Differential Equations based on Fuzzy Rules

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Abstract— *The purpose of this paper is to use the Fuzzy Set Theory, specially fuzzy rule-based systems, as a mathematical tool for modeling dynamical systems given by differential equation, whose direction field is f (IVP-f). As fuzzy systems methodology produces input-output systems, that is, it produces m outputs from n inputs, one can see it as $f_r : \mathbb{R}^n \to \mathbb{R}^m$, where r is the number of rules of the rule base. Under certain assumptions, one can prove that the functions (f_r) have good analytical properties. Beyond it, they can approximate theoretical functions (f). The main result of this paper states that the solutions x of an ODE whose direction field is f (IVP-f) can be approximated by the solutions x_r of an ODE whose direction field is f_r (IVP-f_r). By utilizing important theorems as the Lebesgue's Dominated Convergence Theorem it was possible to prove the proposed theorem. Finally we use a numerical method (Runge-Kutta) to simulate solutions x_r^n that approximates x_r when n increases. The solutions x_r^n approximates the solution x of IVP-f also provided $\{x_r^n\} \overset{n\to\infty}{\longrightarrow} x_r \overset{r\to\infty}{\longrightarrow} x$.*

Keywords— approximation theory, dynamical systems, fuzzy systems, initial value problem, numerical solutions.

1 Introduction

The investigation of a physical (biological) phenomena using differential equations needs knowledgement of the direction field. That is, it is necessary to adopt a theoretical function to represent the variation rates (derivatives) as function of the state variables. For deterministic differential equations, this direction field is a function of real variables that models its variation in time. Traditionally, aleatory uncertainties have been studied from Differential Equations in two ways: first, considering the state variables as aleatory variables. In this case it is necessary to define a variation rate (or derivative) of an aleatory variable. So, it is called stochastic differential equation. Second, considering only the aleatory parameters (therefore, it is not necessary a new definition of variation rate). These are called aleatory differential equations [1, 2]. Recently, the use of Fuzzy Set Theory to incorporate uncertainties in differential equations has been increased, so we have the fuzzy differential equations. Like the stochastic case, there are many proposals for studying these equations. The first one, using the notion of fuzzy derivative, is originated of the Hukuhara's derivative used to multifunctions and introduced in the fuzzy area by Puri and Ralescu [3]. Another notions do not require derivative to fuzzy variables. The variation rate is the same as the classical one, but some proposals have already been suggested: use differential inclusions [4, 5], use some fuzzification procedure for a deterministic solution [6], etc. In each one of these proposals there is an equation. That is, the derivatives are given by direction fields that depend on the state variables. Our research suggests a new methodology to study fuzzy differential equations using the fuzzy set theory. It will not be required to have the variation rates explicitly as a function of the state variables. We propose a fuzzy relation to model the association between the state variables and the direction field. We consider the variation rate and the state variables as linguistic variables. If no defuzzification procedure is used or if the used method produces an interval [7, 8], then the system can be studied with the aid of differential inclusion theory. The methodology we propose here uses some defuzzification method, which produces a crisp solution (real number or real vector) for each instant of time t. Regarding fuzzy differential equations, the methodology we proposed differs from previous methodologies in its formulation. While other methodologies use the concept of derivative (classical or fuzzy), our methodology uses a fuzzy formulation given by a rule base. More specifically, we adopt a fuzzy rule-based system (FRBS) to represent the differential equation. The FRBS methodology has been very important to solve modeling problems, since it is a universal approximator. For universal approximator we mean something that can approximate a continuous functions with the desired accuracy in compact sets. Thus, it is expected that fuzzy rule-based systems can model a direction field when the direction field is only partially known. Works such as Nguyen [9, 10] were extremely useful in our research. The main purpose of this work is to use fuzzy systems to obtain functions (f_r) in accordance with certain properties of the phenomenon being modeled. These families of functions will be constructed in such a way that they will be universal approximators of the function f that represents a theoretical field. Our greatest interest is to study the solutions of these new dynamical systems, given by the direction field (f_r) and establish conditions that ensure convergence of its solutions to the solution of the theoretical problem. The structure of this paper is as follows: in Section 2 we give all basic concepts and definitions the reader needs for a good understanding of our work. In Section 3 we present the problem we must solve, that is, the new IVP given by a fuzzy rule-based system. In Section 4 we present our main result as a theorem and then we prove it. Finally, in Section 5 we show an example to illustrate our result.

