Dispersal of Leaf-Cutting Ants: Fuzzy Mathematical Modeling, Numerical Approximation and Simulations

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Abstract— *The objective of this paper is to propose a model for the occupation, by leaf-cutting ants, of an attractive area using a form of the diffusion-advection partial differential equation, in which the population dispersal is modeled using a fuzzy parameter. In the studied domain there is an attractive region, which can represent areas with an elevated concentration of palatable host-plants with better nutritious qualities. The developed algorithm uses information about the foraging behavior of a leaf-cutting ant colony from the Amazon region, in northern Brazil. The solution of the chosen partial differential equation is approximated using first order finite triangular elements and determines population dispersal with the use of a fuzzy rule-based system, which depends upon the number of individuals in the population and terrain characteristics. The numerical algorithm was developed in a MATLAB environment. The calculated solution is compatible with ant behavior when there is an attraction region nearby, as reported in the literature.*

Keywords— Fuzzy Set; Leaf-Cutting Ants; Mathematical Modeling; Numerical Methods; Partial Differential Equation;

1 Introduction

Ants of the genera *Atta* and *Acromyrmex* (*Hymenoptera: Formicidae*), collectively known as leaf-cutting ants, have the unique habit of culturing fungus on fresh plant materials [1]. These ants are generalist foragers that exploit a large number of plant species, although they usually focus on a subset of these species [2], especially those with low levels of toxic secondary compounds and high nutrient content [3, 4]. Their system of foraging trails influence the spatial organization of the foraging activity. Spatial and temporal heterogeneity in plant resources within the vicinity of the ants' nest produces changes in trail direction and modifications in the geometry of the foraging territories [5, 6]. Trails often lead to sites where, at a given time, plant resources are more attractive and abundant. For instance, a study in Costa Rica showed that, when leaf-cutting ant colonies have access to different plant communities, they concentrate their foraging efforts on the community with the greatest density of their preferred plant species (a monoculture of cassava) [7].

This paper proposes a model for the occupation of an attractive area by leaf-cutting ants, using a form of the diffusion-advection partial differential equation to model population dispersal [8] using a fuzzy parameter. The mathematical model for population dispersal is based on the balance of mass in phenomena of fluid dynamics and their physical principles. In this case, Fick's law; a conservation equation - such as conservation of mass -; and the notion of advection and diffusive flux arising from directed motion (convection) and effective random motion, respectively, are fundamental for the development and understanding of the model. In this work, the dispersion depends on the number of individuals of the population and on the characteristics of the terrain over which ants travel. This dispersion is determined by means of a system based on fuzzy rules. The model is constructed using expert entomological knowledge on behavior of leaf-cutting ants. The domain of this study contains an attractive region, which can represent areas with a high concentration of palatable host plants with good nutritious qualities. In literature, this research usually involve the use of a deterministic or stochastic model. However, mathematical literature on uncertainty has grown considerably over the last decade, especially in system modeling, optimization, control, and pattern recognition areas, to mention just a few. Recently, several authors have proposed the use of fuzzy set theory in epidemiology problems [9, 10, 11, 12] and population dynamics [13]. In this paper, we suggest fuzzy set theory, introduced in the 1960s by Lotfi Zadeh [14], to deal with the uncertain nature of ant population dispersal. In [15], the authors propose fuzzy rules that have input variables given by the difference (D) in the pheromone concentrations of the ants on the left and right trail branches. Based on these rules, the authors are able to determine the probability of choosing the left branch. The behavior of ants has also inspired several optimization techniques, the most exhaustively studied of which is known as ant colony optimization [16], which is also based on ant pheromone. The present study is based on the fact that the availability of plant resources to leaf-cutting ant colonies varies considerably in space and time. Thus, colonies must continually gather information about the current availability or stock of resources within their territories. Once a patch of palatable vegetation (i.e., an attractive area) is found, a chemically and physically marked trail is established and the ant foragers follow this trail in order to retrieve the newly found resource.

The next section presents information about the area where the research on ants was conducted [17].

