Dominance-based Rough Set Analysis of Uncertain Data Tables

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Abstract— *In this paper, we propose a dominance-based rough set approach for the decision analysis of a preference-ordered uncertain data table, which is comprised of a finite set of objects described by a finite set of criteria. The domains of the criteria may have ordinal properties that express preference scales. In the proposed approach, we first compute the degree of dominance between any two objects based on their imprecise evaluations with respect to each criterion. This results in a fuzzy dominance relation on the universe. Then, we define the degree of adherence to the dominance principle by every pair of objects and the degree of consistency of each object. The consistency degrees of all objects are aggregated to derive the quality of the classification, with which we can define the reducts of an uncertain data table. In addition, the upward and downward unions of decision classes are fuzzy subsets of the universe. The lower and upper approximations of the decision classes based on the fuzzy dominance relation are thus fuzzy rough sets. By using the lower approximations of the decision classes, we can derive two types of decision rules that can be applied to new decision cases.*

Keywords— Dominance-based rough set approach, multi-criteria decision analysis, preference-ordered data tables, rough set theory, uncertain data tables.

1 Introduction

The theory of knowledge has long been an important topic in many academic disciplines, such as philosophy, psychology, economics, and artificial intelligence, whereas the storage and retrieval of data is the main concern of information science. In modern experimental science, knowledge is usually acquired from observed data, which is a valuable resource for researchers and decision-makers. However, when the amount of data is large, it is difficult to analyze the data and extract knowledge from it. With the aid of computers, the vast amount of data stored in relational data tables can be transformed into symbolic knowledge automatically. Thus, intelligent data analysis has received a great deal of attention in recent years. The rough set theory proposed in [19] provides an effective tool for extracting knowledge from data tables.

When rough set theory is applied to *multi-criteria decision analysis* (MCDA), it is crucial to deal with preference-ordered attribute domains and decision classes [6, 7, 8, 9, 10, 11, 12, 24]. The original rough set theory cannot handle inconsistencies arising from violations of the dominance principle due to its use of the indiscernibility relation. Therefore, in the above-mentioned works, the indiscernibility relation is replaced by a dominance relation to solve the multi-criteria sorting problem; and the data table is replaced by a pairwise comparison table to solve multi-criteria choice and ranking problems. The approach is called the *dominance-based rough set approach* (DRSA). For MCDA problems, DRSA can induce a set of decision rules from sample decisions provided by decision-makers. The induced rules form a comprehensive preference model and can provide recommendations about a new decision-making environment.

A strong assumption about data tables is that each object takes exactly one value with respect to an attribute. However, in practice, we may only have incomplete information about the values of an object's attributes. Thus, more general data tables are needed to represent incomplete information. For example, set-valued and interval-valued data tables have been introduced to represent incomplete information [15, 16, 17, 18, 25]. DRSA has also been extended to deal with missing values in MCDA problems [10, 24]. Since a data table with missing values is a special case of uncertain data tables, we propose further extending DRSA to the decision analysis of uncertain data tables. In this paper, we investigate such an extension based on the fuzzy dominance principle.

In the proposed approach, we first compute the degree of dominance between any two objects based on their imprecise evaluations with respect to each criterion. This results in a fuzzy dominance relation on the universe. Then, we define the degree of adherence to the dominance principle by every pair of objects and the degree of consistency of each object. The consistency degrees of all objects are aggregated to derive the quality of the classification, with which we can define the reducts of the uncertain data tables. In addition, the upward and downward unions of decision classes are fuzzy subsets of the universe. The lower and upper approximations of the decision classes based on the fuzzy dominance relation are thus fuzzy rough sets. By using the lower approximations of the decision classes, we can derive two types of decision rules that can be applied in new decision-making environments.

In the next section, we review the dominance-based rough set approach. Then, in Section 3, we present the extension of DRSA for decision analysis of uncertain data tables. Section 4 contains some concluding remarks.