2 Basic Concepts and Definitions

2.1 *Fuzzy Set Theory*

An ordinary subset A of a set U is determined by its characteristic function χ_A defined by

$$\chi_A(x) = \begin{cases} 1, & if\ x \in A \\ 0, & if\ x \notin A \end{cases}.$$

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The characteristic function of a subset A of a set U specifies whether or not an element is in A. Zadeh [11] generalized this notion by allowing images of elements to be in the interval $[0,1]$ rather than being restricted to the two element set $\{0,1\}$.

Definition 1 *A fuzzy subset A of a set U is determined by a function $\varphi_A : U \to [0,1]$, where the number $\varphi_A(x)$ represents the degree of membership of the element x to the fuzzy subset A.*

Definition 2 *A binary operation $\Delta : [0,1] \times [0,1] \to [0,1]$ is a* t-norm *if it satisfies the following:*

1. $\Delta(1,x) = 1\Delta x = x$;
2. $\Delta(x,y) = x\Delta y = y\Delta x = \Delta(y,x)$;
3. $x\Delta(y\Delta z) = (x\Delta y)\Delta z$;
4. *If $x \leq u$ and $y \leq v$ then $x\Delta y \leq u\Delta v$.*

The operator t-norm models the connective "and".

Definition 3 *A binary operation $\nabla : [0,1] \times [0,1] \to [0,1]$ is a* t-conorm *if and only if*

1. $\nabla(0,x) = 0\nabla x = x$;
2. $\nabla(x,y) = x\nabla y = y\nabla x = \nabla(y,x)$;
3. $x\nabla(y\nabla z) = (x\nabla y)\nabla z$;
4. *If $x \leq u$ and $y \leq v$ then $x\nabla y \leq u\nabla v$.*

The operator t-conorm models the connective "or".

Definition 4 *By a* defuzzification procedure *$\mathcal{D}$ we mean a mapping that transforms a membership function $\varphi(x)$ into a number and satisfies the following properties:*

1. *if $\varphi(x) = 0$ for all $x \in (-\infty, a)$, then $\mathcal{D}(\varphi) \geq a$;*
2. *if $\varphi(x) = 0$ for all $x \in (-\infty, a]$, then $\mathcal{D}(\varphi) > a$;*
3. *if $\varphi(x) = 0$ for all $x \in (a, +\infty)$, then $\mathcal{D}(\varphi) \leq a$;*
4. *if $\varphi(x) = 0$ for all $x \in [a, +\infty)$, then $\mathcal{D}(\varphi) < a$.*

2.2 Fuzzy Rule-Based System

A Fuzzy Rule-Based System (FRBS) (Fig. 1) is an input-output system based on fuzzy rules. It is composed of four components: an input processor; a collection of linguistic rules, called rule base; a fuzzy inference method and an output processor, which generates a real number as output.

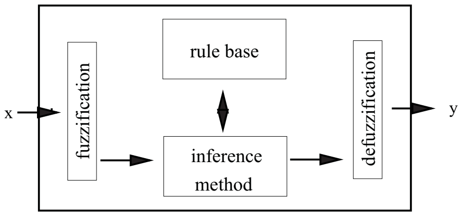

Figure 1: A FRBS scheme.

The fuzzification is the stage where the system's entries are modeled by fuzzy sets. The membership functions are constructed for each fuzzy set involved in the process and, even if the entry is crisp, it will be fuzzified by its characteristic function. The rule base is composed of a set of "if-then" rules describing an input-output relationship. In this component we can find the fuzzy propositions, which are provided in accordance with an expert. The defuzzification translates the result of fuzzy inference into a single numeric value of control (a real number) [12].

A rule base consisting of r rules has the form:

R_1 : "Fuzzy Proposition 1"
or
R_2 : "Fuzzy Proposition 2"
or
⋮
or
R_r : "Fuzzy Proposition r".

In general, each fuzzy proposition has the form

If "$state$" then "$action$",

where every "state" and every "action" are assumed as linguistic variables modeled by fuzzy sets.