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2 Ant Study Area in Amazonas

This study was carried out in a forest reserve located about 70 kilometers north of Manaus, state of Amazonas, Brazil. The vegetation is that of a primary inland rain forest, which, unlike other types of forest in the Amazon basin, is not subject to annual flooding. The climate is tropical and the annual precipitation, which is about 2,100 mm, is seasonal in distribution [18]. The rainy season lasts from November to May, and the dry season from June to October.

Vasconcelos's research [17] involved a study of the foraging activity of an *Atta cephalotes* colony from July 1985 to January 1986 and from September 1986 to March 1987. This study determined the spatial distribution of ant foraging trails and of the plant species exploited by the ants (i.e., their foraging sites).

Fig. 1 shows that the foraging sites of the *Atta cephalotes* were scattered over the entire extent of the trails, though mainly at intermediate distances from the nest (40 - 60m).

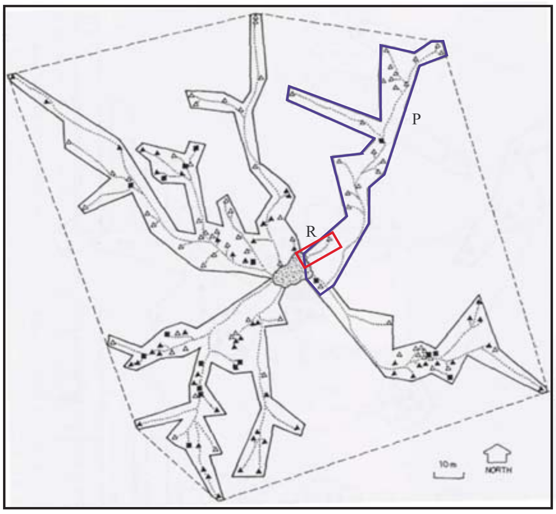

Figure 1: Foraging range (broken line) and foraging territory (continuous line) of the *Atta cephalotes* colony. Solid triangles represent the plants attacked between July 1985 and January 1986, and empty triangles those attacked between September 1986 and March 1987. Solid squares represent plants attacked during both periods. Dotted lines represent the foraging trails radiating from the nest (shaded area) [17].

The next section presents a partial differential equation with a fuzzy parameter for the occupation of leaf-cutting ants.

3 Fuzzy Model

The study of population dynamics involves several uncertain variables, such as the number of individuals in the population, the landscape, the speed at which the population travels, and population dispersal. Fuzzy set theory is a mathematical tool for modeling uncertain phenomena [19], for example, provides key notions for modeling epidemiological phenomena. Thus, the aim of this research is to find a numerical solution for modeling the presence of ant populations, treating dispersal as a fuzzy parameter [20].

The model proposed for the occupation by leaf-cutting ants will be studied by means of the diffusion-advection partial differential equation, given by:

$$\frac{\partial P}{\partial t} + v \cdot \nabla P - \nabla \cdot (\alpha(P, loc_{tot}) \nabla P) = 0. \qquad (1)$$

The functional variable $P = P(\mathbf{x}, t)$ indicates the population at the instant $t \in [0, T]$ and at the point $\mathbf{x} \in \Omega_1 \subset \mathbf{R}^2$ (see Fig. 2). The speed of occupation of the region v is a constant vector (v_1, v_2). Thus, we assume that the population dispersal is represented by parameter $\alpha(P, loc_{tot})$, where total locomotion (loc_{tot}) is determined by a fuzzy rule-based system that depends on the input variables loc_x (horizontal movement) and loc_y (vertical movement). This dispersal depends on the subregions of the domain, indicating the difficulty of locomotion of the ants over the terrain. The fuzzy parameters are modeled as follows:

- the domain is divided into subregions;
- the characteristics of the environment in which the individuals travel are represented by trapezoidal membership functions that indicate the degree of difficulty in locomotion through the domain;

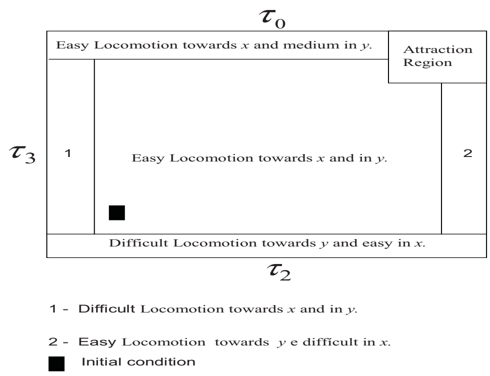

Figure 2: Domain Ω_1.

