Interval AHP for a group of decision makers

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Abstract— *This paper considers Interval Analytic Hierarchy Process (Interval AHP) in group decision making for encouraging communication. Interval AHP is suitable method to handle subjective judgments since the induced results are intervals which can include uncertainty of given information. The decision makers' opinions can be aggregated at some stages of decision making process to reach consensus. One of the approaches is to aggregate the given judgments considering outliers and from them the group preferences are obtained. By the other approach, first the individual preferences are obtained from the respective decision maker's judgments and then they are aggregated. In the sense of reducing communication barriers, obtaining individual priority weights of alternatives beforehand helps decision makers realize their own and the others' opinions. The judgments and preferences can be aggregated based on the possibility view or by introducing importance weights of decision makings.*

Keywords— group decision making, importance weight, interval AHP, intuitive judgments, possibility, uncertainty

1 Introduction

In most organizations decision making is made by two or more people, regardless of whether the organization is public or private and the decision problem is local or global. Although all members do not need to be located in the same physical location, they are aware of one another and act as a member of the group which makes a decision. The apparent purpose of the group is to reach a decision, that is, to choose one alternative which seems to be acceptable and agreeable for all members. However, it is sometimes difficult to reach a consensus among group members [1]. Especially when there are various members who are not face-to-face, there exist some barriers to understand one another. These communication barriers make decision making tough. The technological advancements, such as computers and networks, accelerate such situations. In addition, the decision makers have some difficulties to represent and recognize their own opinions which fit their intuitive judgments and experiences, because the problems are often very complicated. It may happen that some members may exaggerate their preferences to influence the group decision. In this sense, it is also important to support the interpersonal information exchange, as well as to find the optimal alternative, in the group decision making. As a preparation for the consensus, it becomes necessary to remove communication barriers by representing individual opinions simply and clearly [1, 2]. This paper concerns on these design phase more than the choice phase in group decision support system by focusing on the aggregation of decision makers' opinions.

The group decision support system is discussed from the scope of AHP (Analytic Hierarchy Process). AHP is a useful method in multi-criteria decision making problems [3]. It is structured hierarchically as criteria and alternatives. The priority or weight for each element of the hierarchy is obtained by eigenvector method [3]. Then, the priority weight for each alternative is calculated with them. The advantages of AHP are the following two points. It helps decision maker structure complex problems hierarchically. The decision makers can give their subjective judgments directly by comparing a pair of attributes.

The group decision making with AHP is discussed in [4, 5, 6, 7]. On this assumption, more than two comparison matrices are given. The decision makers' individual opinions are aggregated in several stages of decision making process by several approaches. It can be done at the beginning stage by searching consensus on the given judgments. It is also possible to take the average of individually induced preferences with importance weights of the group members at the last stage. Between these stages, the decomposed intermediate judgments may be aggregated by voting and so on [2]. This paper concerns on the aggregations at the beginning and last stages based on two concepts such as possibility and nominal average. Furthermore, in order to reflect uncertainty of the given comparisons, Interval AHP [8, 9], where the interval weights, instead of crisp weights in the conventional AHP, are obtained is applied.

This paper is based on possibility concept in dealing with the given data and obtained results and consists as follows. As a preliminary, the definition and properties of the interval probability which are used for normalization of intervals are explained in Section 2. At first, Interval AHP as a tool to represent each decision maker's preference is shown briefly in Section 3. Then, in Section 4, the approaches to aggregate various opinions from several viewpoints are proposed. Finally, the proposed models are tested with a numerical example in case of a group of four decision makers in Section 5.

2 Interval probability as preliminary

The interval probabilities are defined by a set of intervals as follows. This definition is originally proposed in [10] and also is used in [11, 12]. The conventional crisp probabilities are extended into interval ones.