In mathematical terms, each one of the r propositions is described as

"If x_1 is A_1 and x_2 is A_2 and $\ldots$ x_n is A_n then
y_1 is B_1 and y_2 is B_2 and $\ldots$ u_m is B_m",

where $x = (x_1, x_2, ..., x_n)$ and $y = (y_1, y_2, ..., y_m)$ are the input and output variables, respectively. A_i, $1 \leq i \leq n$ and B_j, $1 \leq j \leq m$ are fuzzy sets which model each one of the terms assumed by the input and output variables, respectively.

It is important to note that a FRBS maps $\mathbb{R}^n$ in $\mathbb{R}^m$ of a specific manner. Given an n-tuples $(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$, the FRBS produces an output $y = y(y_1, y_2, \ldots, y_m) \in \mathbb{R}^m$. For a set of rules of the form "If x_i is A_{ij} then y is B_j, $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, r$", where $x_i,\ y_i \in \mathbb{R}$, A_{ij} and B_j are fuzzy sets, n is the number of input variables and r is the number of rules, we obtain $y_r = f_r(x_1, x_2, \ldots, x_n)$. This map always depends on: a) the membership functions A_{ij} and B_j; b) the t-norm and t-conorm chosen and c) the defuzzification method.

Many authors proved that, for particular methodologies, a fuzzy rule-based system is a universal approximator [13, 14, 15, 16], but Nguyen [9, 10] generalized and extended these results. Due to importance of his work we will report some aspects of it in here.

Let a general class of functions $\mathcal{F}(\mathcal{M}, \mathcal{L}, \mathcal{D})$, where $\mathcal{M}$ consists of those membership functions φ such that $\varphi(x) = \varphi_0(ax + b)$ for some $a, b \in \mathbb{R}$, and $a \neq 0$, and $\varphi_0(x)$ is continuous, positive on some interval of $\mathbb{R}$, and 0 outside that interval. $\mathcal{L}$ consists of continuous t-norms and t-conorms. $\mathcal{D}$ is a defuzzification procedure transforming each membership function into a real number in such a way that if $\varphi(x) = 0$ outside an interval (α, β), then $\mathcal{D}(\varphi) \in [\alpha, \beta]$.

Theorem 1 *For any design methodology $(\mathcal{M}, \mathcal{L}, \mathcal{D})$ and any compact subset K of $\mathbb{R}^n$, $\mathcal{F}(\mathcal{M}, \mathcal{L}, \mathcal{D})|_K$ is dense in $C(K)$ with respect to the sup-norm.*

Proof: See [10].

According to Nguyen [10], for a FRBS with a $(\mathcal{M}, \mathcal{L}, \mathcal{D})$ methodology, the output f_r is continuous (therefore integrable) and it can be written as

$$f_r(x) = \frac{\sum_{j=1}^{r} y_j \ \Delta(\varphi_{A_{1j}}(x_1) \ \Delta \ ... \ \Delta \ \varphi_{A_{nj}}(x_n))}{\sum_{j=1}^{r}(\varphi_{A_{1j}}(x_1) \ \Delta \ ... \ \Delta \ \varphi_{A_{nj}}(x_n))}, \quad (1)$$

where r is the number of rules of the rule base, $\varphi_{A_{ij}}(x_i)$ is the degree of membership of x_i in the fuzzy set A_{ij} and Δ is a t-norm (see Definition 2).

Besides, the function f_r is bounded, since it is continuous and defined in a compact set.

Note that $f_r(K)$ is a compact set of $\mathbb{R}$, that is, $f_r(K)$ is a closed interval of $\mathbb{R}$.

3 The Problem

3.1 Construction of an Initial Value Problem and its solution

Suppose that the Initial Value Problem (IVP) models a particular phenomenon. Moreover, suppose that it is an Autonomous Ordinary Differential Equation, that is, the rate of change does not depend explicitly of the time. The IVP is, therefore,

$$\begin{cases} \dfrac{dx}{dt} = f(x(t)) \\ x(t_0) = x_0 \end{cases}, \quad (2)$$

where f is a known function.

The solution of the IVP (2) is guaranteed by the following proposition:

Proposition 1 *Let $f : \mathbb{R}^n \to \mathbb{R}$ be continuous. The function $x : [a, b] \to \mathbb{R}^n$ is a solution of the IVP (2) if, and only if, it is continuous and satisfies the integral equation*

$$x(t) = x_0 + \int_{t_0}^{t} f(x(s))ds,$$

$t \in [a, b]$.

