A Comparative Analysis of Symbolic Linguistic Computational Models

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Abstract— *There are many situations in which problems deal with vague and imprecise information. In such cases, the information could be modelled by means of numbers, however, it doesn't seem logical to model imprecise information in a precise way. Therefore, the use of linguistic modelling have been used with successful results in these problems. The use of linguistic information involves the need of carrying out processes which operate with words, so called Computing with Words (CW). In the literature exists different linguistic approaches and different computational models. We focus in this contribution on the use of the fuzzy linguistic approach (FLA) to model vague and imprecise information, but more specifically we focus on their computational models paying more attention on different symbolic computational models that have been defined to deal with linguistic information. We are going to review their main features and make a comparative analysis among them.*

Keywords— Computing with words (CW), fuzzy linguistic approach, linguistic 2-tuple, linguistic variable.

1 Introduction

Many problems in the real world deal with vague and imprecise information. There exist different kinds of tools to manage this type of information. The probability theory can be a powerful tool in order to treat the uncertainty and can be applied in different areas, like decision making, evaluation, planning, scheduling and so on. However, it is easy to see that many aspects of uncertainties have a non-probabilistic character since they are related to imprecision and vagueness of meanings. The use of the fuzzy linguistic approach [18] to model this kind of information provided successful results because the experts involved in such situations provide linguistic values rather than numbers. The linguistic modelling implies processes of CW, in the specialized literature can be found two classical linguistic computational models that provide linguistic operators for CW:

i) Model based on the extension principle (semantic model) [2, 4].

ii) Symbolic model [6, 8, 16].

The former provides accuracy but their results cannot be expressed by linguistic terms without an approximation process. The latter also needs an approximation process to express the results in a linguistic way, but the computational process is simpler and easier to understand by the experts involved in the problems.

Due to the previous facts, different symbolic approaches based on the fuzzy linguistic approach have been defined to improve the classical computational model. These approaches have modified the representation of the linguistic information from different points of view in order to improve the computational results. The symbolic models in which we are interested are: the 2-tuple linguistic representation model [9], the virtual linguistic model [15] and the proportional 2-tuple model [13]. These models improve the classical symbolic model by avoiding the approximation in processes of CW in order to improve the precision in the final results.

The aim of this contribution is to make a comparative analysis of the three aforementioned symbolic models to show their features and discuss their correctness regarding the fuzzy linguistic approach as their basis.

This paper is structured as follows: In Section 2, we introduce in short the fuzzy linguistic approach and its classical computational models. In Section 3, we shall review the different symbolic computational models, such as, the 2-tuple linguistic representation model, the virtual linguistic model and the proportional 2-tuple representation model. In Section 4, we shall make a comparative analysis among the different symbolic computational models, and finally we shall point out some concluding remarks.

2 Fuzzy Linguistic Approach

Many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case, a better approach might be to use linguistic assessments instead of numerical values. The fuzzy linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables [18]. We have to choose the appropriate linguistic descriptors for the linguistic term set and their semantics. To do so, an important aspect to be analyzed is the *granularity of uncertainty* i.e., the level of discrimination among different degrees of uncertainty. Typical values of cardinality used in the linguistic models are odd ones, such as 7 or 9, where the mid term represents an assessment of "roughly 0.5" and the rest of the terms being placed symmetrically around it [2]. Once the cardinality of the linguistic term set has been established, we must provide the linguistic terms and their semantics. There exist different possibilities to accomplish this task [1, 8, 17]. One of them consists in supplying directly the term set by considering all the terms distributed on a scale on which a total order is defined [8, 17]. For example, a set of seven terms S, could be:

$$S = \{s_0 : N, s_1 : VL, s_2 : L, s_3 : M, s_4 : H, s_5 : VH, s_6 : P\}$$

Usually, in these cases, it is required that in the linguistic term set there exists:

1. A negation operator Neg$(s_i) = s_j$ such that $j = g - i$ ($g + 1$ is the cardinality)

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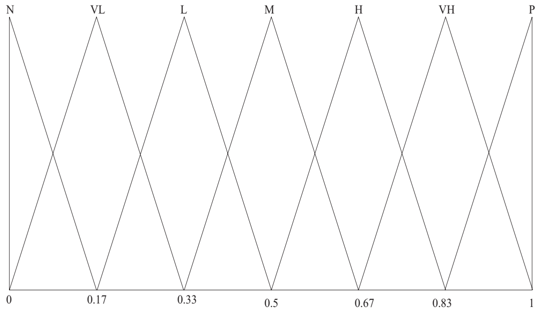

Figure 1: A Set of 7 Terms with its Semantic

2. A max operator: $\max(s_i, s_j) = s_i \; if \; s_i \geq s_j$

3. A min operator: $\min(s_i, s_j) = s_i \; if \; s_i \leq s_j$

The semantics of the terms are represented by fuzzy numbers, defined in the interval $[0, 1]$, described by membership functions. A way to characterize a fuzzy number is to use a representation based on parameters of its membership function [2]. The linguistic assessments given by the users are just approximate ones, then linear trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments [5]. This representation is achieved by the 4-tuple (a, b, c, d), where b and d indicate the interval in which the membership value is 1, with a and c indicating the left and right limits of the definition domain of the trapezoidal membership function [2]. A particular case of this kind of representation are the linguistic assessments whose membership functions are triangular, i.e., $b = d$, so we represent this kind of membership function by 3-tuples (a, b, c). An example can be:

$$\begin{array}{ll} P = (.83, 1, 1) & VH = (.67, .83, 1) \\ H = (.5, .67, .83) & M = (.33, .5, .67) \\ L = (.17, .33, .5) & VL = (0, .17, .33) \\ N = (0, 0, .17). & \end{array}$$

which is graphically shown in Figure 1.

2.1 Classical Computational Models

The use of linguistic variables implies processes of computing with words such as their fusion, aggregation, comparison, etc. To perform these computations in the fuzzy linguistic approach appeared two classical computational models:

1. **Model based on the Extension Principle (Semantic Model):** This model carries out operations with linguistic terms by means of operations associated to their membership functions based on the Extension Principle. The Extension Principle is a basic concept in the fuzzy sets theory [7] which is used to generalize crisp mathematical concepts to fuzzy sets. The use of extended arithmetic based on the Extension Principle [7] increases the vagueness of the results. Therefore, the results obtained by the fuzzy linguistic operators based on the Extension Principle are fuzzy numbers that usually do not match with any linguistic term in the initial term set. For this reason, it is necessary to carry out a linguistic approach in order to express the results in the original expression domain. In the literature, we can find different linguistic approximation operators [2, 4]. A linguistic aggregation operator based on the extension principle acts according to:

$$S^n \xrightarrow{\tilde{F}} F(\mathcal{R}) \xrightarrow{app_1(\cdot)} S \qquad (1)$$

where S^n symbolizes the n Cartesian product of S. $\tilde{F}$ is an aggregation operator based on the extension principle, $F(R)$ the set of fuzzy sets over the set of real numbers R, $app_1 : F(R) \rightarrow S$ is a linguistic approximation function that returns a label from the linguistic term set S, being S the initial term set.

2. **Symbolic Model:** This model uses the ordered structure of the linguistic terms set, $S = \{s_0, s_1, \ldots, s_g\}$ where $s_i < s_j$ if $i < j$, to operate [6, 8, 16]. The intermediate results of these operations are numeric values, $\alpha \in [0, g]$, which must be approximated in each step of the process by means of an approximation function $app_2 : [0, g] \rightarrow \{0, \ldots, g\}$ that obtains a numeric value, such that, it indicates the index of the associated linguistic term, $s_{app_2(\alpha)} \in S$. Formally, it can be expressed as:

$$S^n \xrightarrow{C} [0, g] \xrightarrow{app_2(\cdot)} \{0, \ldots, g\} \longrightarrow S \qquad (2)$$

where C is a symbolic linguistic aggregation operator, $app_2(\cdot)$ is an approximation function used to obtain an index $\{0, \ldots, g\}$ associated to a term in $S = \{s_0, \ldots, s_g\}$ from a value in $[0, g]$

Both models when operate with linguistic information produce loss of information due to the approximation processes and hence a lack of precision in the results. This loss of information is produced because the information representation model of the fuzzy linguistic approach is discrete in a continuous domain.

