Production and Transportation Planning – A fuzzy Approach for Minimizing the Total Cost

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Abstract— *In this paper, we deal with the production and transportation planning of a household appliances manufacturer that has production facilities and central stores for resellers in several sites in Europe. Each store can receive products from all production plants and it is not necessary that all products are produced in all production units. The transport between any two bases is done by trucks. For simplicity we assume, that each truck has the same capacity of M EURO-pallets, and for each product the unit is EURO-pallet. The target of this paper is to determine a production and transportation plan that minimize the total sum of the production cost and the transportation cost. For working in a realistic environment we assume that the production capacities in the different plants and the demand in the sales bases are not known exactly but the management can describe the data in form of fuzzy numbers. By using an inter-active algorithm for solving the fuzzy linear programming system we achieve a stable production and a satisfactory supply of the products. Moreover, we demonstrate that this integer programming problem can adequately be solved without using computation-intensive integer programming algorithms. Additionally, in the course of the inter-active solution process the production bottlenecks get clearly visible. A numerical example illustrates the efficiency of the proposed procedure.*

Keywords *— Fuzzy LP-systems, integer programming, FULPAL, Production and transportation planning, inter-active process.*

1 Introduction

Due to globalisation and the involved international expansion of companies numerous firms in Europe produce their products in several European and Overseas countries. On the one hand they take advantage of the different production cost and of higher lot sizes, on the other hand they endeavour to minimize the total cost of manufacturing and transportation.

In literature it is usually proposed to determine a minimal cost production plan by means of an integer programming system. But this procedure is very computationally intensive. Moreover, it is neglected that real production capacities in the different plants and the demand for the products are not known exactly. In this paper we look on the more realistic case that the management can describe the production capacities in the different plants and the demand in the sales bases in form of fuzzy numbers.

The paper is organized as follows: In Section 2 the problem is formulated in detail. In Section 3 we demonstrate that the integer programming problem can adequately be solved by using an inter-active algorithm for solving the fuzzy linear programming system. A numerical example in Section 4 illustrates the efficiency of the proposed procedure. Finally, conclusions and possible extensions are presented in Section 5.

2 Problem formulation

We deal with the production and transportation planning of a household appliances manufacturer HAM that has production facilities and central stores for resellers in several sites in Europe. Each store can receive products from all production plants and it is not necessary that all products are produced in all production units.

- The number of production and sales bases of HAM is N. Obviously, it is not necessary to differ between production plants and sales stores. A base without any demand is a pure production location and a base where nothing is produced is a pure sales shop.
- The number of products of HAM is K.
- The transport between any two bases is done by trucks. For simplicity we assume, that each truck has the same capacity of M EURO-pallets independent of the sort of products.
- For each product the unit is EURO-pallet.

In the production and transportation planning model, we use the following notations:

x_{ki} output of the product k at the base i,

y_{kij} total number of units of the product k transported from the base i to the base j,

w_{ij} number of the trucks from base i to base j,

$\tilde{D}_{ki}$ demand for the product k at the base i,

$\tilde{P}_{ki}$ production capacity for the product k at the base i,

C_{ki} production cost of one unit of product k at the base i,

T_{ij} cost for transporting a truck from the base i to the base j, where $T_{ii} = 0$.

$k \in \{1,\ldots,K\}$, $i, j \in \{1,\ldots,N\}$

The items $x_{ki}, y_{kij}, w_{ij}, \tilde{D}_{ki}, \tilde{P}_{ki}$ are referred to the same time period, e. g. one week or one month.