In practice, depending on the complexity of the field f, the solution $x(t)$ may not have an analytical expression. However, there are several numerical methods that estimate the unknown solution with the desired accuracy, that is, we can use a numerical method to obtain a numerical solution to the IVP (2) so that $\{x^n\} \stackrel{n\to\infty}{\longrightarrow} x$. Note that knowing the direction field f (or at least knowing it in some points) is necessary to use a numerical method.

We propose an alternative way to obtain the solution of the IVP (2) or at least an approximation of it, without knowing the field f explicitly. Our goal is to take advantage of the qualitative information available to construct a rule base that will represent the properties that characterize the phenomenon. To obtain the output f_r we use the methodology presented in Section 2.2 and therefore we guarantee that f_r is a good approach to f.

Thus, we have a new IVP to represent the studied phenomenon, which is given by

$$\begin{cases} \dfrac{dx}{dt} = f_r(x(t)) \\ x_r(t_0) = x_0 \end{cases}, \quad (3)$$

whose solution will be denoted by x_r and it will be studied later.

In the next subsection we will make an analysis about the existence and uniqueness of the solution x_r of the IVP (3). Since we can guarantee its existence, we will describe the main result of this work: the theorem ensures that the solution of the IVP (3) is arbitrarily close to the solution of the IVP (2) as r increases.

3.2 Existence and uniqueness of the new IVP's solution, x_r

Based on Proposition 1, if $f_r : \mathbb{R}^n \to \mathbb{R}$ is continuous, then the solution of (3) is given by

$$x_r(t) = x_0 + \int_{t_0}^{t} f_r(x(s))ds,$$

$t \in [a, b]$.

To ensure the existence and uniqueness of the solution we have the following result:

Proposition 2
- *If $f \in C_0(\mathbb{R}^n, \mathbb{R})$ then, for all $x_0 \in \mathbb{R}^n$, there is an interval $I_{x_0} \equiv (\alpha_{x_0}, \beta_{x_0})$ containing t_0 and a solution $x(., x_0)$ of the initial value problem that satisfies the initial condition $x(t_0, x_0) = x_0$.*
- *Besides, if $f \in C^1(\mathbb{R}^n, \mathbb{R})$, then $x(., x_0)$ is unique in I_{x_0} and it is a function of C^1 class.*

As we have seen in section 2.2, f_r is continuous and integrable. Therefore, the function f_r satisfies both conditions of the Proposition 2 and so we can guarantee that the solution of the IVP (3), x_r, exists and it is unique.

4 Main Result

In this section we will state the theorem that ensures the convergence of solution of the IVP (3), x_r, to the solution of the IVP (2), x, as r increases.

For a demonstration of this result we will make use of the following Lebesgue's Dominated Convergence Theorem:

Theorem 2 *Lebesgue's Dominated Convergence Theorem (LDCT): Let X be a measure space with a σ-algebra $\mathcal{A}$ and a measure μ on it. Let $f, f_1, f_2, \ldots : X \to C$ be measurable functions such that $f_n \stackrel{n\to\infty}{\longrightarrow} f$ for almost everywhere (that is, $\mu(\{x \in X; f_n(x) \nrightarrow f(x)\}) = 0$). Suppose there is an integrable function $g : X \to [0, +\infty]$ such that $|f_n(x)| \leq g(x)$ for almost every $x \in X$. Then $\int f_n d\mu \stackrel{n\to\infty}{\longrightarrow} \int f d\mu$.*

Proof: See [17].

In the following we state the main result of this work.

Theorem 3 *Let f be a continuous function in a compact set $K \subset \mathbb{R}^n$. Let $\{f_r\}$ be class of functions defined in K such that $f_r \stackrel{r\to\infty}{\longrightarrow} f$, for example, the fuzzy system output as in (1). Consider the IVPs (2) and (3) with x and x_r their solutions, respectively.*

Then $x_r \stackrel{r\to\infty}{\longrightarrow} x$.

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Proof: We know that the solutions x and x_r are given, respectively, by the equations:

$$x(t) = x_0 + \int_{t_0}^{t} f(x(s))ds,$$

and

$$x_r(t) = x_0 + \int_{t_0}^{t} f_r(x(s))ds.$$

Using the LDCT we can guarantee that the solution x_r converges to x as r increases if there is a positive and integrable function g such that $|f_r(x)| \leq g(x)$ in μ - for almost $x \in K$.

