2 fuzzy sets"," *IEEE Trans. Fuzzy Systems,* to appear.

[11] Klir, G. J. and B. Yuan, *Fuzzy Sets and Fuzzy Logic: Theory and Applications*. Upper Saddle River, NJ: Prentice-Hall, 1995.

[12] Karnik, N. N. and J. M. Mendel, "Centroid of a type-2 fuzzy set," *Information Sciences*, vol. 132, pp. 195-220, 2001.

[13] Mendel, J. M., *Uncertain Rule-Based Fuzzy Logic Systems*. Upper Saddle River, NJ: Prentice-Hall, 2001.

[14] Mendel, J. M., "Computing with words and its relationship with fuzzistics," *Information Sciences*, vol. 177, pp. 988-1006, 2007.

[15] Mendel, J. M. and H. Wu, "Type-2 fuzzistics for symmetric interval type-2 fuzzy sets: Part 1: forward problem," *IEEE Trans. Fuzzy Systems*, vol. 14, pp. 781-792, 2006.

[16] Mendel, J. M. and H. Wu, "Type-2 fuzzistics for symmetric interval type-2 fuzzy sets: Part 2: inverse problem," *IEEE Trans. Fuzzy Systems*, vol. 15, pp. 301-308, 2007.

[17] Liu, D. and J. M. Mendel, "Encoding words into interval Type-2 fuzzy sets using an *interval* approach," *IEEE Trans. Fuzzy Systems*, vol. 16, pp. 1503-1521, Dec. 2008.

ISBN: 978-989-95079-6-8