Approximate Reasoning in Surgical Decisions

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Abstract*—Approximate reasoning is one of the most effective fuzzy systems. The compositional rule of inference founded on the logical law modus ponens is furnished with a true conclusion, provided that the premises of the rule are true as well. Even though there exist different approaches to an implication, being the crucial part of the rule, we modify the early implication proposed by Zadeh* [1] *in our practical model concerning a medical application. The approximate reasoning system presented in this work considers evaluation of a risk in the situation when physicians weigh necessity of the operation on a patient. The patient's clinical symptom levels, pathologically heightened, indicate the presence of a disease possible to recover by surgery. We wish to evaluate the extension of the operation danger by involving particularly designed fuzzy sets in the algorithm of approximate reasoning.*

Keywords—Approximate reasoning, compositional rule of inference, Zadeh's implication, operation risk, symptom levels, parametric membership functions.

1 Introduction

The technique of approximate reasoning, earliest evolved by Zadeh [1, 2] quickly found many adherents who differentiated the foundations of the theory. Especially, the changes concerned the implication IF…THEN…ELSE…, which constitutes an important factor of the reasoning system. In [3, 4, 5] we can trace the discussion revealing definitions of the implication generated by Kleene and Denies, Willmot, Mamdani and Assilian, Larsen, Gödel et al. The trials of inserting individually created operations on fuzzy sets discern the approaches mentioned above. Even the item of compositional rule of inference was debated from separate points of views [6, 7, 8, 9, 10]. We can mention the Yager conception [10] and the Sugeno design [3] as the most original modifications of the initial version of the rule.

For a practitioner an applicable meaning of approximate reasoning is essential, especially in technique and natural sciences where vagueness of input and output is often expected. Although some technical trials of applications are remarkable, it can happen coincidentally to counterpart the approximate reasoning in medicine. The only contribution in the topic, found by the author in [11], is a discussion of the model employing a pharmacological example.

Since members of surgical staff make decisions about operations on severely-ill patients with the highest care then we wish to support these verdicts by results coming from reasoning systems. We adopt Zadeh's approach to the rule [1, 2, 12], which is slightly modified by us and based on Lukasiewicz's definition of the fuzzy implication [1, 6, 12]. We still find this rule to be the most appealing for the reason of simply performed operations and clearly interpretable results. Then we build an own original apparatus accommodated to medical assumptions. Particular fuzzy sets that contain input data and output effects are designed in compliance with the physician's hint. The discussion about how to find the objective of reasoning, i.e. operation risk, is accomplished in Section 2. Fuzzy sets, taking place in the model, are furnished with appropriate membership degrees in Section 3. Section 4, added as a presentation of efficiency of the algorithm, reveals some risks in cancer surgery.

2 Adoption of approximate reasoning to operation decisions

For patients, who suffer from e.g. cancer, decisions concerning their operations are made with the highest thoughtfulness. In the later or the last stage of the disease the possibility to cure the patient totally of cancer by operating him/her for tumors is rather little. As a physician does not want the patient to run the risk to suffer even more after an unnecessary operation, he ought to judge thoroughly the consequences of the surgery.

We intend to involve approximate reasoning to support mathematically the extraction of a proper decision when discerning the operation danger. The most decisive clinical symptoms found in an individual patient will be taken into consideration to evaluate the risk.

Let us ponder a logical compound statement

$$\begin{aligned}&\text{IF}\,(p\ \text{AND}\,((\text{IF}\ p\ \text{THEN}\ q)\ \text{ELSE}\,(\text{IF}\,(\text{NOT}\ p)\\&\text{THEN}\,(\text{NOT}\ q)))\,)\ \text{THEN}\ q\end{aligned} \tag{1}$$

whose primitive statements p and q are included in the equivalent form of (1) derived as

$$p \wedge ((p \rightarrow q) \wedge (\neg p \rightarrow \neg q)) \rightarrow q. \tag{2}$$

The logical joint ELSE is interpreted in (2) as the conjunction $\wedge$ in compliance with the suggestions made by Lukasiewicz and Zadeh [1, 6].