Let $g : K \to [0, +\infty]$ be such that $g(x) = max\{|\alpha|, |\beta|\}$, where $[\alpha, \beta] \subset \mathbb{R}$ is an interval such that $\mathcal{D}(\varphi(x)) \in [\alpha, \beta]$, for all $x \in K$ and $\mathcal{D}$ is a defuzzification procedure as we have seen in Section 2.2. Moreover g is a constant function in a compact set, then it is integrable. Therefore we have $|f_r(x)| \leq g(x)$ for every $x \in K$ and all r.

So, the hypothesis of LDCT are satisfied and we can conclude that

$$\int_{t_0}^{t} f_r(x(s))ds \overset{r\to\infty}{\longrightarrow} \int_{t_0}^{t} f(x(s))ds$$

and, therefore,

$$x_r \overset{r\to\infty}{\longrightarrow} x.$$

4.1 Numerical Solution

In practice, the solution, x, of a theoretical dynamical system cannot have an analytical expression due to the complexity of the direction field f. In this case it is necessary to use numerical methods, and then find solutions $\{x^n\}$ such that $\{x^n\} \overset{n\to\infty}{\longrightarrow} x$.

However $\{x^n\}$ will only be obtained if the direction field f is known, that is, knowing the direction field directions f is a restriction of the numerical method to produce the estimated $\{x^n\}$.

When there is no explicit knowledge of the field f, we can use the fuzzy sets theory and, as we have seen before, we can use fuzzy rule-based system to produce a function f_r that can replace the field f, getting the IVP (3).

Knowing f_r in a table form (for each x, we have $f_r(x)$), we can use stable and convergent numerical methods to obtain $\{x_r^n\}$ such that $\{x_r^n\} \overset{n\to\infty}{\longrightarrow} x_r$.

Therefore, as long as the fuzzy rule-based system used to produce f_r satisfies the assumptions of the main theorem, we have

$$\{x_r^n\} \overset{n\to\infty}{\longrightarrow} x_r \overset{r\to\infty}{\longrightarrow} x,$$

that is

$$x_r^n \overset{\substack{n\to\infty \\ r\to\infty}}{\longrightarrow} x.$$

Now we will show a practical example of the theory discussed here. More details can be found in [18].

5 Example

Suppose a population that has its dynamics following the qualitative characteristics of the Verhulst model (Logistic Equation). Thus, this population grows as the IVP

$$\begin{cases} \dfrac{1}{x}\dfrac{dx}{dt} = a(P_\infty - x) \\ x(t_0) = x_0 \end{cases}, \tag{4}$$

where a is the relative growth rate and P_∞ is the carrying capacity of the population (inhibition factor).

The classical solutions of (4), which represent the population $x(t)$ at every instant t, are given by

$$x(t) = \frac{P_\infty}{\left(\dfrac{P_\infty}{x_0} - 1\right) e^{-at} + 1}, \tag{5}$$

from where we conclude that:

- x increases if $x_0 < P_\infty$
- x is constant if $x_0 = P_\infty$
- x decreases if $x_0 > P_\infty$,

and is illustrated in Fig. 2.

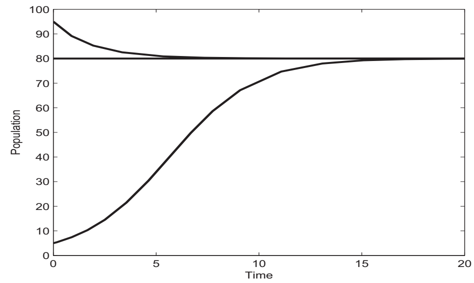

Figure 2: Possible solutions of the Verhulst model.

Considering the relative growth rate we have, rewriting (4) that

$$\frac{1}{x}\frac{dx}{dt} = a(P_\infty - x) = f(x), \tag{6}$$

where f is linear (decreasing).

Consider a FRBS with the Mamdani inference and the centroid as the defuzzification procedure. The input variable is *population* and the output variable is *relative growth rate* (here we will denote it by variation rate). The solution $x_r(t)$ is obtained using a Runge-Kutta method for each output $f_r(x)$.