Definition 1 (Interval probability) *The set of intervals denoted as* $\{W_1, ..., W_n\}$ *where* $W_i = [\underline{w}_i, \overline{w}_i]$ *are called interval probabilities if and only if*

$$\begin{array}{ll} 1) & 0 \leq \underline{w}_i \leq \overline{w}_i \ \forall i \\ 2) & \sum_{i \neq j} \overline{w}_i + \underline{w}_j \geq 1 \forall j \\ 3) & \sum_{i \neq j} \underline{w}_i + \overline{w}_j \leq 1 \ \forall j. \end{array} \quad (1)$$

From Definition 1, following two inequalities hold

$$\sum_i \underline{w}_i \leq 1 \ \text{and} \ \sum_i \overline{w}_i \geq 1.$$

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Definition 1 is regarded as a normality condition of intervals corresponding to the conventional one $\sum_i w_i = 1$. It is noted that in interval probabilities there are many combinations of crisp values whose sum is one.

The combination of a pair of interval probability sets is denoted as follows.

Property 1 Combination rule *Assuming a pair of interval probabilities on* n *elements as* $\{W_1^A, ..., W_n^A\}$ *and* $\{W_1^B, ..., W_n^B\}$ *which satisfy Definition 1, their combination is denoted as* $\{W_1^{AB}, ..., W_n^{AB}\}$*. Each of elements is an interval* $W_i^{AB} = [\underline{w}_i^{AB}, \overline{w}_i^{AB}]$ *denoted as follows.*

$$\begin{array}{l} \underline{w}_i^{AB} = \min\{\underline{w}_i^A, \underline{w}_i^B\} \\ \overline{w}_i^{AB} = \max\{\overline{w}_i^A, \overline{w}_i^B\} \end{array} \quad (2)$$

The set of combined intervals $\{W_1^{AB}, ..., W_n^{AB}\}$ also satisfies Definition 1 so that it is interval probability.

Property 2 Weighted sum of interval probabilities *Assuming* m *sets of interval probabilities on* n *elements as* $\{W_1^{A_k}, ..., W_n^{A_k}\}$ $k = 1, ..., m$*, and the weights for the sets as* $\{p_k \forall k\}$ *where* $p_k \geq 0$ *and* $\sum_k p_k = 1$*, the weighted sum of interval probabilities is* $\{W_1, ..., W_n\}$*. Each of elements is an interval* $W_i = [\underline{w}_i, \overline{w}_i]$ *denoted as follows.*

$$\begin{array}{l} \underline{w}_i = \sum_k p_k \underline{w}_i^{A_k} \\ \overline{w}_i = \sum_k p_k \overline{w}_i^{A_k} \end{array} \quad (3)$$

As far as the sum of weights is 1, the weighted sum of interval probabilities becomes also interval probability. It is verified as follows. They apparently satisfy the condition 1) in Definition 1. The condition 2) holds as follows.

$$\begin{array}{l} \sum_{i \neq j} \sum_k p_k \overline{w}_i^{A_k} + \sum_k p_k \underline{w}_j^{A_k} \\ = \sum_k p_k (\sum_{i \neq j} \overline{w}_i^{A_k} + \underline{w}_j^{A_k}) \text{ [Definition 1-2)]} \\ \geq \sum_k p_k = 1 \end{array} \quad (4)$$

The other condition 3) is satisfied by the similar way.

$$\begin{array}{l} \sum_{i \neq j} \sum_k p_k \underline{w}_i^{A_k} + \sum_k p_k \overline{w}_j^{A_k} \\ = \sum_k p_k (\sum_{i \neq j} \underline{w}_i^{A_k} + \overline{w}_j^{A_k}) \\ \leq \sum_k p_k = 1 \end{array} \quad (5)$$

Then, the weighted sum of interval probabilities satisfy the conditions in Definition 1.

3 Interval AHP

AHP is an approach to multi-criteria decision making problems. The problem is decomposed into hierarchy by criteria and alternatives. The choice or preferences of alternatives are induced as a final decision from the decision maker's judgments given as pariwise comparison matrix. The decision maker compares all pairs of alternatives and gives the pairwise comparison matrix for n alternatives as follows [3].

$$A = \begin{pmatrix} 1 & \cdots & a_{1n} \\ \vdots & a_{ij} & \vdots \\ a_{n1} & \cdots & 1 \end{pmatrix} \quad (6)$$

where a_{ij} shows the importance ratio of alternative i comparing to alternative j. The comparison matrix satisfies the following relations so that the number of given comparisons is $n(n-1)/2$.

$$\begin{array}{ll} a_{ii} = 1 & \text{identical} \\ a_{ij} = 1/a_{ji} & \text{reciprocal} \end{array} \quad (7)$$

The decision maker can give his/her judgment intuitively without caring about the relative relations of comparisons. Therefore, the given comparisons are not always consistent each other. The consistent comparisons satisfy the following transitivity relations.

$$a_{ij} = a_{ik} a_{kj} \ \forall (i, j) \quad (8)$$

In the following, inconsistency means that (8) is not satisfied. The proposed models in this paper deals with such inconsistency from the possibility view [13].