Now, a production and transportation plan that minimize the total sum of the production cost and the transportation cost can be determined by solving the following integer programming problem:

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$$z(\mathbf{x},\mathbf{w}) = \sum_{i=1}^{N}\sum_{k=1}^{K} C_{ki}x_{ki} + \sum_{j=1}^{N}\sum_{i=1}^{N} T_{ji}w_{ji} \quad \to \quad \text{Min}$$

subject to (1)

$$x_{ki} \mathrel{\tilde{\le}} \tilde{P}_{ki}, \quad k=1,\ldots,K;\ i=1,\ldots,N$$

$$s_{ik} = x_{ki} - \sum_{\substack{j=1\\ j\ne i}}^{N} y_{kij} + \sum_{\substack{j=1\\ j\ne i}}^{N} y_{kji} \mathrel{\tilde{\ge}} \tilde{D}_{ki}, \quad k=1,\ldots,K;\ i=1,\ldots,N$$

$$\sum_{k=1}^{K} y_{kij} \le M\,w_{ij}, \quad i,j=1,\ldots,m$$

$$x_{ij}, y_{kij}, w_{ij} \in \mathbf{N}\cup\{0\}, \quad k=1\ldots,K,\ i,j=1,\ldots,N$$

$$\mathbf{x} = (x_{11},\ldots,x_{K1},x_{12},\ldots,x_{K2},\ldots,x_{1N},\ldots,x_{KN})$$

$$\mathbf{w} = (w_{11},\ldots,w_{N1},w_{12},\ldots,w_{N2},\ldots,w_{1N},\ldots,w_{NN})$$

The first constraint expresses that the output of the product k at the base i is smaller than or equal to its production capacity $\tilde{P}_{ki}$; the second constraint means that the supply s_{ki} of the product k at the base i is larger than or equal to the its demand $\tilde{D}_{ki}$; the third constraint indicates that the total amount of products transported from the base i to the base j is smaller than or equal to the transportation capacity of w_{ij} trucks; the fourth constraint means that the output of the product k at the base i, the number of products transported from the base i to the base j and the number of trucks from the base i to base j are nonnegative integer.

Concerning the imprecise right-hand sides $\tilde{P}_{ki}$ and $\tilde{D}_{ki}$ we assume that management is able to describe the production capacities in the different plants and the demand in the sales bases in form of fuzzy numbers $\tilde{P}_{ki} = (p_{ki}, 0, \pi_{ki})$ and $\tilde{D}_{ki} = (d_{ki}, \delta_{ki}, 0)$. Here, p_{ki} and $d_{ki} - \delta_{ki}$ d_{ki} are the production capacities and the demands respectively that are expected in any case. Furthermore the management estimate the maximal production capacities and the maximal demands as $p_{ki} + \pi_{ki}$ and d_{ki} respectively.

3 Solution process

As shown in Rommelfanger [3], fuzzy integer programming LP-problem can be effectively solved by interactive algorithms for solving fuzzy LP-systems. The present system (1) is a relative simple model with one crisp objective function and soft constraints. Therefore we can use a special form of the algorithm FULPAL (**FU**zzy **L**inear **P**rogramming Based on **A**spiration **L**evels) for a stepwise calculation of an efficient compromise solution of the system (1).

At first, we have to calculate the smallest and the highest total cost by solving the two crisp LP-systems:

$$\underline{z} = \text{Min}\ \Big(\sum_{i=1}^{N}\sum_{k=1}^{K} C_{ki}x_{ki} + \sum_{i=1}^{N}\sum_{j=1}^{N} T_{ji}w_{ji}\Big)$$

subject to (2)

$$x_{ki} \le p_{ki} + \pi_{ki}, \quad k=1,\ldots,K;\ i=1,\ldots,N$$

$$s_{ki} = x_{ki} - \sum_{\substack{j=1\\ j\ne i}}^{N} y_{kij} + \sum_{\substack{j=1\\ j\ne i}}^{N} y_{kji} \ge d_{ki} - \delta_{ki}, \quad \begin{matrix} k=1,\ldots,K \\ i=1,\ldots,N \end{matrix}$$