3 New Symbolic Computational Models

The necessity of dealing with linguistic information in many real world problems has driven the researchers to develop models in order to improve the processes of CW. Different models have been presented in the literature recently. In this section we focus our attention on different symbolic approaches that have developed new representation and computational models for the linguistic information in order to improve the accuracy of the results of processes of CW. Such approaches are the 2-tuple linguistic model [9], the virtual linguistic model [15] and the proportional 2-tuple linguistic model [13], which improve the limitations that present the classical symbolic computational model, regulating the loss of information and imprecision in its computations. These approaches have been widely used in problems dealing with linguistic information such as, Decision making [12, 14], Evaluation [3], Recommender Systems [11] and so on.

Due to the fact that, these symbolic approaches based on the fuzzy linguistic approach have modified the linguistic representation in order to improve the processes of CW, their review implies the study of their linguistic representation models and their computational models to accomplish the processes of CW in a symbolic and precise way.

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3.1 2-Tuple linguistic representation model

This model was presented in [9] to avoid the loss of information and to express symbolically any counting of information in the universe of the discourse.

(a) Representation model:

This representation is based on the concept of *symbolic translation* and uses it for representing the linguistic information by means of a pair of values, called *2-tuples*, (s_i, α) where s is a linguistic term and α is a numerical value representing the symbolic translation. Let $S = \{s_0, \ldots, s_g\}$ be a term set, and $\beta \in [0, g]$ a numerical value in its interval of granularity (e.g.: let β be a value obtained from a symbolic aggregation operation).

Definition 1 *The symbolic translation is a numerical value assessed in $[-0.5, 0.5)$ that supports the "difference of information" between a counting of information β assessed in the interval of granularity $[0, g]$ of the term set S and the closest value in $\{0, \ldots, g\}$ which indicates the index of the closest linguistic term in S.*

From this concept, is developed a linguistic representation model which represents the linguistic information by means of 2-tuples (s_i, α), $s_i \in S$ and $\alpha_i \in [-0.5, 0.5)$

- s_i represents the linguistic label of the information
- α is a numerical value expressing the value of the translation

This representation model defines a set of functions to facilitate computational processes with 2-tuples [9].

Definition 2 *Let $S = \{s_0, \ldots, s_g\}$ be a set of linguistic terms. The 2-tuple set associated with S is defined as $\langle S \rangle = S \times [-0.5, 0.5)$. We define the function $\Delta : [0, g] \longrightarrow \langle S \rangle$ given by*

$$\Delta(\beta) = (s_i, \alpha), \ \ \textit{with} \ \left\{ \begin{array}{l} i = \textit{round}\,(\beta), \\ \alpha = \beta - i, \end{array} \right. \tag{3}$$

where round *assigns to β the integer number $i \in \{0, 1, \ldots, g\}$ closest to β.*

We note that Δ is bijective [9, 10] and $\Delta^{-1} : \langle S \rangle \longrightarrow [0, g]$ is defined by $\Delta^{-1}(s_i, \alpha) = i + \alpha$. In this way, the 2-tuples of $\langle S \rangle$ will be identified with the numerical values in the interval $[0, g]$.