2 Dominance-based Rough Set Approach

2.1 *Rough set theory*

The basic construct of rough set theory is an *approximation space*, which is defined as a pair (U, R), where U is a finite universe and $R \subseteq U \times U$ is an equivalence relation on U. A binary relation R is an equivalence relation if it is (1)reflexive (i.e., $(x, x) \in R$ for all $x \in U$); (2) symmetric (i.e., for all $x, y \in U$, if $(x, y) \in R$, then $(y, x) \in R$); and (3) transitive

ISBN: 978-989-95079-6-8

(i.e., for all $x, y, z \in U$, if $(x, y) \in R$ and $(y, z) \in R$, then $(x, z) \in R$). An equivalence relation partitions the universe U into a family of equivalence classes so that each element of U belongs to exactly one of these equivalence classes. In other words, there exist $U_1, U_2, \cdots, U_k \subseteq U$ such that $U = \cup_{i=1}^{k} U_i$, $U_i \cap U_j = \emptyset$ for $i \neq j$; and for $x, y \in U$, $(x, y) \in R$ iff there exists i such that both x and $y \in U_i$. Thus, we can write an equivalence class of R as $[x]_R$ if it contains the element x. Note that $[x]_R = [y]_R$ iff $(x, y) \in R$.

According to Pawlak, knowledge is deep-seated in the classification capabilities of human beings. A classification is simply a partition of the universe, so an approximation space can construct basic knowledge about the objects in the universe. In philosophy, the *extension* of a concept is defined as the objects that are instances of the concept. For example, the extension of the concept "bird" is simply the set of all birds in the universe. Pawlak identified a concept by its extension. Thus, a subset of the universe is called a *concept* or a *category* in rough set theory.

Given an approximation space (U, R), each equivalence class of R is called an *R-basic category* or *R-basic concept*, and any union of R-basic categories is called an R-category. Now, for an arbitrary concept $X \subseteq U$, we are interested in defining X by using R-basic categories. We say that X is *R-definable*, if X is an R-category; otherwise X is *R-undefinable*. The R-definable concepts are also called *R-exact* sets, whereas R-undefinable concepts are said to be *R-inexact* or *R-rough*. When the approximation space is explicit from the context, we simply omit the qualifier R and call a set an exact set or a rough set.

A rough set can be approximated from below and above by two exact sets. The lower approximation and upper approximation of X are denoted by $\underline{R}X$ and $\overline{R}X$ respectively, and defined as follows:

$$\underline{R}X = \{x \in U \mid [x]_R \subseteq X\}, \tag{1}$$

$$\overline{R}X = \{x \in U \mid [x]_R \cap X \neq \emptyset\}. \tag{2}$$

2.2 Classical data tables

Although an approximation space is an abstract framework used to represent classification knowledge, it can easily be derived from a concrete data table (DT). The following formal definition of a data table is given in [20].

Definition 1 *A data table*[1] *is a tuple*

$$T = (U, A, \{V_i \mid i \in A\}, \{f_i \mid i \in A\}), \tag{3}$$

where U is a nonempty finite set, called the universe; A is a nonempty finite set of primitive attributes; for each $i \in A$, V_i is the domain of values for i; and for each $i \in A$, $f_i : U \to V_i$ is a total function.

An attribute in A is usually denoted by the lower-case letters i or a. In decision analysis, we assume the set of attributes is partitioned into $\{d\} \cup (A - \{d\})$, where d is called the *decision attribute*, and the remaining attributes in $A - \{d\}$ are called *condition attributes*.

[1] Also called knowledge representation systems, information systems, or attribute-value systems

Let $T = (U, A, \{V_i \mid i \in A\}, \{f_i \mid i \in A\})$ be a data table and $B \subseteq A$ be a subset of attributes. Then, we can define an equivalence relation, called the *indiscernibility relation* based on B, as

$$ind(B) = \{(x, y) \mid x, y \in U, f_i(x) = f_i(y) \forall i \in B\}. \tag{4}$$

In other words, x and y are B-indiscernible if they have the same values with respect to all the attributes in B. Consequently, for each $B \subseteq A$, $(U, ind(B))$ is an approximation space.