In the conventional AHP, crisp priority weights are obtained from the given comparison matrix. They are extended to intervals in Interval AHP [8, 9]. The given comparisons are inconsistent each other, that is, they do not always satisfy (8). In order to reflect such inconsistency, the priority weight of alternative is denoted as the following interval.

$$W_i = [\underline{w}_i, \overline{w}_i] \ \forall i \quad (9)$$

For their normalization, they are represented as interval probabilities so that they satisfy (1) in Definition 1.

The pairwise comparison is an intuitive ratio of two alternatives so that they are approximated by the following interval.

$$\frac{W_i}{W_j} = \left[\frac{\underline{w}_i}{\overline{w}_j}, \frac{\overline{w}_i}{\underline{w}_j} \right] \quad (10)$$

where $0 < \underline{w}_i \forall i$ and the upper and lower bounds of the approximated comparison are defined as the maximum range with respect to the two intervals.

In the approximation model the probabilities are determined so as to include the given pairwise comparisons. Thus, from the possibility view, the obtained interval probabilities satisfy the following inclusion relations.

$$a_{ij} \in \frac{W_i}{W_j} \ \forall (i, j) \quad (11)$$

which leads to the following inequalities.

$$\frac{\underline{w}_i}{\overline{w}_j} \leq a_{ij} \leq \frac{\overline{w}_i}{\underline{w}_j} \Leftrightarrow \begin{cases} \underline{w}_i \leq a_{ij} \overline{w}_j \ \forall (i, j) \\ \overline{w}_i \geq a_{ij} \underline{w}_j \ \forall (i, j) \end{cases} \quad (12)$$

The approximations by the obtaining interval priority weights include the given inconsistent comparisons.

For any inconsistent comparisons, assuming $[\underline{w}_i, \overline{w}_i] = [0, 1] \ \forall i$, the above inclusion relation (12) is apparently satisfied. A decision maker does not need to revise their intuitive judgments so as to be consistent. When a decision maker gives completely inconsistent judgments, the obtained priority weights of all alternatives are equally $[0, 1]$. It represents complete ignorance and fits our natural sense. Inconsistency among the given comparisons is reflected in the uncertainty of interval probabilities.

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The constraint conditions for determining the interval probabilities are (1) and (12). In order to obtain the least uncertain probabilities, the uncertainty of interval probabilities should be minimized. The uncertainty of interval probabilities can be measured by several indices, such as widths of intervals and entropy [14]. For simplicity, the sum of widths of intervals is used in this paper. The problem to determine the interval priority weights is formulated as follows.

$$\begin{array}{rl} I = & \min \ \sum_i (\overline{w}_i - \underline{w}_i) \\ \text{s.t.} & \sum_{i \neq j} \overline{w}_i + \underline{w}_j \geq 1 \ \forall j \\ & \sum_{i \neq j} \underline{w}_i + \overline{w}_j \leq 1 \ \forall j \\ & \underline{w}_i \leq a_{ij} \overline{w}_j \ \forall (i,j) \\ & \overline{w}_i \geq a_{ij} \underline{w}_j \ \forall (i,j) \\ & \underline{w}_i \geq \epsilon \ \forall i \end{array} \quad (13)$$

The greater optimal objective function value is, the more uncertain the given interval priority weight becomes.

4 Group of Decision Makers

Interval AHP is introduced to the group decision making by aggregating individual opinions. Each decision maker gives pairwise comparisons for alternatives based on his/her subjective judgments. The comparison matrix given by the decision maker k, where $k = 1, ..., m$, is denoted as follows.

$$A_k = \begin{pmatrix} 1 & \cdots & a_{1nk} \\ \vdots & a_{ijk} & \vdots \\ a_{n1k} & \cdots & 1 \end{pmatrix} \ \forall k \quad (14)$$

In the following sections, the given judgments and the obtained priority weights are aggregated at the beginning and last stages of the decision making process, respectively. The procedures of these approaches are illustrated in Figure 1.