Remark 1 *We can consider the injective mapping $S \longrightarrow \langle S \rangle$ that allows us to transform a linguistic term s_i into a 2-tuple: $(s_i, 0)$. On the other hand, $\Delta_S(i) = (s_i, 0)$ and $\Delta_S^{-1}(s_i, 0) = i$, for every $i \in \{0, 1, \ldots, g\}$.*

Let's suppose a symbolic aggregation operation over labels assessed in $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$ that obtains as its result $\beta = 2.8$, then the representation of this counting of information by means of a 2-tuple will be:

$$\Delta(2.8) = (s_3, -0.2)$$

b) Computational model:

Together the representation model, a linguistic computational approach based on the functions Δ and Δ^{-1} was also defined in [9] with the following computations and operators:

1. Comparison of 2-tuples

The comparison of linguistic information represented by 2-tuples is carried out according to an ordinary lexicographic order.

Let (s_k, α_1) and (s_l, α_2) be two 2-tuples, with each one representing a counting of information:

- if $k < l$ then $(s_k, \alpha_1) < (s_l, \alpha_2)$
- if $k = l$ then
  1. if $\alpha_1 = \alpha_2$ then (s_k, α_1), (s_l, α_2) represents the same information
  2. if $\alpha_1 < \alpha_2$ then $(s_k, \alpha_1) < (s_l, \alpha_2)$
  3. if $\alpha_1 > \alpha_2$ then $(s_k, \alpha_1) > (s_l, \alpha_2)$

2. Negation operator of a 2-tuple

The negation operator over 2-tuples was defined as:

$$Neg((s_i, \alpha)) = \Delta(g - (\Delta^{-1}(s_i, \alpha))) \tag{4}$$

where $g + 1$ is the cardinality of S, $S = \{s_0, ..., s_g\}$.

3. Aggregation of 2-tuples

The aggregation of information consists of obtaining a value that summarizes a set of values, therefore, the result of the aggregation of a set of 2-tuples must be a 2-tuple. There exists several 2-tuple aggregation operators [9]. For instance, the 2-tuple arithmetic mean is defined as:

Definition 3 *Let $x = \{(s_1, \alpha_1), \ldots, (s_n, \alpha_n)\}$ be a set of 2-tuples, the 2-tuple arithmetic mean $\overline{x}^e$ is computed as,*

$$\overline{x}^e = \Delta(\sum_{i=1}^{n} \frac{1}{n} \Delta^{-1}(s_i, \alpha_i)) = \Delta(\frac{1}{n} \sum_{i=1}^{n} \beta_i) \tag{5}$$

The arithmetic mean for 2-tuples allows to compute the mean of a set of linguistic values in a precise way without any approximation process.

Example

Let's suppose an example where we have the following linguistic preference vector:

s_2	s_3	s_3	s_2

and $S = \{s_0 : N, s_1 : VL, s_2 : L, s_3 : M, s_4 : H, s_5 : VH, s_6 : P\}$ is the term set shown in the Figure 1.

We want to aggregate these values by using the arithmetic mean as aggregation operator. We follow the process below:

- The preference vector is transformed into 2-tuples as follows:

$(s_2, 0)$	$(s_3, 0)$	$(s_3, 0)$	$(s_2, 0)$

- The linguistic aggregated value obtained by the arithmetic 2-tuple is:
  $\overline{x} = \Delta(\frac{1}{4}(\Delta^{-1}(s_2, 0) + \Delta^{-1}(s_3, 0) + \Delta^{-1}(s_3, 0) + \Delta^{-1}(s_2, 0))) = \Delta(2.5) = (s_3, -0.5)$

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3.2 *Virtual linguistic model*

This model was presented by Xu in [15] to avoid the loss of information in processes of CW and increase the operators in processes of CW.

a) Representation model:

In this symbolic model, Xu extended the discrete term set S to a continuous linguistic term set $\bar{S} = \{s_\alpha | s_l < s_\alpha \leq s_t, \alpha \in [1, t]\}$, where, if $s_\alpha \in S$, s_α is called *the original linguistic term*, otherwise, s_α is called *virtual linguistic term* which **does not have assigned any semantics**.