Let B be a subset of attributes. Then, an object x is *B-consistent* (with respect to the decision attribute d) if $[x]_B \subseteq [x]_d$, where $[x]_B = [x]_{ind(B)}$ and $[x]_d = [x]_{ind(\{d\})}$; otherwise, x is *B-inconsistent*. In other words, x is a B-consistent object in a data table if it satisfies the following *indiscernibility principle* for all $y \in U$:

$$(x, y) \in ind(B) \Rightarrow (x, y) \in ind(\{d\}). \tag{5}$$

That is, the objects that have the same evaluations as x on the condition attributes should have the same decision class assignment as x. Note that a *B-consistent* object corresponds to a decision rule whose decision class can be determined consistently based on the values of the attributes in B.

In [20], a decision logic (DL) is proposed for the representation of knowledge discovered from data tables. The basic alphabet of a DL consists of a finite set of attribute symbols A, and a finite set of value symbols V_i for $i \in \mathsf{A}$. Thus, the syntax of DL can be defined as follows.

Definition 2

1. *An atomic formula of DL is a descriptor (i, v), where $i \in \mathsf{A}$ and $v \in \mathsf{V}_i$.*
2. *The set of well-formed formulas (wffs) of a DL is the smallest set that contains the atomic formulas and is closed under the Boolean connectives $\neg$, $\wedge$, and $\vee$.*
3. *If φ and ψ are wffs of a DL, then $\varphi \longrightarrow \psi$ is a rule in the DL, where φ is called the antecedent of the rule and ψ is the consequent.*

A data table $T = (U, A, \{V_i \mid i \in A\}, \{f_i \mid i \in A\})$ relates to a given DL if there is a bijection $\tau : \mathsf{A} \to A$ such that, for every $a \in \mathsf{A}$, $V_{\tau(a)} = \mathsf{V}_a$. Thus, by somewhat abusing the notation, we usually denote an atomic formula as (i, v), where $i \in A$ and $v \in V_i$ if the data tables are clear from the context. Intuitively, each element in the universe of a data table corresponds to a data record, and an atomic formula (which is in fact an attribute-value pair) describes the value of some attribute in the data record. Thus, the atomic formulas (and therefore the wffs) can be satisfied or not satisfied by each data record. This results in a satisfaction relation between the universe and the set of wffs.

Definition 3 *Given a DL and a data table $T = (U, A, \{V_i \mid i \in A\}, \{f_i \mid i \in A\})$ related to it, the satisfaction relation $\models_T$ between U and the wffs of the DL is defined inductively as follows (the subscript T is omitted for brevity).*

1. *$x \models (i, v)$ iff $f_i(x) = v$,*
2. *$x \models \neg\varphi$ iff $x \not\models \varphi$,*

ISBN: 978-989-95079-6-8

3. $x \models \varphi \wedge \psi$ iff $x \models \varphi$ and $x \models \psi$,

4. $x \models \varphi \vee \psi$ iff $x \models \varphi$ or $x \models \psi$.

If φ is a DL wff, the set $m_T(\varphi)$ defined by

$$m_T(\varphi) = \{x \in U \mid x \models \varphi\} \tag{6}$$

is called the meaning set of the formula φ in T. If T is understood, we simply write $m(\varphi)$.

In terms of DL, each equivalence class of $ind(B)$ is characterized by a DL formula $\wedge_{i \in B}(i, v_i)$ and any formula φ of DL can be considered as a concept $m_T(\varphi)$. Thus, for each B-consistent object x, we can derive a valid rule $\wedge_{i \in B}(i, f_i(x)) \longrightarrow (d, f_d(x))$.