The logical statement (2) is a tautology, which can be easily confirmed by the method of truth tables. We also prove that thesis q in (2) will become true if the premises p and $(p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$ constitute true statements as well. In order to accomplish the last proof we utilize the method of denying the truth of the thesis q. Let $\nu(p)$ and $\nu(q)$ denote the truth values of p and q according to the convention of binary logic. If, on behalf of the proof, we

ISBN: 978-989-95079-6-8

assume that the thesis q is not true then $\nu(q) = 0$. From the previous assumption $\nu((p \to q) \wedge (\neg p \to \neg q)) = 1$ if $\nu(p \to q) = 1$ and $\nu(\neg p \to \neg q) = 1$. But $\nu(q) = 0$, which suggests that $\nu(p) = 0$ as well to warrant $\nu(p \to q) = 1$. On the other hand we have already assumed that premise p is true. As the suggestion $\nu(q) = 0$ leads to the contradiction "p is false" against "p is true" then we will accept $\nu(q) = 1$.

In accordance with the extended law *modus ponens* proposed by Zadeh [1, 2] we interpret (2) as a sentence

IF
p` (premise)
AND (3)
(IF *p* THEN *q*) ELSE (IF (NOT *p*) THEN (NOT *q*)) (premise)
THEN
q` (thesis)

provided that the semantic meaning of *p* and *p*` (*q* and *q*` respectively) is very close.

Let *p* be visualized by a fuzzy set *P* in the universe *X* and let *q* be expressed by another fuzzy set *Q* in the universe of discourse *Y*. Analogously, the fuzzy set $P` \subset X$ constitutes a mathematical formalization of the primitive statement *p*` whereas $Q` \subset Y$ replaces formally the sense of *q*`. The modus ponens rule thus becomes

IF
p` = *P*` (premise)
AND (4)
(IF *p* = *P* THEN *q* = *Q*) ELSE (IF (NOT *p* = *CP*) THEN (NOT *q* = *CQ*)) (premise)
THEN
q`= *Q*` (thesis)

The sets *CP* and *CQ* are complements of *P* and *Q*.

When making a feedback to the medical task previously outlined, we wish to use a technique of accommodating actual theoretical assertions to concrete formulations letting us evaluate the operation decision in some grades of risk.

Let *S* denote a symptom possessing the most decisive power in the evaluation of the operation risk. We regard *S* as either the complex qualitative symptom or the symptom whose intensity is assimilated with level codes. These codes, determined for both descriptions of *S*'s complexion, form the universe *X* = "*symptom levels*" = {1,…,*k*,…,*n*}. Let us assume that level 1 is associated with the slightly heightened symptom values whereas level *n* indicates their critical status.

The statement *p*`

p` = "symptom *S* is found in patient on level *k*"

is now addressed to a fuzzy set *P*` introduced by

$$P` = \frac{\mu_{P`}(1)}{1} + \cdots + \frac{\mu_{P`}(k)}{k} + \cdots + \frac{\mu_{P`}(n)}{n}. \quad (5)$$

The sentence *p* built by

"*p* = "rising levels of *S* are essential for operation risk"

is dedicated to a fuzzy set *P* given by

$$P = \frac{\mu_P(1)}{1} + \cdots + \frac{\mu_P(k)}{k} + \cdots + \frac{\mu_P(n)}{n}. \quad (6)$$

Another category of elements, constituting a content of the universe *Y*, is determined in the model as risk grades. We set risk grades in *Y* = "*operation risk grades*" = {L_0 = "*none*", L_1 = "*little*", L_2 = "*moderate*", L_3 = "*great*", L_4 = "*total*"}, on condition that *Y* is experimentally restricted to five risk grades only.