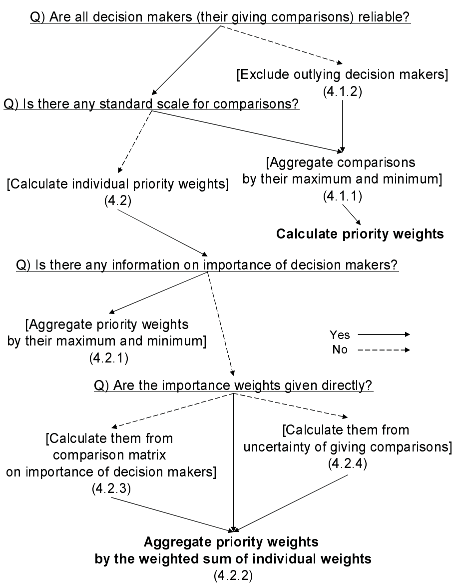

Figure 1: Steps for group decision making

4.1 *Aggregation of given judgments*

When the judgments which are the comparisons on alternatives in this paper are given, they can be aggregated at the beginning stage of group decision making process. It is usually said that the smaller group is more efficient since all the given judgments are reliable and tend not to be too much diverse. As for the well experienced group, for instance, the standard of judgments might be previously required or satisfied and then it is useful to aggregate the given judgments at the beginning stage. From the view of possibility concept, this is a natural and simple aggregation method. The decision makers can see their differences on their giving comparisons.

4.1.1 *Directly aggregation of comparisons*

The approach where the geometric mean of comparisons is taken has been proposed in [4, 5, 15, 16]. Since the aggregated comparison matrix satisfies (7), the eigenvector method can be also used to obtain the priority weights. In this section, the comparisons given by m decision makers are aggregated from the possibility view by taking their minimum and maximum.

$$A_{ij} = [\underline{a}_{ij}, \overline{a}_{ij}] = [\min_k a_{ijk}, \max_k a_{ijk}] \ \forall (i,j) \quad (15)$$

Since the aggregated comparisons are intervals, the inclusion constraints (12) can be written as follows.

$$A_{ij} \in \frac{W_i}{W_j} \ \Leftrightarrow \ \frac{\underline{w}_i}{\overline{w}_j} \leq \underline{a}_{ij}, \overline{a}_{ij} \leq \frac{\overline{w}_i}{\underline{w}_j} \quad (16)$$

where the aggregated comparisons are interaval and included in the approximated ones by the interval priority weights. The inclusion relation (16) with interval comparisons is an extension of (12) with crisp ones.

By solving the revised problem (13) from constraint (12) into (16), the bounds of the interval priority weights are obtained. The approximated comparison with them surely includes individually given comparisons.

4.1.2 *Finding and excluding outliers*

The decision making with a big group is also required. Instead of including opinions of all the decision makers in Section 4.1.1, the way to find outliers is proposed in this section. The decision makers whose opinions are apparently different from the others is called outliers. Such outliers are removed one by one till the number of removed decision makers reaches the pre-fixed number. It is noted that the number of appropriate decision makers should be determined beforehand. Then, the following function is used to measure his/her outlierness.

$$J_o = \sum_{(i,j)} \frac{\max_k a_{ijk}}{\max_{k \neq o} a_{ijk}} + \frac{\min_{k \neq o} a_{ijk}}{\min_k a_{ijk}} \geq n(n-1) \quad (17)$$

The evaluation function J_o represents the amount of extension by adding the decision maker o into the group. The increase of their upper bounds is measured as a ratio since the comparisons are ratio measure. As for the lower bounds, the decrease ratio is measured by the same way. The decision maker $\{o^* | J^*_o = \max_o J_o\}$, who gives the maximum of J is removed. If there are more than two decision makers who give the maximum, all of them are excluded. When the number of excluded decision makers reach the pre-fixed number, the comparisons given by the others are aggregated by (15) into the interval comparison matrix. The following process is the same as in Section 4.1.1.