2.3 Preference-ordered data tables

In this subsection, we consider the DRSA in [10]. For MCDA problems, each object in a data table or decision table can be seen as a sample decision, and each condition attribute is a criterion for the decision. Since the domain of values of a criterion is usually ordered according to the decision-maker's preferences, we define a preference-ordered data table (PODT) as a tuple

$$T = (U, A, \{(V_i, \succeq_i) \mid i \in A\}, \{f_i \mid i \in A\}), \tag{7}$$

where $T = (U, A, \{V_i \mid i \in A\}, \{f_i \mid i \in A\})$ is a classical data table; and for each $i \in A$, $\succeq_i \subseteq V_i \times V_i$ is a binary relation over V_i. The relation $\succeq_i$ is called a *weak preference relation* or *outranking* on V_i, and represents a preference over the set of objects with respect to the criterion i [24]. For $x, y \in U$, $f_i(x) \succeq_i f_i(y)$ means "x is at least as good as y with respect to criterion i". The weak preference relation $\succeq_i$ is supposed to be a complete preorder, i.e., a complete, reflexive, and transitive relation. In addition, we assume that the domain of the decision attribute is a finite set $V_d = \{1, 2, \cdots, n\}$ such that r is strictly preferred to s if $r > s$ for any $r, s \in V_d$. Thus, the weak preference relation $\succeq_d$ is defined as $r \succeq_d s$ iff $r \geq s$; consequently, $\succeq_d$ is a total order.

For a condition criterion i and an object x, $f_i(x)$ denotes the *evaluation* of the object with respect to the criterion i; and for the decision attribute d, $f_d(x)$ is the *assignment* of x to a decision label in V_d. Let P be a subset of criteria. We can then define the *P-dominance relation* $D_P \subseteq U \times U$ as follows:

$$(x, y) \in D_P \Leftrightarrow f_i(x) \succeq_i f_i(y) \forall i \in P. \tag{8}$$

When $(x, y) \in D_P$, we say that x P-dominates y, and that y is P-dominated by x. We usually use the infix notation xD_Py to denote $(x, y) \in D_P$. Although each $\succeq_i$ is a complete preorder, the dominance relation may simply be a preorder. If $P = \{i\}$ is a singleton, we write D_i instead of $D_{\{i\}}$. The most basic principle underlying DRSA is called the *dominance principle*. Let P denote a subset of condition criteria. Then, the dominance principle with respect to P can be expressed for $x, y \in U$ as follows:

$$xD_Py \Rightarrow xD_dy. \tag{9}$$

The principle states that if x P-dominates y (i.e., x is at least as good as y with respect to all criteria in P), then x should be assigned to a decision class at least as good as the class assigned to y.

In the classical rough set approach, a consistent object must satisfy the indiscernibility principle; however, in DRSA, we also require adherence to the dominance principle. Thus, an object x is *P-consistent* in the PODT $T = (U, A, \{(V_i, \succeq_i) \mid i \in A\}, \{f_i \mid i \in A\})$ if for all $y \in U$, we have

$$(xD_Py \Rightarrow xD_dy) \wedge (yD_Px \Rightarrow yD_dx); \tag{10}$$

otherwise, x is *P-inconsistent*. Note that the dominance principle implies the indiscernibility principle because of the reflexivity of the dominance relation and the antisymmetry of $\succeq_d$.

Given the dominance relation D_P, the P-dominating set and P-dominated set of x are defined, respectively, as

$$D_P^+(x) = \{y \in U \mid yD_Px\} \tag{11}$$

and

$$D_P^-(x) = \{y \in U \mid xD_Py\}. \tag{12}$$

In addition, for each $t \in V_d$, we define the decision class Cl_t as $\{x \in U \mid f_d(x) = t\}$. Then, the *upward and downward unions of classes* are defined as

$$Cl_t^{\geq} = \bigcup_{s \geq t} Cl_s \tag{13}$$

and

$$Cl_t^{\leq} = \bigcup_{s \leq t} Cl_s \tag{14}$$

respectively. Based on the P-dominating sets and P-dominated sets, we can define the P-lower and P-upper approximations of $Cl_t^{\geq}$ and $Cl_t^{\leq}$ for each $t \in V_d$ as follows:

$$\underline{P}(Cl_t^{\geq}) = \{x \in U \mid D_P^+(x) \subseteq Cl_t^{\geq}\}, \tag{15}$$

$$\overline{P}(Cl_t^{\geq}) = \{x \in U \mid D_P^-(x) \cap Cl_t^{\geq} \neq \emptyset\}, \tag{16}$$

$$\underline{P}(Cl_t^{\leq}) = \{x \in U \mid D_P^-(x) \subseteq Cl_t^{\geq}\}, \tag{17}$$

$$\overline{P}(Cl_t^{\leq}) = \{x \in U \mid D_P^+(x) \cap Cl_t^{\geq} \neq \emptyset\}. \tag{18}$$

The P-boundaries of $Cl_t^{\geq}$ and $Cl_t^{\leq}$ are then defined as

$$Bn_P(Cl_t^{\geq}) = \overline{P}(Cl_t^{\geq}) - \underline{P}(Cl_t^{\geq}) \tag{19}$$

and

$$Bn_P(Cl_t^{\leq}) = \overline{P}(Cl_t^{\leq}) - \underline{P}(Cl_t^{\leq}) \tag{20}$$

respectively. Let $\mathbf{Cl} = \{Cl_t \mid t \in V_d\}$ denote the partition of the universe into decision classes. Then, the *quality of the approximation* of the partition $\mathbf{Cl}$ based on the set of criteria P is defined as the ratio

$$\gamma_P(\mathbf{Cl}) = \frac{|U - (\bigcup_{t \in V_d} Bn_P(Cl_t^{\geq}) \cup \bigcup_{t \in V_d} Bn_P(Cl_t^{\leq}))|}{|U|}. \tag{21}$$

Note that $\gamma_P(\mathbf{Cl})$ is equal to the ratio of P-consistent objects in the universe U. Let $C = A - \{d\}$ be the set of all condition criteria; then every minimal subset $P \subseteq C$ such that $\gamma_P(\mathbf{Cl}) = \gamma_C(\mathbf{Cl})$ is a *reduct* of C.

ISBN: 978-989-95079-6-8

2.4 *Preference-ordered decision logic*

To represent the rules induced from a PODT, preference-ordered decision logic (PODL) is proposed in [5]. The syntax of PODL is the same as that of DL, except for the form of the atomic formulas. An atomic formula in PODL is a descriptor in the form of $(\geq_i, v)$ or $(\leq_i, v)$, where $i \in A$ and $v \in V_i$. The satisfaction relation between U and the set of PODL wffs is defined in the same way as the relation for DL wffs, except that the satisfaction of an atomic formula is defined by $x \models (\geq_i, v)$ iff $f_i(x) \succeq v$, and by $x \models (\leq_i, v)$ iff $v \succeq f_i(x)$.

By using DRSA, two kinds of rules can be induced from a PODT explicitly. Let P be a reduct and $t \in V_d$. Then, for an object $x \in \underline{P}(Cl_t^{\geq})$, we can derive

$$\bigwedge_{i\in P} (\geq_i, f_i(x)) \longrightarrow (\geq_d, t), \tag{22}$$

and for an object $x \in \underline{P}(Cl_t^{\leq})$, we can derive

$$\bigwedge_{i\in P} (\leq_i, f_i(x)) \longrightarrow (\leq_d, t). \tag{23}$$

Note that if x is P-consistent, these two formulas also hold for $t = f_d(x)$.