In general, experts use the original linguistic terms to assess the linguistic variables, and the virtual linguistic terms appear in operations.

b) Computational model:

To accomplish processes of CW with this representation model, Xu introduced the following operational laws:

Let $s_\alpha, s_\beta \in \bar{S}$, be any two linguistic terms and $\mu, \mu_1, \mu_2 \in [0, 1]$.

1. $(s_\alpha)^\mu = s_{\alpha^\mu}$
2. $(s_\alpha)^{\mu_1} \otimes (s_\alpha)^{\mu_2} = (s_\alpha)^{\mu_1+\mu_2}$
3. $(s_\alpha \otimes s_\beta)^\mu = (s_\alpha)^\mu \otimes (s_\beta)^\mu$
4. $s_\alpha \otimes s_\beta = s_\beta \otimes s_\alpha = s_{\alpha\beta}$
5. $s_\alpha \oplus s_\beta = s_{\alpha+\beta}$
6. $s_\alpha \oplus s_\beta = s_\beta \oplus s_\alpha$
7. $\mu s_\alpha = s_{\mu\alpha}$
8. $(\mu_1 + \mu_2)s_\alpha = \mu_1 s_\alpha \oplus \mu_2 s_\alpha$
9. $\mu(s_\alpha \oplus s_\beta) = \mu s_\alpha \oplus \mu s_\beta$

Example

Let's suppose the example presented previously. In order to compute the arithmetic mean with this model, we have to apply the above operational rules to the linguistic terms:

- The arithmetic mean according to Xu is defined as:

$$\overline{x}^e = \frac{\sum_{i=1}^n s_i}{n} = \frac{1}{n} s_{\sum_{i=1}^n i} \tag{6}$$

- We aggregate the preference vector and we obtain the following collective preference value

$$\overline{x} = \frac{1}{4} s_{(2+3+3+2)} = \frac{1}{4} s_{10} = s_{2.5}$$

3.3 *Proportional 2-Tuples representation model*

This model presented by Wang and Hao in [13] develops a new way to represent the linguistic information that is a generalization and extension of 2-tuple linguistic representation model [9]. This model deals with linguistic labels in a precise way, but it does not require that the labels are symmetrically distributed around a medium label and either having "equal distance" between them. Besides, it describes the initial linguistic information by members of a "continuous" linguistic scale domain which does not necessarily require the ordered linguistic terms of a linguistic variable being equidistant.

a) Representation model:

This model represents the linguistic information by means of proportional 2-tuples, such as $(0.2A, 0.8B)$ for the case when someone's grades in the answer scripts of a whole course are distributed as $20\%A$ and $80\%B$. The authors point out that if B were used as the approximative grade then some performance information would be lost. This approach, proportional 2-tuples, is based on the concept of *symbolic proportion* [13].

Definition 4 *Let $S = \{s_0, s_1, ..., s_g\}$ be an ordinal term set, $I = [0, 1]$ and*

$$IS \equiv I \times S = \{(\alpha, s_i) : \alpha \in [0, 1] \text{ and } i = 0, 1, ..., g\} \tag{7}$$

where S is the ordered set of $g + 1$ ordinal terms $\{s_0, ..., s_g\}$. Given a pair (s_i, s_{i+1}) of two successive ordinal terms of S, any two elements (α, s_i), (β, s_{i+1}) of IS is called a symbolic proportion pair and α, β are called a pair of symbolic proportions of the pair (s_i, s_{i+1}) if $\alpha+\beta = 1$. A symbolic proportion pair (α, s_i), $(1 - \alpha, s_{i+1})$ is denoted by $(\alpha s_i, (1 - \alpha)s_{i+1})$ and the set of all the symbolic proportion pairs is denoted by $\overline{S}$, i.e., $\overline{S} = \{(\alpha s_i, (1 - \alpha)s_{i+1}) : \alpha \in [0, 1] \text{ and } i = 0, 1, \ldots, g - 1\}$.