For sentence *q*

q = "operation risk exists for patient"

a creation of a fuzzy set *Q* is supported by

$$Q = \frac{\mu_Q(L_0)}{L_0} + \frac{\mu_Q(L_1)}{L_1} + \frac{\mu_Q(L_2)}{L_2} + \frac{\mu_Q(L_3)}{L_3} + \frac{\mu_Q(L_4)}{L_4}. \quad (7)$$

At last, we define *q*` containing the final risk judgment as a statement

q` = "patient runs estimated risk of being operated",

where *risk* is graded by membership degrees of the corresponding fuzzy set *Q´* proposed as

$$Q` = \frac{\mu_{Q`}(L_0)}{L_0} + \frac{\mu_{Q`}(L_1)}{L_1} + \frac{\mu_{Q`}(L_2)}{L_2} + \frac{\mu_{Q`}(L_3)}{L_3} + \frac{\mu_{Q`}(L_4)}{L_4}. \quad (8)$$

In the next paragraph we accomplish the discussion about an apparatus providing us with membership degrees of sets (5)–(8).

Due to *modus ponens* rule (4) we set all decision data in the scheme

IF
"symptom *S* is found in patient on level *k*" = *P*` (premise)
AND
(IF "rising levels of *S* are essential for operation risk" = *P* THEN "operation risk exists for patient" = *Q*) ELSE (IF ("rising levels of *S* are not essential for operation risk" = *CP* THEN operation risk does not exists for patient = *CQ*) (premise)
THEN
"patient runs estimated risk of being operated" = *Q*` (thesis)

In conformity with [1, 6, 12] we first prognosticate a mathematical expression of the implication

ISBN: 978-989-95079-6-8

(IF "rising levels of S are essential for operation risk" = P THEN "operation risk exists for patient" = Q) ELSE (IF ("rising levels of S are not essential for operation risk" = CP THEN operation risk does not exists for patient = CQ)

performed as matrix R. Even though several approaches to membership functions of implications were made [1, 2, 3, 5, 6, 8, 12] we still feel attracted by the Lukasiewicz [6, 12] conception of fuzzy implication R with a membership function derived as

$$\begin{aligned}\mu_R(k,L_l) &= 1 \wedge ((1-\mu_P(k)) + \mu_Q(L_i)) \\ &\wedge (\mu_P(k) + (1-\mu_Q(L_i))),\end{aligned} \tag{9}$$

$k = 1,\ldots,n$, $l = 0,\ldots,4$, for all $x \in X$ and all $y \in Y$.

The membership degrees of set $Q`$ will be visualized after composing set $P`$ with relation R due to Zadeh's compositional rule [1]

$$Q` = P' \circ R \tag{10}$$

designated by the membership function

$$\mu_{Q`}(L_l) = \max_{k \in X}(\min(\mu_{P`}(k), \mu_R(k,L_l))) . \tag{11}$$

The comparisons of magnitudes of membership degrees in set $Q`$ yield indications referring to judgments of the risk grades after consideration of symptom level k verified in the patient.

As the operations of maximum and minimum have a tendency to filter the input data, which sometimes does not result in a clear-cut decision, then we will propose another set of composition operations in (10). In accordance with [13] we propose

$$Q` = P' \underset{\cdot +}{\circ} R \tag{12}$$

assisted by membership degrees

$$\mu_{Q`}(L_l) = \frac{\sum_{k=1}^{n} \mu_{P`}(k) \cdot \mu_R(k,L_l)}{\sum_{k=1}^{n} \mu_R(L_l)} . \tag{13}$$

To be able to apply (13) we ought to prove that the value of the quotient $\mu_{Q`}(L_l)$ is a number belonging to the interval [0, 1]. To verify this we first notice that $\mu_{P`}(k) \cdot \mu_R(k,L_l) \le \mu_R(k,L_l)$ since both $\mu_{P`}(k)$ and $\mu_R(k,L_l)$ are less than one for all k and l, $k = 1,\ldots,n$, $l = 0,\ldots,4$. This causes the value of a product to be lesser than the values of both factors. We thus conclude that the numerator is less than or equal to the denominator, which guarantees that the entire value of the quotient is a member of [0, 1]; therefore it can be approved as a membership degree of L_l coming from the support of $Q`$.

We also notice that the sum placed in the denominator of the quotient never becomes equal to zero, since almost all risk grades will be designed as positive quantities. This assumption prohibits membership degrees of the risk grades from being undefined structures.