3 DRSA for Uncertain Data Tables

3.1 *Preference-ordered Uncertain Data Tables*

Although the PODT can represent multi-criteria decision cases effectively, it inherits the restriction of the classical DT so that uncertain information can not be represented. An uncertain data table is a generalization of a DT such that the values of some or all of its attributes are imprecise [4, 1]. An analogous generalization can be applied to PODT to define preference-ordered uncertain data tables (POUDT). Formally, a POUDT is a tuple

$$T = (U, A, \{(V_i, \succeq_i) \mid i \in A\}, \{f_i \mid i \in A\}), \tag{24}$$

where $U, A, \{(V_i, \succeq_i) \mid i \in A\}$ are defined as above, and for each $i \in A$, $f_i : U \to 2^{V_i} - \{\emptyset\}$. The intuition about a POUDT is that the evaluation of criterion i for an object x belongs to $f_i(x)$, although the evaluation is not known exactly. When $f_i(x)$ is a singleton, we say that the evaluation is precise. If all evaluations of T are precise, then T is said to be precise. Furthermore, we assume that for each criterion i, the Cartesian plane $V_i \times V_i$ is endowed with a uniform measure μ_i. Thus, for each subset $S \subseteq V_i \times V_i$, $\mu_i(S)$ is a non-negative real number. When V_i is a finite set, we take $\mu_i(S)$ as the cardinality of S; and when V_i is a real interval, we take $\mu_i(S)$ as the area of S.

3.2 *Fuzzy dominance relation*

In a POUDT, the objects may have imprecise evaluations with respect to the condition criteria and imprecise assignments to decision classes. Thus, the dominance relation between objects can not be determined with certainty. Instead, we define a degree of dominance between two objects with respect to each criterion i based on the associated measures μ_i. Formally, the dominance relation with respect to the criterion i is a fuzzy relation $D_i : U \times U \to [0, 1]$ such that for all $x \neq y$,

$$D_i(x, y) = \frac{\mu_i(\{(v_1, v_2) \mid v_1 \succeq_i v_2, v_1 \in f_i(x), v_2 \in f_i(y)\})}{\mu_i(f_i(x) \times f_i(y))}, \tag{25}$$

and $D_i(x, x) = 1$ for any $x \in U$.

An example of computing the degree of dominance is shown in Figure 1, where the evaluation of x with respect to criterion i, denoted by $s(x)$, is in a continuous interval $f_i(x) = [l_x, u_x]$. In this example, $D_i(x, y)$ is the ratio of the area of ABC over the area of $ABDE$, i.e., $\frac{u_x - l_y}{2(u_y - l_y)}$.

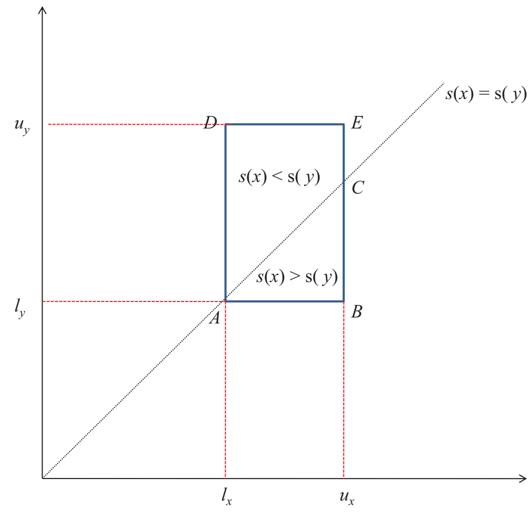

Figure 1: The degree of dominance between x and y

After deriving the fuzzy dominance relation for each criterion, we can aggregate all the relations into P-dominance relations for any subset of criteria P. Let $\otimes$, $\oplus$ and $\to$ denote, respectively, a t-norm, an s-norm and an implication operation[2] on $[0, 1]$. Then, the fuzzy P dominance relation $D_P : U \times U \to [0, 1]$ is defined as

$$D_P(x, y) = \bigotimes_{i\in P} D_i(x, y). \tag{26}$$

Since the dominance relation is a fuzzy relation, the satisfaction of the dominance principle is a matter of degree. Thus, the *degree of adherence* of (x, y) to the dominance principle with respect to a subset of condition criteria P is defined as

$$\delta_P(x, y) = D_P(x, y) \to D_d(x, y), \tag{27}$$

and the degree of P-consistency of x is defined as

$$\delta_P(x) = \bigotimes_{y\in U} (\delta_P(x, y) \otimes \delta_P(y, x)). \tag{28}$$