Remark 2 *Since for $i = \{2, \ldots, g - 1\}$, ordinal term s_i can use either $(0s_{i-1}, 1s_i)$ or $(1s_i, 0s_{i+1})$ as its representative in $\overline{S}$, by abuse of notation.*

$\overline{S}$ is called the *ordinal proportional 2-tuple set* generated by S and the members of $\overline{S}$, *ordinal proportional 2-tuples*, which is used to represent the ordinal information for CW.

The notion of proportional 2-tuple allows experts to express their opinions using two adjacent ordinals.

In a similar way to the symbolic 2-tuple Wang and Hao introduced functions in order to facilitate the computations with this type of representation.

Definition 5 *Let $S = \{s_0, s_1, \ldots, s_g\}$ be an ordinal term set and $\overline{S}$ be the ordinal proportional 2-tuple set generated by S. The function $\pi : \overline{S} \to [0, g]$ was defined by*

$$\pi((\alpha s_i, (1 - \alpha)s_{i+1})) = i + (1 - \alpha), \tag{8}$$

where $i = \{0, 1, \ldots, g - 1\}, \alpha \in [0, 1]$ and π is called the position index function of ordinal 2-tuples.

Note that, under the identification convention which was remarked after the definition 4, the position index function π becomes a bijection from $\overline{S}$ to $[0, g]$ and its inverse $\pi^{-1} : [0, g] \to \overline{S}$ is defined by

$$\pi^{-1}(x) = ((1 - \beta)s_i, \beta s_{i+1}) \tag{9}$$

where $i = E(x)$, E is the integer part function, $\beta = x - i$.

b) Computational model:

To operate with linguistic information under proportional 2-tuple contexts, Wang and Hao expanded the computational

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techniques for symbolic to proportional 2-tuples and defined the following operators:

1. Comparison of Proportional 2-tuples

The comparison of linguistic information represented by proportional 2-tuples is carried out as follows: Let $S = \{s_0, \ldots, s_g\}$ be an ordinal term set and $\bar{S}$ be the ordinal proportional 2-tuple set generated by S. For any $(\alpha s_i, (1-\alpha)s_{i+1}), (\beta s_j, (1-\beta)s_{j+1}) \in \overline{S}$, define $(\alpha s_i, (1-\alpha)s_{i+1}) < (\beta s_j, (1-\beta)s_{j+1}) \Leftrightarrow \alpha i + (1-\alpha)(i+1) < \beta j + (1-\beta)(j+1) \Leftrightarrow i + (1-\alpha) < j + (1-\beta)$.

Thus, for any two proportional 2-tuples $(\alpha s_i, (1-\alpha)s_{i+1})$ and $(\beta s_j, (1-\beta)s_{j+1})$:

- if $i < j$, then
  1. $(\alpha s_i, (1-\alpha)s_{i+1}), (\beta s_j, (1-\beta)s_{j+1})$ represents the same information when $i = j-1$ and $\alpha = 0, \beta = 1$
  2. $(\alpha s_i, (1-\alpha)s_{i+1}) < (\beta s_j, (1-\beta)s_{j+1})$ otherwise
- if $i = j$, then
  1. if $\alpha = \beta$ then $(\alpha s_i, (1-\alpha)s_{i+1}), (\beta s_j, (1-\beta)s_{j+1})$ represents the same information
  2. if $\alpha < \beta$ then $(\alpha s_i, (1-\alpha)s_{i+1}) < (\beta s_j, (1-\beta)s_{j+1})$
  3. if $\alpha > \beta$ then $(\alpha s_i, (1-\alpha)s_{i+1}) > (\beta s_j, (1-\beta)s_{j+1})$

2. Negation operator of a Proportional 2-Tuple

The negation for proportional 2-tuples is defined as:

$$Neg((\alpha s_i, (1-\alpha)s_{i+1})) = ((1-\alpha)s_{g-i-1}, \alpha s_{g-i}), \quad (10)$$

where $g+1$ is the cardinality of S, $S = \{s_0, s_1, \ldots, s_g\}$

3. Aggregation of Proportional 2-Tuple

Wang and Hao defined many aggregation operators to handle processes of CW. The definitions of these aggregation operators are based on canonical characteristic values of linguistic labels. To do so, they used the similar corresponding aggregation operators developed in [9] in order to aggregate ordinal 2-tuples through their position indexes [13].