Values $\mu_R(k,L_l)$ are adaptable to be treated as weights of level importance assigned to a distinct risk. These, as the entries of matrix R are invariants in the system promoting the same diagnostic model, contrary to information concerning different patients that is changeable. And, additionally, we can prove that operation (13) satisfies the criteria of OWA operators [13].

3 Mathematical design of data sets

The decision model designed in Section 2 includes operations on fuzzy sets furnished with symbolically established membership degrees. In the current paragraph we put some life into theoretical symbols by assigning to them mathematical structures. The set $P`$ a.k.a. (5) now gets assigned

$$\begin{aligned}P` &= \frac{\mu_{P`}(1)}{1} + \cdots + \frac{\mu_{P`}(k)}{k} + \cdots + \frac{\mu_{P`}(n)}{n} \\ &= \cdots + \frac{\frac{n-2}{n}}{k-2} + \frac{\frac{n-1}{n}}{k-1} + \frac{1}{k} + \frac{\frac{n-1}{n}}{k+1} + \frac{\frac{n-2}{n}}{k+2} + \cdots\end{aligned} \tag{14}$$

for the k^{th} symptom level certified in the patient examined.

Another set P, concerning the same symptom levels in the support, is found by (6) and modified as

$$\begin{aligned}P &= \frac{\mu_P(1)}{1} + \cdots + \frac{\mu_P(k)}{k} + \cdots + \frac{\mu_P(n)}{n} \\ &= \frac{\frac{1}{n}}{1} + \cdots + \frac{\frac{k}{n}}{k} + \cdots + \frac{\frac{n}{n}}{n},\end{aligned} \tag{15}$$

due to the previously made assumptions, which suggest the tendency to ascending values of the membership degrees in P.

The set Q is more sophisticated to design as a fuzzy set whose support consists of other fuzzy sets L_l, $l = 0,\ldots,4$, commonly defined in a symbolic risk reference set Z = [0, 1]. We also intend to determine the membership degrees of Q as some characteristic quantities from [0, 1]. Evaluation of these numbers is founded on a procedure involving a linguistic variable

"*operation risk grades*" = {L_0 = "*none*", L_1 = "*little*", L_2 = "*moderate*", L_3 = "*great*", L_4 = "*total*"},

experimentally restricted to five risk grades only.

We first fuzzify the expressions concerning the items of the list to continue further with their defuzzification in order to attach numerical equivalents to the words from the list. Each word assists now a fuzzy set L_l, l = 0, 1, 2, 3, 4, whose constraint is grounded on an s-class mapping defined for z in Z = [0, 1] as [14]

ISBN: 978-989-95079-6-8

$$\mu_{L_l}(z) = \mu_{L_0(l)}(z) = \begin{cases} \text{left}(\mu_{L_0(l)}(z)) = \\ \text{right}(\mu_{L_0(l)}(z)) = \end{cases} \tag{16}$$

$$s(z, \alpha_{L_0}, \beta_{L_0}, \gamma_{L_0}, l \cdot h), \qquad \text{for} \quad z \le \gamma_{L_0},$$
$$1 - s(z, \alpha_{L_0} + h, \beta_{L_0} + h, \gamma_{L_0} + h, l \cdot h) \quad \text{for} \quad z > \gamma_{L_0}.$$

We clarify the fact that formulas of all membership functions are derived from only one predetermined subject defining $\mu_{L_0}(z)$. The equality $\mu_{L_l}(z) = \mu_{L_0(l)}(z)$ reveals that $\mu_{L_l}(z)$ is dependent on a parameter l equal to level number l, l = 0,…,4. The h unit determines a distance between α_{L_l} and $\alpha_{L_{l+1}}$ (respectively β_{L_l} and $\beta_{L_{l+1}}$ or γ_{L_l} and $\gamma_{L_{l+1}}$) for symmetric functions s.