The adopted boundary conditions considered are:

$$\alpha(P, loc_{tot}) \frac{\partial P}{\partial y} = P v_2, \quad \forall \mathbf{x} \in \tau_0, \quad \forall t \in [0, T] \qquad (2)$$

$$\alpha(P, loc_{tot}) \frac{\partial P}{\partial x} = P v_1, \quad \forall \mathbf{x} \in \tau_1, \quad \forall t \in [0, T] \qquad (3)$$

$$\alpha(P, loc_{tot}) \frac{\partial P}{\partial y} = P v_2, \quad \forall \mathbf{x} \in \tau_2, \quad \forall t \in [0, T] \qquad (4)$$

$$\alpha(P, loc_{tot}) \frac{\partial P}{\partial x} = P v_1, \quad \forall \mathbf{x} \in \tau_3, \quad \forall t \in [0, T] \qquad (5)$$

The initial condition for this problem is given by:

$$P(\mathbf{x}, 0) = P_0(\mathbf{x}), \quad \forall \mathbf{x} \in \Omega_1. \qquad (6)$$

To determine the numerical solution for (1), (2)-(5), the finite element method was used with a triangular uniform grid and linear functions [8]. The Crank-Nicolson Method was employed for the time discretization.

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3.1 Boundary Conditions

The spatial domain of the partial differential equation is rectangular: $\Omega_1 = [0, X] \times [0, Y]$, where X is the longitudinal length (axis x) and Y is the transversal length (axis y) of domain Ω_1. The domain boundary, $\partial\Omega_1$, is divided into parts which are indicated by: τ_0: the upper edge; τ_1: the right edge; τ_2: the lower edge and τ_3: the left edge. The unitary vector with an external direction normal to $\partial\Omega_1$ is given by: $n = (0, 1)$, in τ_0; in τ_1, $n = (1, 0)$, at the right edge; in τ_2, $n = (0, -1)$, at the lower edge; and $n = (-1, 0)$, in τ_3. In order to preserve the population mass as a function of time $M(t)$, the proper boundary conditions must be identified [**?**, 21]. Initially, the equation (1) is written as follows:

$$\frac{\partial P}{\partial t} + div(vP) - div(\alpha(P, loc_{tot})\nabla P) = 0. \quad (7)$$

Integrating equation (7) throughout the domain Ω_1, with $\int_{\Omega_1} P d\mathbf{x} = \mathbf{M(t)}$ and using the Divergence Theorem, one has:

$$\frac{d}{dt}M(t) + \int_{\partial\Omega_1} (Pv \cdot n - \alpha(P, loc_{tot})\nabla P \cdot n)dS = 0 \quad (8)$$

To ensure that $M(t)$ is constant, one must have: $\frac{d}{dt}M(t) = 0$. Therefore:

$$(Pv \cdot n - \alpha(P, loc_{tot})\nabla P \cdot n) = 0. \quad (9)$$

Thus, one obtains the boundary conditions for the Ω_1, from (2) to (5).

3.2 Parameters for Computational Simulation

Fig. 2 represents domain Ω_1. To find the solution for equation (1) through first-order finite elements, this domain was subdivided into small triangles. The discrete solution will be obtained at the vertices (or nodes) of these triangles. The objective of computational simulations was to describe region R, given in Fig. 1. The solution for equation (1) was obtained from the finite triangular elements, using 3600 elements with 1860 nodes. The region of attraction has eight triangles and is located in the upper right-hand side of domain Ω_1, Fig. 2. The *A. cephalotes* colony is represented by four initial nodes close to boundaries τ_2 and τ_3, where an initial population of 400 ants was placed, Fig. 10. At each time step, two individuals are added to the calculated population, simulating ants present at the entrance of the ant nest at all times.