Example

By using the example presented previously, if we apply the proportional 2-tuples we obtain the following results:

- The arithmetic mean according to Wang and Hao is defined as:

$$\begin{aligned}\overline{x} = \pi^{-1}(\textstyle\sum_{i=1}^{n} \frac{1}{n}\pi(\alpha s_i, (1-\alpha)s_{i+1})) = \\ = \pi^{-1}(\frac{1}{n}\textstyle\sum_{i=1}^{n}(i + (1-\alpha))\end{aligned} \quad (11)$$

- The preference vector is transformed into proportional 2-tuple as follows:

$(1s_2, 0s_3)$	$(1s_3, 0s_4)$	$(1s_3, 0s_4)$	$(1s_2, 0s_3)$

- The collective preference value obtained under ordinal proportional 2-tuple contexts is
  $\overline{x} = \pi^{-1}(\frac{1}{4}(\pi(1s_2, 0s_3) + \pi(1s_3, 0s_4) + \pi(1s_3, 0s_4) + \pi(1s_2, 0s_3))) = \pi^{-1}(\frac{1}{4}(2+3+3+2)) = \pi^{-1}(2.5) = ((1-0.5)s_2, 0.5s_3) = (0.5s_2, 0.5s_3)$

4 Comparative Analysis

The aim of this contribution is to make a comparative analysis among the symbolic approaches presented in section 3. This analysis will consist of studying all the approaches from different points of view such as: representation model, computations, accuracy, comprehension and so on.

- *The Representation*

  Here, we want to point out that although the authors of the three approaches said that they are based on the fuzzy linguistic approach. It is clear that the *Virtual model* does not follow this approach because its representation does not have any semantics to interpret the linguistic information. Nevertheless, the 2-tuple and the proportional 2-tuple in spite of using additional information to the representation of the linguistic information both, keep the basis of the definition of linguistic variable provided in the fuzzy linguistic approach by using fuzzy numbers to represent the semantics of the linguistic terms.

- *The accuracy*

  By comparing the results provided by the three different approaches in the example showed we can see that the results are similar although the representation of the information would be different. The reason of similar results in this case is because we have considered a linguistic term set symmetrically distributed. However, if we go in deep through the three approaches we can see that the 2-tuple approach guarantees the accuracy when the labels are symmetrically distributed and there is only point with maximum height. The proportional 2-tuple guarantees the accuracy when the terms have the same width in their support. Finally regarding the virtual model is difficult to say anything about accuracy because it does not have semantics to represent the information.

- *The computations*

  By comparing the computational models provided by the three approaches we can observe an important difference. The Virtual model proposes many symbolic operations directly on the linguistic terms obtaining results without any meaning. However the 2-tuple and proportional 2-tuple approaches propose their symbolic computational models with symbolic operators and additionally provide transformation functions to facilitate the computations.

- *The comprehension*

  An important point that should be taken into account regarding the linguistic information is, that this type of information is not only used to represent the information but also to facilitate the comprehension of the results to the users in many different problems.

  Therefore, we can say that the virtual model is only valid for ranking issues because due to the fact the virtual terms do not have semantics are hard to understand apart of a simple order. The proportional 2-tuple provides a clear representation but comparing with the 2-tuple is a little bit more complex due to the fact of the use of four values to represent a single one.

In the table 1 we show the main differences among models.

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Table 1: Comparative analysis among models

	2-Tuple	Virtual Linguistic	Proportional 2-Tuple
Representation	symbolic and semantics	No semantics	symbolic and semantics
Accuracy	equidistant labels	always because no semantics	same width
Computation	symbolic	no symbolic	symbolic
Comprehension	easy to understand	only ranking issues	understandable

5 Concluding Remarks

In this paper we have reviewed the classical computational models and we have made a brief review of recent symbolic computational models, like the 2-tuple model, the proportional 2-tuple model and the virtual linguistic model.