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4.2 Aggregation of obtained individual preferences

The pariwise comparisons are relative values and each decision maker gives comparisons based on his/her own standard. In order to deal with these variations of comparisons, the decision makers' opinions are aggregated after obtaining individual preferences from respective given comparison matrix. First, the priority weights are obtained from the individually given pairwise comparison matrix. The interval priority weights based on the comparison matrix given by kth decision maker are denoted as $[\underline{w}_{ik}, \overline{w}_{ik}]$. Each decision maker can realize his/her priority weights on the alternatives, as well as others' ones. Then, in order to reach a consensus of the group, the obtained individual priority weights are aggregated.

4.2.1 Directly aggregation of priority weights

When there is not enough information on the importance of the decision makers, they are aggregated simply by taking their maximum and minimum.

$$W_i^1 = [\underline{w}_i, \overline{w}_i] = [\min_k w_{ik}, \max_k w_{ik}]\ \forall i \tag{18}$$

The obtained set of intervals are also interval probabilities by Property 1. The aggregated interval can be said to give an agreeable result, since the priority weights by all members are included in it. This is based on possibility concept and the assumption that all the decision makers give reasonable information. When the individual preferences are very different one another such as in a big group, the widths of the aggregated priority weights become large so that they are uncertain. The diversity of opinions is reflected in uncertainty of the aggregated weights. Based on the individual preferences, it is also possible to exclude outliers similarly in Section 4.1.2.

4.2.2 Aggregation with directly given importance weights

When the importance of decision makers and/or their confidence degrees of the given judgments are different, it is better to reflect such information. Therefore, the weighted sum of priority weights is proposed in the following sections.

When the importance of each decision maker is given by a supervisor, it is introduced as his/her weight directly. Denoting the given importance weight as $p_k \geq 0 \forall k$, where $\sum_k p_k = 1$, the aggregated priority weights are obtained as follows.

$$W_i^2 = [\sum_k p_k \underline{w}_{ik}, \sum_k p_k \overline{w}_{ik}]\ \forall i \tag{19}$$

As far as the weights are normalized so as to make the sum one, the set of the aggregated priorities is also interval probability by Property 2. The situation that there is not enough information is also represented by giving all the decision makers the same importance weights $1/m$. In the next section, it is represented differently from the view of interval probabilities.

4.2.3 Aggregation with importance weights by AHP

It is also possible to use conventional AHP and Interval AHP in order to induce the importance weights of decision makers. It is easier for a supervisor to give pariwise comparisons on importance of decision makers than to give their importance weights directly. The similar idea has been proposed in [6, 16], where a level of decision makers is added to the existing criteria levels. The importance weights of decision makers are determined at the level. Then, they are used as weights in the way of summing the local weights up to the overall priority weights. Using Interval AHP in obtaining weights, the overall weights are obtained as intervals in [17].

The supervisor gives comparisons on the importance of decision makers.

$$B = \begin{pmatrix} 1 & \cdots & b_{1m} \\ \vdots & b_{kl} & \vdots \\ b_{m1} & \cdots & 1 \end{pmatrix} \tag{20}$$

where each comparison represents how important is the decision maker comparing to the other.

Applying the conventional AHP in [3], the importance weights are obtained so as to make the sum one. Then, the following process is the same as in Section 4.2.2.

The Interval AHP formulated by (13) can be used to determine importance weights. The given comparisons on alternatives a_{ij} and the obtained priority weights $W_i = [\underline{w}_i, \overline{w}_i]$ in (13) are replaced into those on decision makers b_{kl} and their interval importance weights $P_k = [\underline{p}_k, \overline{p}_k]$, respectively. By solving the revised problem (13), the interval importance weights of decision makers are obtained. The problems to obtain the upper and lower bounds of the aggregated priority weight of alternative i are formulated as follows, respectively.

$$\begin{aligned} \underline{w}_i = & \min \textstyle\sum_k p_{ik} \underline{w}_{ik} \\ & \text{s.t. } \textstyle\sum_k p_{ik} = 1 \\ & \underline{p}_k \leq p_{ik} \leq \overline{p}_k\ \forall k \\ \overline{w}_i = & \max \textstyle\sum_k p_{ik} \overline{w}_{ik} \\ & \text{s.t. } \textstyle\sum_k p_{ik} = 1 \\ & \underline{p}_k \leq p_{ik} \leq \overline{p}_k\ \forall k \end{aligned} \tag{21}$$