Let T be a POUDT. Then, the *quality of the classification* of T based on the set of criteria P is defined as

$$\gamma_P(T) = \frac{\sum_{x\in U} \delta_P(x)}{|U|}. \tag{29}$$

Note that $\gamma_P(T)$ is monotonic with respect to P, i.e., $\gamma_Q(T) \leq \gamma_P(T)$ if $Q \subseteq P$. Thus, we can define every minimal subset $P \subseteq C$ such that $\gamma_P(T) = \gamma_C(T)$ as a *reduct* of C, where

[2] For the properties of these operations, see a standard reference on fuzzy logic, such as [14]

ISBN: 978-989-95079-6-8

$C = A - \{d\}$ is the set of all condition criteria. In addition, the degree of P-consistency is monotonic with respect to P, so a reduct is also a minimal subset $P \subseteq C$ such that $\delta_P(x) = \delta_C(x)$ for all $x \in U$. However, because $\delta_P(x)$ is less sensitive to individual changes in $\delta_P(x, y)$, we can not guarantee that a reduct will preserve the degree of adherence to the dominance principle for each pair of objects. Thus, an alternative definition of the quality of the classification is

$$\eta_P(T) = \frac{\sum_{x,y \in U} \delta_P(x, y)}{|U|^2}. \quad (30)$$

The reducts can also be defined with respect to this kind of definition.

3.3 Dominance-based fuzzy rough approximations

In a POUDT, the assignment of a decision label to an object may be imprecise, so the decision classes may be fuzzy subsets of the universe. First, for each decision label $t \in V_d$, the decision class $Cl_t : U \to [0, 1]$ is defined by

$$Cl_t(x) = \begin{cases} \frac{1}{|f_d(x)|}, & \text{if } t \in f_d(x), \\ 0, & \text{otherwise.} \end{cases} \quad (31)$$

Second, the upward and downward unions of classes are defined by

$$Cl_t^{\geq}(x) = \frac{|f_d(x) \cap \{v \in V_d : v \geq t\}|}{|f_d(x)|} \quad (32)$$

and

$$Cl_t^{\leq}(x) = \frac{|f_d(x) \cap \{v \in V_d : v \leq t\}|}{|f_d(x)|} \quad (33)$$

respectively. Note that $Cl_t^{\geq} = \bigcup_{s \geq t} Cl_s$ and $Cl_t^{\leq} = \bigcup_{s \leq t} Cl_s$ only hold when we take the Łukasiewicz s-norm as the union operation, i.e., only when $(F \cup G)(x) = F(x) \oplus G(x)$, where $a \oplus b = \min(1, a + b)$. Finally, since our dominance relation is a fuzzy relation and the decision classes are fuzzy sets, the lower and upper approximations of these classes are defined in the same way as those for fuzzy rough sets[3, 21]. More specifically, the P-lower and P-upper approximations of $Cl_t^{\geq}$ and $Cl_t^{\leq}$ for each $t \in V_d$ are defined as fuzzy subsets of U with the following membership functions:

$$\underline{P}(Cl_t^{\geq})(x) = \bigotimes_{y \in U} (D_P(y, x) \to Cl_t^{\geq}(y)), \quad (34)$$

$$\overline{P}(Cl_t^{\geq})(x) = \bigoplus_{y \in U} (D_P(x, y) \otimes Cl_t^{\geq}(y)), \quad (35)$$

$$\underline{P}(Cl_t^{\leq})(x) = \bigotimes_{y \in U} (D_P(x, y) \to Cl_t^{\leq}(y)), \quad (36)$$

$$\overline{P}(Cl_t^{\leq})(x) = \bigoplus_{y \in U} (D_P(y, x) \otimes Cl_t^{\leq}(y)). \quad (37)$$

3.4 Decision rules

To represent knowledge discovered from a POUDT, we generalize PODL to a kind of preference-ordered uncertain decision logic (POUDL). The syntax of POUDL is same as that of PODL, except that its atomic formulas are of the form $(\geq_i, s_i)$ or $(\leq_i, s_i)$, where $i \in A$ and $s_i \subseteq V_i$. When $s_i = \{v_i\}$ is a singleton, we abbreviate $(\geq_i, s_i)$ (resp. $(\leq_i, s_i)$) as $(\geq_i, v_i)$ (resp. $(\leq_i, v_i)$).