We prepare constraints for L_0, which are affected by $\alpha_{L_0} = -0.25$, $\beta_{L_0} = -0.125$ and $\gamma_{L_0} = 0$ as

$$\text{left}(\mu_{L_0}(z)) = \begin{cases} 2\left(\frac{z-(-0.25)}{0-(-0.25)}\right)^2 \\ \text{for} \quad -0.25 \le z < -0.125, \\ 1 - 2\left(\frac{z-0}{0-(-0.25)}\right)^2 \\ \text{for} \quad -0.125 \le z < 0, \end{cases} \tag{17}$$

and

$$\text{right}(\mu_{L_0}(z)) = \begin{cases} 1 - 2\left(\frac{z-0}{0.25-0}\right)^2 \\ \text{for} \quad 0 \le z < 0.125, \\ 2\left(\frac{z-0.25}{0.25-0}\right)^2 \\ \text{for} \quad 0.125 \le z < 0.25. \end{cases} \tag{18}$$

By inserting in (17) and (18) the current value l, l = 0,…,4, and the distance h, casually determined as h = 0.25, we obtain a formula of the left branch of L_l

$$\text{left}(\mu_{L_l}(z)) = \begin{cases} 2\left(\frac{z-(-0.25+l\cdot 0.25)}{0-(-0.25)}\right)^2 \\ \text{for} \ -0.25 + l \cdot 0.25 \le z < -0.125 + l \cdot 0.25, \\ 1 - 2\left(\frac{z-(0+l\cdot 0.25)}{0-(-0.25)}\right)^2 \\ \text{for} \ -0.125 + l \cdot 0.25 \le z < 0 + l \cdot 0.25, \end{cases} \tag{19}$$

and a function shaping its right branch

$$\text{right}(\mu_{L_l}(z)) = \begin{cases} 1 - 2\left(\frac{z-(0+l\cdot 0.25)}{0.25-0}\right)^2 \\ \text{for} \ 0 + l \cdot 0.25 \le z < 0.125 + l \cdot 0.25, \\ 2\left(\frac{z-(0.25+l\cdot 0.25)}{0.25-0}\right)^2 \\ \text{for} \ 0.125 + l \cdot 0.25 \le z < 0.25 + l \cdot 0.25. \end{cases} \tag{20}$$

Figure 1 collects plots of L_0–L_4 in conformity with different values of l included in (19) and (20).

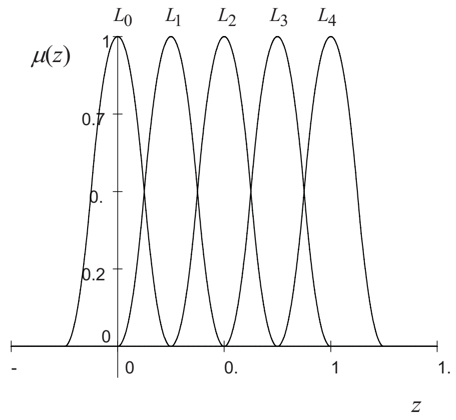

Figure 1: The terms of "*operation risk grades*" as fuzzy sets L_0–L_4

Actually, we have an intension to emphasize the meaning of parametric nature of the L_l membership functions, which deprives the model of many distinct formulas. Apart from this advantage we focus on generating the functions that represent elegant structures mathematically expressed.

In the process of defuzzification we consider only z-values for which the sets L_0–L_4 get the status of normal sets, i.e., z = 0, z = 0.25, z = 0.5, z = 0.75 and z = 1. For these, another fuzzy set "*numerical operation risk*" is projected by developing its membership function in the form of

$$\mu_{\text{"numerical operation risk"}}(z) = \begin{cases} 2\left(\frac{z-0}{1-0}\right)^2 \\ \text{for} \ 0 \le z < 0.5, \\ 1 - 2\left(\frac{z-1}{1-0}\right)^2 \\ \text{for} \ 0.5 \le z < 1. \end{cases} \tag{21}$$

Via the selected z-quantities above, we tie their membership degrees calculated by means of (21) to expressions from the list in order to establish relations between words and their numerical replacements. Therefore, the set Q finally obtains a shape of

$$Q = \frac{0}{L_0} + \frac{0.125}{L_1} + \frac{0.5}{L_2} + \frac{0.875}{L_3} + \frac{1}{L_4}. \tag{22}$$

We now wish to demonstrate the action of approximate reasoning accustomed to the judgment of surgical risk.