To calculate the dispersal several calculations were made based on Fig. 1. Initially, we measured the dotted trails in region P (large polygon) of Fig. 1. These trails were about 173 meters (m) in length, and the ants took approximately 40 days to occupy this area [17]. The dotted trail in region R (small rectangle) of Fig. 1 was $12m$ long. The rectangle of region R is approximately $13m$ by $5m$ and, to increase the performance of the computational program, let us consider that domain Ω_1 is $6m$ by $3m$. Because the computational program was implemented in a region whose sides measured half of the original region R, we considered that the trail was also half as long as the original one, i.e., $6m$. Therefore, we determined that the ants took 1.3873 days to occupy the $6m$ trail. According to the entomologist, ant trails have a width of approximately $0.15m$. Thus, the area occupied by the ants in the region for the implementation is approximately $4.7971m^2$. Since the dispersion (α) is determined by the area/time ratio, one has: $\alpha \cong 0.14m^2$/hour. We considered the speed in direction y to be equal to $0.07m$/hour. The speed in direction x was considered to be equal to $0.14m$/hour, i.e., double the speed in direction y.

3.3 Linguistic variables and rule base

Fuzzy sets are a way to represent imprecise information and knowledge. The values of locomotion along the x and y directions are expressed as {*constant*, *difficult*, *medium*, *easy*} while those of the population (P) are expressed in term sets {*very small*, *small*, *medium*, *large*, *very large*}. The membership functions that specify the meaning of the linguistic values are shown in Fig. 3, 4, 6, 7 and 8 for locomotion along the x direction, locomotion along the y direction, total locomotion, population and dispersal, respectively. Total locomotion is the output variable in the fuzzy rule-based system (FRBS 1) that depends on the variables of locomotion along the x and y directions. Dispersal is the output variable in the fuzzy rule-based system (FRBS 2) that depends on the variables of total locomotion and population number, Fig. 5. Population dispersal varies from 0 to $0.14m^2$/hours, according to the result of the calculations of this part of region R of Fig. 1. Total locomotion is expressed in a term set {*constant*, *very difficult*, *difficult*, *medium*, *easy*}. The membership functions that designate the meaning of the linguistic values are given in Figs. 6 and 8 for total locomotion and dispersal, respectively. The rule base that encodes the relationship of locomotion along the x and y directions; and total locomotion is summarized in Table 1. The rule base that encodes the relationship of total locomotion, population and dispersal is shown in Table 2.

All the rule bases are processed using Mamdani's Inference Method with center of gravity defuzzification [22]. There are other inference methods that could be adopted, such as Takagi-Sugeno's Inference Method [23]. Nevertheless, we could not use this method because the FRBS output variables are not precise and also they can not be written as input variables.

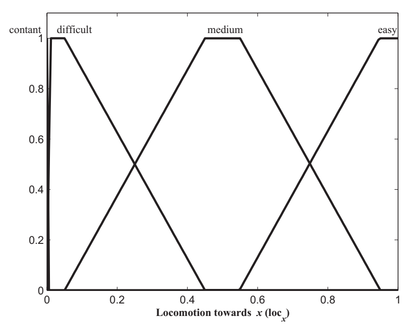

Figure 3: Membership functions for locomotion along the x direction (loc_x).

3.4 Fuzzy Computational Simulation

In order to define the type of ant locomotion as a function of the type of ground (e.g., rivers, hills, mountains and rocks)

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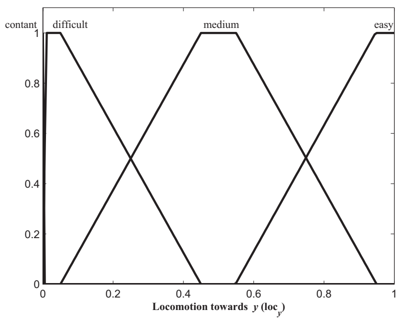

Figure 4: Membership functions for locomotion along the y direction (loc_y).

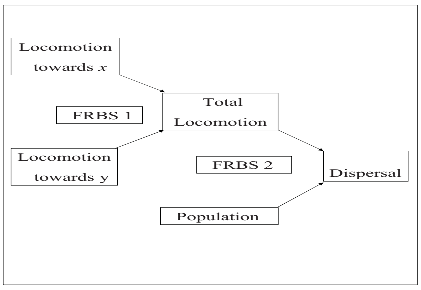

Figure 5: Fuzzy rule-based systems 1 and 2.

in the area of study to each node is allotted a value $[0, 1]$ for movement in x and y according to its location in the area, see Fig. 2.