Let P denote a reduct of a POUDT and $t \in V_d$. Then, for each object x where $\underline{P}(Cl_t^{\geq})(x) > 0$ (or above some predetermined threshold), we can derive the first type of fuzzy rule:

$$\underline{P}(Cl_t^{\geq})(x) : \bigwedge_{i \in P} (\geq_i, f_i(x)) \longrightarrow (\geq_d, t); \quad (38)$$

and for each object x where $\underline{P}(Cl_t^{\leq})(x) > 0$ (or above some predetermined threshold), we can derive the second type of fuzzy rule:

$$\underline{P}(Cl_t^{\leq})(x) : \bigwedge_{i \in P} (\leq_i, f_i(x)) \longrightarrow (\leq_d, t), \quad (39)$$

where $\underline{P}(Cl_t^{\geq})(x)$ and $\underline{P}(Cl_t^{\leq})(x)$ are the respective degrees of truth of the rules.

Now, for a new decision case with (possibly imprecise) evaluations on the condition criteria P, we can apply these two types of rules to derive its decision label assignment. Specifically, let x be a new object such that for each criterion $i \in P$, $f_i(x) \subseteq V_i$ is given; and let α be a rule $c : \bigwedge_{i \in P} (\geq_i, s_i) \longrightarrow (\geq_d, t)$ discovered by the above-mentioned approach. Then, we can derive that the degree of satisfaction of $f_d(x) \succeq_d t$, according to the rule α and denoted by $\varepsilon(\alpha, f_d(x) \succeq_d t)$, is

$$c \otimes \bigotimes_{i \in P} \frac{\mu_i(\{(v_1, v_2) \mid v_1 \succeq_i v_2, v_1 \in f_i(x), v_2 \in s_i\})}{\mu_i(f_i(x) \times s_i)}. \quad (40)$$

Let $\mathcal{R}_t^{\geq}$ denote the set of all rules with the consequent $(\geq_d, t)$. Then, the final degree of $f_d(x) \succeq_d t$ is

$$\bigoplus_{\alpha \in \mathcal{R}_t^{\geq}} \varepsilon(\alpha, f_d(x) \succeq_d t). \quad (41)$$

Analogously, we can derive the degree of $f_d(x) \preceq_d t$ from the second type of rule.

4 Conclusion

The work reported in this paper extends DRSA to a dominance-based fuzzy rough set approach (DFRSA), which can be applied to the reduction of criteria and the induction of rules for decision analysis in a POUDT. Unlike other approaches that deal with imprecise evaluations and assignments, DFRSA induces fuzzy rules instead of qualitative rules. Thus, it would be worthwhile to compare DFRSA with other extensions of DRSA for handling uncertain data tables, like those proposed in [2, 13, 22]. In addition, the proposed approach may be useful for sparse data sets[23][3], so we will explore possible applications of DFRSA to such data sets in a future work. Moreover, since DFRSA is a general framework, we do not specify the t-norms used in the aggregation of consistency degrees or the implications used in the definition of adherence to the dominance principle. Consequently, we do not present detailed algorithms for the computation of reducts. The computational aspects of DFRSA for specialized t-norms and implication operations will aslo be addressed in a future work.

[3]We would like to thank the anonymous referee for pointing out this possibility.

ISBN: 978-989-95079-6-8

Acknowledgment

This work was partially supported by the National Science Council of Taiwan under grant 95-2221-E-001-029-MY3.

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ISBN: 978-989-95079-6-8