4 Risks grades in cancer surgery

In patients, who suffer from cancer as the recognized diagnosis, one of the symptoms, namely, *CRP* (*C*-reactive proteins) is carefully measured and discussed with a view to make a decision about accomplishing a successful operation. The heightened values of *CRP* (measured in milligrams per liter) are theoretically discerned in four levels stated as

1 = "*almost normal*" for *CRP* < 10,
2 = "*heightened*" if 10 ≤ *CRP* ≤ 20,
3 = "*very heightened*" if 20 ≤ *CRP* ≤ 25,

ISBN: 978-989-95079-6-8

4 = "*dangerously heightened*" for *CRP* > 25.

Due to (15) set *P* is expressed as

$$P = \frac{0.25}{1} + \frac{0.5}{2} + \frac{0.75}{3} + \frac{1}{4} \qquad (23)$$

in $X = \{1,\ldots,4\}$.

Suppose that an individual patient examined reveals the *CRP*-value to be 23. *CRP* is thus classified in level 3 and set *P*` characteristic of the patient is stated in the form of

$$P` = \frac{0.5}{1} + \frac{0.75}{2} + \frac{1}{3} + \frac{0.75}{4} \; . \qquad (24)$$

according to (14).

The sets (23) and (22) together with

$$\mathrm{C}P = \frac{0.75}{1} + \frac{0.5}{2} + \frac{0.25}{3} + \frac{0}{4} \qquad (25)$$

and

$$\mathrm{C}Q = \frac{1}{L_0} + \frac{0.875}{L_1} + \frac{0.5}{L_2} + \frac{0.125}{L_3} + \frac{0}{L_4} \qquad (26)$$

generate matrix *R* with the entries computed in compliance with (9). *R* is expanded as a two-dimensional table

$$R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{ccccc} L_0 & L_1 & L_2 & L_3 & L_4 \end{array} \\ \begin{bmatrix} 0.75 & 0.875 & 0.75 & 0.375 & 0.25 \\ 0.5 & 0.625 & 1 & 0.625 & 0.5 \\ 0.25 & 0.375 & 0.75 & 0.875 & 0.75 \\ 0 & 0.125 & 0.5 & 0.875 & 1 \end{bmatrix} \end{array} \qquad (27)$$

which, inserted in (10) for *P*` determined by (24), provides us with

$$Q` = \frac{0.5}{L_0} + \frac{0.625}{L_1} + \frac{0.75}{L_2} + \frac{0.875}{L_3} + \frac{0.75}{L_4} \; . \qquad (28)$$

By interpreting the meaning of (28) we understand that there exists a risk when considering an operation in patient whose *CRP*-index is evaluated on the third level. The most possible risk is evaluated as "*great*" according to the highest quantity of the membership degree. The total danger of accomplishing the surgical operation is evaluated as essential with the membership degree 0.75.

Even the results of implementing (13) given as

$$Q` = \frac{0.66}{L_0} + \frac{0.69}{L_1} + \frac{0.75}{L_2} + \frac{0.795}{L_3} + \frac{0.725}{L_4} \; . \qquad (29)$$

fully confirm the risk extension judged by (28).

We hope that the classical model of approximate reasoning, modified by us and adapted to the problem of operation decision can constitute its complementary solution, especially when a decision of saving somebody's life via surgery is crucial.

5 Conclusions

Via the way of forming the text of this paper we have already come to substantial conclusions. We only summarize that we have used approximated reasoning to introduce the own initial interpretation of the system to approximate the operation risk concerning patients with rising values of a biological index. The formulas of membership degrees and membership functions have been expanded by applying a formal mathematical design. We expect that the study makes a contribution in the domain of mathematical models projected for medical applications.

In future works we wish to examine a model consisted of several symptoms that are divided in different numbers of levels. The symptoms should be included in the pattern simultaneously, which may expose some internal interactions among them. In other words, the operation risk will be a criterion that can employ many data factors. We count on finding some helpful remarks in [15] to implement an algorithm supporting the method newly planned.

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ISBN: 978-989-95079-6-8