The chosen trapezoidal membership function, which is associated to a fuzzy set A, $u_A(x)$, involving four parameters $[c_1 \ \ c_2 \ \ c_3 \ \ c_4]$, see Fig. 9, is given by:

$$u_A(x) = \begin{cases} \dfrac{x - c_1}{c_2 - c_1} & \text{if } c_1 \leq x < c_2 \\ 1 & \text{if } c_2 \leq x \leq c_3 \\ \dfrac{-x + c_4}{c_4 - c_3} & \text{if } c_3 < x \leq c_4 \end{cases} \tag{10}$$

If the triangular elements are located in the area of difficult locomotion, the value for locomotion in those triangular elements is calculated by $c_1 + (c_4 - c_1) * rand$, where rand is equal to a random value in interval $(0, 1)$, $c_1 = 0$ e $c_4 = 0.45$. If the triangular elements are located in the area of medium locomotion, the value for locomotion in those triangular elements is calculated by $c_1 + (c_4 - c_1) * rand$, where rand is equal to a random value of $(0, 1)$, $c_1 = 0.05$ and $c_4 = 0.95$. If the triangular elements are located in the area of easy locomotion, the value for locomotion in those triangular elements is calculated by $c_1 + (1 - c_1) * rand$, since the values of x that represent the degree of membership between 0 and 1 for easy locomotion vary from $c_1 = 0.548$ to 1. Thus, locomotion in directions x and y in each triangle is determined randomly depending on the domain of the function of pertinence corresponding to the classification of the region of Fig.2. For horizontal and vertical locomotion, the triangular elements of the region of attraction are allotted the value of zero (0), which

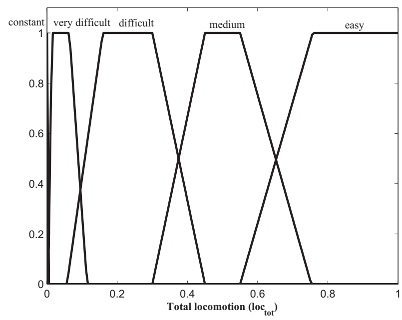

Figure 6: Membership functions for total locomotion (loc_{tot}).

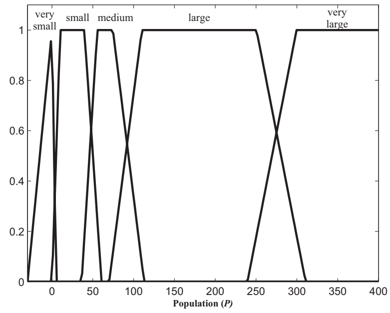

Figure 7: Membership functions for population (P).

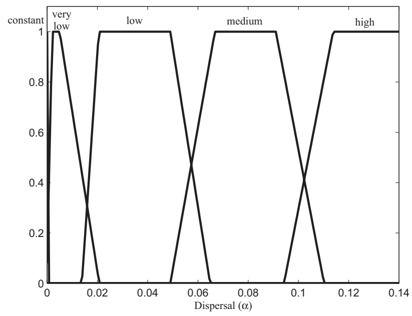

Figure 8: Membership functions for dispersal (α).

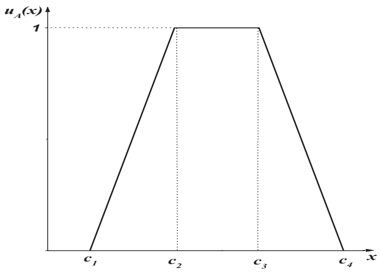

Figure 9: Parameters of trapezoidal membership functions.

has a unitary fuzzy set. Therefore, each triangular element received a value for locomotion along x and y.

To obtain the fuzzy output for each triangular element, for FRBS 2, we use the average of population values on each of three nodes.

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Table 1: Fuzzy rules for total locomotion (loc_{tot}).

(loc_x) \ (loc_y)	*constant*	*difficult*	*medium*	*easy*
constant	*constant*	*very difficult*	*difficult*	*medium*
difficult	*very difficult*	*difficult*	*difficult*	*medium*
medium	*difficult*	*difficult*	*medium*	*medium*
easy	*medium*	*medium*	*medium*	*medium*

Table 2: Fuzzy rules for dispersal (α).