Next, we have two simulations given by a fuzzy system with 3 and 8 rules in the rule base, respectively. Note that in all simulations we have a rule base that presents semantic opposition in the consequents, that is, there exists change of the consequents signals. That is what guarantees the "auto-inhibition" characteristic.

5.1 Simulation 1

In this first simulation we have three rules in the rule base. They are as follows:

R_1: If the population is *"small"* then the variation rate is *"positive medium"*
R_2: If the population is *"medium"* then the variation rate is *"positive small"*
R_3: If the population is *"large"* then the variation rate is *"negative small"*

The membership functions of the input and output variables are in Figs. 3 and 4, respectively.

The solution $x_r(t)$ is showed in Fig. 5.

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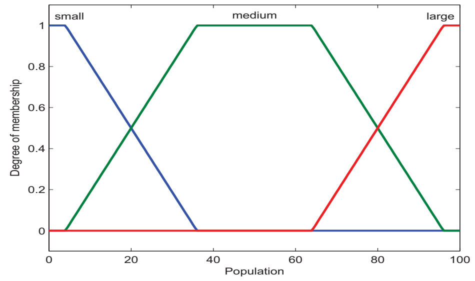

Figure 3: Input of variable *Population*.

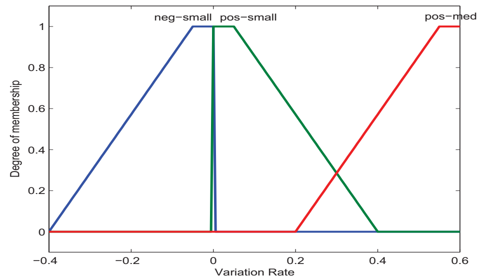

Figure 4: Input of variable *Variation Rate*.

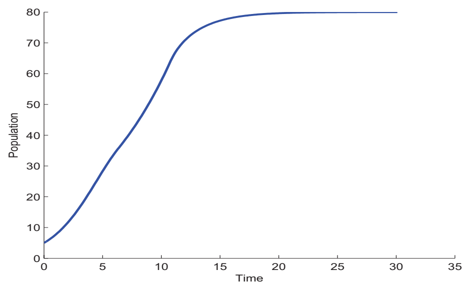

Figure 5: Solution $x_r(t)$ of simulation 1.

5.2 *Simulation 2*

In this second simulation we have eight rules in the rule base. They are as follows:

R_1: If the population is *"very small"* then the variation rate is *"positive very large"*
R_2: If the population is *"small A"* then the variation rate is *"positive large"*
R_3: If the population is *"small B"* then the variation rate is *"positive large"*
R_4: If the population is *"medium A"* then the variation rate is *"positive medium"*
R_5: If the population is *"medium B"* then the variation rate is *"positive medium"*
R_6: If the population is *"medium C"* then the variation rate is *"positive small"*
R_7: If the population is *"large"* then the variation rate is *"negative small"*
R_8: If the population is *"very large"* then the variation rate is *"negative medium"*

The membership functions of the input and output variables are in Figs. 6 and 7, respectively.

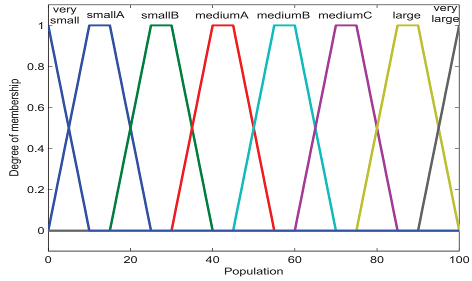

Figure 6: Input of variable *Population*.

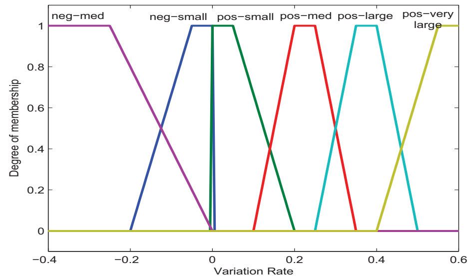

Figure 7: Input of variable *Variation Rate*.

The solution $x_r(t)$ is showed in Figure 8

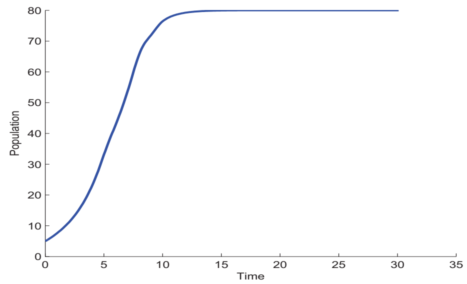

Figure 8: Solution $x_r(t)$ of simulation 2.