In both problems, the variables are the importance weights p_{ik} included in the obtained interval importance weights. For each alternative i and its upper and lower bounds, the optimal solutions might be different. Denoting their optimal solutions for the alternative i as p_{ik*} and p_{ik}^*, respectively, the aggregated priority weight is obtained as the following intervals.

$$W_i^3 = [\sum_k p_{ik*} \underline{w}_{ik}, \sum_k p_{ik}^* \overline{w}_{ik}]\ \forall i \tag{22}$$

They are also interval probabilities by Properties 1 and 2.

From the view of interval probability, complete ignorance is represented as giving all decision makers the same interval weights $P_k = [0, 1]$. With them, the priority weights (22) by solving (21) is the same as the directly aggregated ones (18). Apparently, the aggregated priority weights with importance weights are included in the directly aggregated priority weights, that is, $W_i^2, W_i^3 \in W_i^1$. The results with importance weights are less uncertain. If any information on importance of decision makers is available, it is reasonable to introduce them as their importance weights.

4.2.4 Aggregation with importance weights based on uncertainty of information

As shown in Section 3, the sum of widths represents the uncertainty of the interval weights. The uncertainty of the given information may depend on the confidence or expert levels of decision makers on the problem. It is possible to assume such uncertainty degree of judgments as the decision maker's

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importance among the group. The importance weights of the decision makers are determined by reflecting uncertainty of their information. The decision maker who gives less uncertain comparisons will be evaluated as more important. The importance weight of decision maker l is as follows.

$$p_l = \frac{\sum_{k \neq l} I_k}{(m-1)\sum_k I_k} \tag{23}$$

where I_k is the optimal objective value of (13) with the given information by the kth decision maker and represents his/her uncertainty degree. It is noted that their sum is one, that is, $\sum_l p_l = (\sum_l \sum_{k \neq l} I_k)/((m-1)\sum_k I_k) = 1$. The importance weight of the decision maker who gives interval weights with great widths becomes small.

The aggregated priority weights are denoted as follows and also interval probabilities by Property 2.

$$W_i^4 = [\sum_k p_l \underline{w}_{ik}, \sum_k p_l \overline{w}_{ik}] \ \forall i \tag{24}$$

5 Numerical example

Assuming four decision makers, $k = 1, 2, 3, 4$, each of them gives the pairwise comparison matrix on four alternatives, $i = 1, 2, 3, 4$, shown in Table 1. The interval priority weights W_k and their uncertainty indices I_k are obtained by (13) and its optimal objective function value, respectively.

Table 1: Comparison matrices by four decision makers

A_1				W_1	
1	2	3	4	0.500	J_1=12
	1	2	3	0.250	I^1=0.083
		1	2	[0.125,0.167]	p_1=0.297
			1	[0.083,0.125]	
A_2				W_2	
1	2	4	6	0.545	J_2=13.2
	1	3	4	0.273	I^1=0.109
		1	4	[0.091,0.145]	p_2=0.285
			1	[0.036,0.091]	
A_3				W_3	
1	3	3	4	0.571	J_3=13.5
	1	2	4	0.190	I^1=0.190
		1	1	[0.095,0.190]	p_3=0.250
			1	[0.048,0.143]	
A_4				W_4	
1	1	2	2	0.375	J_4=16.5
	1	3	1	[0.219,0.375]	I^1=0.375
		1	3	[0.125,0.188]	p_4=0.168
			1	[0.063,0.219]	

5.1 Aggregation of comparisons

By (15), the comparisons given by four decision makers are aggregated and shown in Table 2. The interval priority weights are obtained by solving (13) with constraints (16). Each decision maker can accept the results more or less, since they are based on his/her giving comparisons by aggregating them into interval comparisons.