(loc_{tot}) \ *(P)*	*very small*	*small*	*medium*	*large*	*very large*
constant	*constant*	*very low*	*very low*	*very low*	*very low*
very difficult	*constant*	*constant*	*very low*	*low*	*low*
difficult	*constant*	*very low*	*very low*	*medium*	*low*
medium	*constant*	*very low*	*very low*	*medium*	*medium*
easy	*constant*	*low*	*very low*	*medium*	*high*

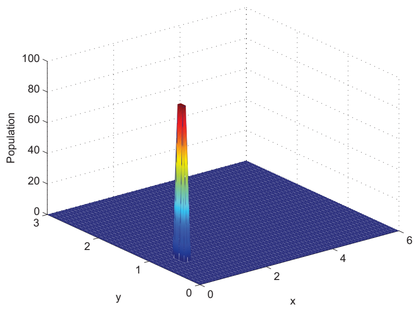

Figure 10: Initial population.

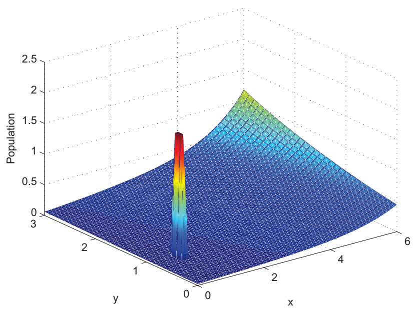

Figure 11: Population after 33 hours.

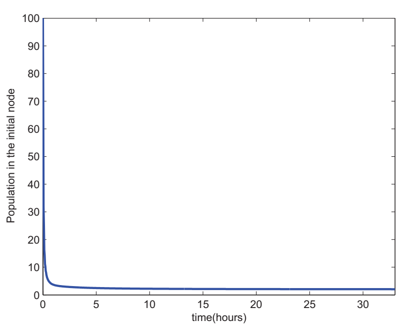

Figure 12: Population in the initial node.

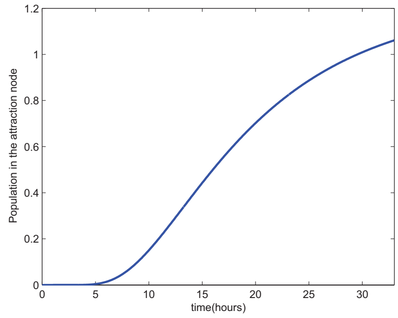

Figure 13: Population in one of the nodes of the region of attraction.

To determine the dispersal in each triangular element, the two FBRS were used: FRBS 1 and FRBS 2, Fig.5. Using FRBS 1, the value for total locomotion in each triangle was determined based on the values of locomotion along x and y. The dispersal in the node of each triangle was determined based on the values of total locomotion and medium population, using FRBS 2.

Thus, x and y components of locomotion were determined only once for each triangle of each region, prior to the first iteration, and total locomotion was determined based on the speed in directions x and y. The dispersion was calculated in each iteration for each triangle, since it depends on the total locomotion and on the population. The latter varies in each iteration.

Fig.10 illustrates the initial condition, i.e., the *A. cephalotes* colony before it occupied the region of attraction. The graph in Fig.11 shows the population after 33 hours, since ants take 33 hours to occupy the considered region of Fig.2. Fig.12 represents ant population in one of the initial nodes as a function of time. As it can be seen, the population in this node is close to two, since there will always be insects at the entrance to the ant nest. Fig. 13 shows the behavior of the number of ants as a function of time in one of the nodes of the region of attraction. Note that the population in this region grows logistically over time.

4 Conclusions

This paper proposes a model for the occupation of an attractive area by leaf-cutting ants, using a form of the diffusion-advection partial differential equation that models the population dispersal using a fuzzy parameter. Dispersion is this fuzzy parameter that depends on the number of the population and on its total locomotion, which is associated with the ease or difficulty of locomotion of the ants as a function of the terrain they traverse. The main difference between the classic models and the fuzzy model (1) is that the latter model exploits uncertainty parameters whereas the classic model does not. Thus, authors believe that, due to the uncertainty of biological phenomena, the combination of differential equations and fuzzy set theory enables computational simulations to much more faithfully portray the phenomenon under study.

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