Fig. 9 shows the two solutions obtained in the simulations and the classical solution of the model, when adopted $K = 80$, $a = 0,006$ and $x_0 = 5$.

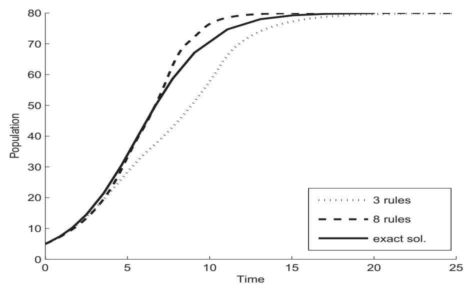

Figure 9: Comparison of the solutions obtained through simulations and the exact solution, using $x_0 = 5$.

Fig. 10 shows the two solutions obtained in the simulations

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and the classical solution of the model, when adopted $K = 80$, $a = 0,006$ and $x_0 = 90$.

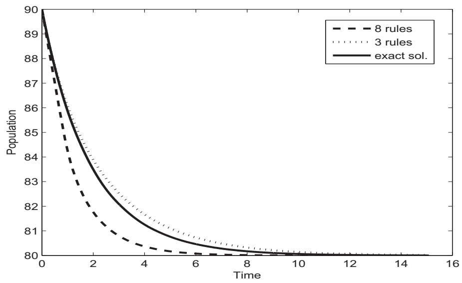

Figure 10: Comparison of the solutions obtained through simulations and the exact solution, using $x_0 = 90$.

Note that Figs. 9 and 10 show that the solutions of the IVP (3) converge to the exact solution of Verhulst Model. Moreover, the bigger the number of rules is, the better the approximation is. This is in accordance with Theorem 3, which ensures convergence as r increases. Table 1 shows the values obtained in the simulations and the value obtained by the solution of the Verhulst Model when $x_0 = 5$, for some values of t.

Table 1: Values obtained by simulation and by the equation.

Time	Theoretical Value (TV)	Sim. 1 (S1) (3 rules)	Sim. 2 (S2) (8 rules)
0	5	5	5
0,2	5,4152	5,4644	5,4460
0,9	7,4423	7,3920	7,2719
2,5	14,5304	13,7874	13,2925
4,6	30,4040	25,8578	28,7976
7,7	58,6421	42,7152	62,3510
11	74,6871	65,6782	78,3973
15,1	79,2707	77,3232	79,9304
17,1	79,7398	78,7905	79,9850
20	79,9407	79,6222	79,9984

The norm of the error of these approximations can be seen in Table 2. It is the difference between the exact value and the simulated values.

Table 2: Errors obtained comparing simulations 1 and 2 with the theoretical value.

Time	Error 1 ($\|TV - S1\|$)	Error 2 ($\|TV - S2\|$)
0	0	0
0,2	0,0491	0,0308
0,9	0,0504	0,1704
2,5	0,7430	2,2487
4,6	4,5416	1,6063
7,7	15,9269	3,7089
11	9,0059	3,7132
15,1	1,9475	0,6597
17,1	0,9493	0,2452
20	0,3185	0,0577

6 Conclusions

In this work we proposed a fuzzy rule-based system methodology to represent the direction field of differential equations. It is applicable when we have only partial knowledgement about the direction field in question. A typical case is when the informations about it are qualitative, as example, low, medium, high, conform the values of the state variable. As we have seen, this methodology was able to construct an input-output system from qualitative information by means of fuzzy sets. Thus, it was possible to adopt a direction field (f_r) (IVP (3)) obtained from theoretical direction field (f) (IVP (2)), that is given by Theorem 1, since f_r converges to f. Moreover, from theorem 3 and using the Lebesgue's Dominated Convergence Theorem we have proved that the solution of the IVP (3) converges to the solution of the IVP (2). Through a simple example we illustrated the methodology we proposed, confirming the obtained results. From the point of view of modeling this result is very important because in many cases, the available informations are qualitative. That is the case of Peixoto's work [19] that applies this methodology in a predator-prey model.

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