Table 2: Aggregated comparison matrix

A (aggregated)				W
1	[1,3]	[2,4]	[2,6]	[0.333,0.500]
	1	[2,3]	[1,4]	[0.167,0.333]
		1	[1,4]	[0.111,0.167]
			1	[0.042,0.167]

In this example, there are only four decision makers so that there is no need to exclude any of them. However, in order to make sure how the outliers are found, the appropriate number of decision makers is assumed as three so that one of four decision makers will be excluded. The evaluation function values J_k by (17) are shown in the right column of Table 1. They represent the changes of aggregated comparisons' bounds by excluding each decision maker and are measured as the sum of increase and decrease ratios. The fact, $J_1 = 12 = n(n-1)$, tells that A_1 does not effect the bounds of aggregated comparisons. Since J_4 is the maximum of the four, the others, A_1, A_2 and A_3, are aggregated and the comparison matrix is transformed from Table 2 into Table 3. The obtained interval priority weights are also shown in Table 3. Since outlier A_4 is excluded, the interval priority weights in Table 3 is less uncertain than those in Table 2.

Table 3: Aggregated comparison matrix without outlier A_4

A (aggregated)				W
1	[2,3]	[3,4]	[4,6]	0.522
	1	[2,3]	[3,4]	[0.174,0.261]
		1	[1,4]	[0.087,0.174]
			1	[0.043,0.130]

5.2 Aggregation of priority weights

Instead of aggregating comparisons at the beginning stage, it is possible to aggregate the calculated individual preferences at the last stage. The obtained individual priority weights shown in Table 1 help the decision makers realize their own preferences, as well as understand one another.

The aggregated priority weights with and without the importance weights of decision makers are shown in Table 5. Its left column represents the case with no information on the importance of decision makers so that they are aggregated directly by (18). The results equal to those obtained by (21) with $P_k = [0, 1] \forall k$. All decision makers' individual priority weights are included in the aggregated ones, $W_k \in W^1$ $k = 1, 2, 3, 4$. In this sense, the results are easily acceptable for all decision makers. On the other hand, the right two columns represent the cases where the importance weights of decision makers are determined based on Interval AHP and the uncertainty of their giving judgments, respectively. The comparison matrix for importance of decision makers, B, is given by a supervisor and the obtained interval importance weights, P, are shown in Table 4. With the importance weights, the priority weight of each alternative W^3 is obtained as an interval by (21). The uncertainty of the given judgments, I_k, which is the

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optimal function value of (13) and the respective importance weight, p_k, calculated by (23) are shown in Table 1. The A_1 and A_4 who give the least and most uncertain information are considered as the most and least important, respectively. With the importance weights, the priority weights of alternatives W^4 are obtained as intervals by (24). The uncertainty of W^3 and W^4 is less than that of W^1 which represents complete ignorance on decision makers' importance, since their widths of intervals are smaller. The individually obtained priory weights are reflected depending on their importance among the group.

Table 4: Comparison matrix on importance of decision makers

B				P
1	2	2	4	0.462
	1	3	4	[0.231,0.317]
		1	3	[0.106,0.231]
			1	[0.077,0.115]

Table 5: Aggregated interval priority weights with and without importance weights of decision makers

W^1	Interval AHP W^3	uncertainty W^4
[0.375,0.571]	[0.508,0.517]	0.510
[0.190,0.375]	[0.239,0.265]	[0.236,0.263]
[0.091,0.190]	[0.110,0.169]	[0.108,0.170]
[0.036,0.219]	[0.062,0.131]	[0.058,0.136]

6 Conclusion

The group decision support system based on Interval AHP has been discussed. Interval AHP is particularly suitable for dealing with uncertainty of human intuitive judgments. By Interval AHP the priority weights of alternatives and importance weights of decision makers are obtained as interval from the given pariwise comparison matrix. The obtained interval priority weights reflect all the possibilities in the given information. The approaches of aggregating group members' opinions at two different stages have been proposed. At the beginning stage, the given judgments are aggregated from the possibility view and the outliers are excluded if there are some. From the aggregated comparison matrix whose elements are intervals, the priority weights of alternatives are obtained by Interval AHP as intervals. The other approach is to aggregate individual preferences at the last stage. At first, from each given comparison matrix the interval priority weights are obtained by Interval AHP. They represent the decision makers' individual preferences. Then, they are aggregated based on the possibility view or the average with the importance weights of the group members. The importance weights of decision makers can be given by a supervisor directly, from a pariwise comparison matrix for their importance or based on uncertainty of their giving information. The aggregated preferences by the proposed approaches help the group members to understand one another and reach consensus.

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