We have also made a comparative analysis among them in which the most remarkable finding obtained is that the virtual model can not be considered a linguistic model in the sense of the fuzzy linguistic approach due to the fact that it does not require semantics to their linguistic terms. In fact, it could be considered a crisp approach because it does not use any real element to represent the vagueness of the qualitative information.

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References

[1] P.P. Bonissone. *A fuzzy sets based linguistic approach: Theory and applications*, pages 329–339. Approximate Reasoning in Decision Analysis, North-Holland, 1982.

[2] P.P. Bonissone and K.S. Decker. *Selecting Uncertainty Calculi and Granularity: An Experiment in Trading-Off Precision and Complexity*. In L.H. Kanal and J.F. Lemmer, Editors., Uncertainty in Artificial Intelligence. North-Holland, 1986.

[3] G. Buyukozkan, D. Ruan, and O. Feyzioglu. Evaluating e-learning web site quality in a fuzzy environment. *International Journal of Intelligent System*, 22:567–586, 2007.

[4] R. Degani and G. Bortolan. The problem of linguistic approximation in clinical decision making. *International Journal of Approximate Reasoning*, 2:143–162, 1988.

[5] M. Delgado, J.L. Verdegay, and M.A Vila. Linguistic decision making models. *International Journal of Intelligent Systems*, 7:479–492, 1992.

[6] M. Delgado, J.L. Verdegay, and M.A Vila. On aggregation operations of linguistic labels. *International Journal of Intelligent Systems*, 8:351–370, 1993.

[7] D. Dubois and H. Prade. *Fuzzy Sets and Systems: Theory and Applications*. Kluwer Academic, New York, 1980.

[8] F. Herrera, E. Herrera-Viedma, and J.L. Verdegay. A sequential selection process in group decision making with linguistic assessment. *Information Sciences*, 85:223–239, 1995.

[9] F. Herrera and L. Martínez. A 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Transactions on Fuzzy Systems*, 8(6):746–752, 2000.

[10] F. Herrera and L. Martínez. The 2-tuple linguistic computational model. Advantages of its linguistic description, accuracy and consistency. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 9(Suppl.):33–49, 2001.

[11] L. Martínez, M.J. Barranco, L.G. Perez, and et al. A knowledge based recommender system with multigranular linguistic information. In *Proceedings of the international conference on intelligent systems and knowledge engineering (ISKE 2007)*, pages 312–319, Chengdu, China, 2007.

[12] J.M. Merigo and A.A. Gil-Lafuente. The linguistic generalized owa operator and its application instrategic decision making. In *Proceedings of the tenth international conference on enterprise information systems (ICEIS 2008)*, pages 219–224, Barcelona, Spain, 2008.

[13] J. Wang and J. Hao. A new version of 2-tuple fuzzy linguistic representation model for computing with words. *IEEE transactions on fuzzy systems*, 14:435, 2006.

[14] Y.J. Xu and L. Huang. An approach to group decision making problems based on 2-tuple linguistic aggregation operators. In *International Colloquium on Computing, Communication, Control, and Management*, pages 73–77, Guangzhou, China, 2008.

[15] Z.S Xu. A method based on linguistic aggregation operators for group decision making with linguistic preference relations. *Information Science*, 166:19–30, 2004.

[16] R.R. Yager. *Fuzzy Logic: State of the Art*, chapter Fuzzy screening systems, pages 251–261. Kluwer Academic Publishers, 1993.

[17] R.R. Yager. An approach to ordinal decision making. *International Journal of Approximate Reasoning*, 12:237–261, 1995.

[18] L.A. Zadeh. The concept of a linguistic variable and its applications to approximate reasoning. *Information Sciences, Part I, II, III*, 8,8,9:199–249,301–357,43–80, 1975.

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