$$\sum_{k=1}^{K} y_{kij} - M\,w_{ij} \le 0, \quad i,j=1,\ldots,m$$

$$x_{ij}, y_{kij}, w_{ij} \ge 0, \quad k=1\ldots,K,\ i,j=1,\ldots,N$$

and

$$\overline{z} = \text{Min}\ \Big(\sum_{i=1}^{N}\sum_{k=1}^{K} C_{ki}x_{ki} + \sum_{i=1}^{N}\sum_{j=1}^{N} T_{ji}w_{ji}\Big)$$

subject to (3)

$$x_{ki} \le p_{ki}, \quad k=1,\ldots,K;\ i=1,\ldots,N$$

(In case of $\sum_{i=1}^{N} p_{ki} < \sum_{i=1}^{N} d_{ki}$ the capacities p_{ki} must be increased up to the existence of an feasible solution of (2), starting with the plants that have the highest production costs; $k = 1,\ldots,K$.)

$$s_{ki} = x_{ki} - \sum_{\substack{j=1\\ j\ne i}}^{N} y_{kij} + \sum_{\substack{j=1\\ j\ne i}}^{N} y_{kji} \ge d_{ki}, \quad k=1,\ldots,K;\ i=1,\ldots,N$$

$$\sum_{k=1}^{K} y_{kij} - M\,w_{ij} \le 0, \quad i,j=1,\ldots,N$$

$$x_{ij}, y_{kij}, w_{ij} \ge 0, \quad k=1\ldots,K,\ i,j=1,\ldots,N$$

In accordance with FULPAL, the objective function and the soft constraints of the system (1) are transformed in utility functions, where $z^A[r], p^A_{ki}[r], d^A_{ki}[r]$ with $r = 1$ are the crisp aspiration levels that are specified by the management for the time being.

In the course of the inter-active solution process the management can change the aspiration levels step by step. For getting an effective comparability of the utilities, the same utility (membership degree) λ_A is assigned to all crisp aspiration levels. The goal of the management is to get a solution that satisfies all aspiration levels.

$$\mu_z(\mathbf{x},\mathbf{w}) = \tag{4}$$

$$\begin{cases} 1 & \text{if} \quad z(\mathbf{x},\mathbf{w}) < \underline{z} \\ 1 - \dfrac{z(\mathbf{x},\mathbf{w}) - \underline{z}}{z^A[r] - \underline{z}}\cdot(1-\lambda_A) & \text{if} \quad \underline{z} \le z(\mathbf{x},\mathbf{w}) < z^A[r] \\ \lambda_A + \dfrac{z(\mathbf{x},\mathbf{w}) - z^A[r]}{\overline{z} - z^A[r]}\cdot(1-\lambda_A) & \text{if} \quad z^A[r] \le z(\mathbf{x},\mathbf{w}) \le \overline{z} \\ 0 & \text{if} \quad \overline{z} < z(\mathbf{x},\mathbf{w}) \end{cases}$$

$$\mu_{xki}(x_{ki}) = \tag{5}$$

$$\begin{cases} 1 & \text{if} \quad x_{ki} < p_{ki} \\ 1 - \dfrac{x_{ki} - p_{ki}}{p^A_{ki}[r] - p_{ki}}\cdot(1-\lambda_A) & \text{if} \quad p_{ki} \le x_{ki} \\ \lambda_A + \dfrac{x_{ki} - p^A_{ki}[r]}{p_{ki} + \pi_{ki} - p^A_{ki}[r]}\cdot(1-\lambda_A) & \text{if} \quad p^A_{ki}[r] < x_{ki} \\ 0 & \text{if} \quad p_{ki} + \pi_{ki} < x_{ki} \end{cases}$$

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$$\mu_{ski}(s_{ki}) = \begin{cases} 0 & \text{if} \quad s_{ki} < d_{ki} - \delta_{ki} \\ \dfrac{s_{ki} - (d_{ki} - \delta_{ki})}{d_{ki}^A[r] - (d_{ki} - \delta_{ki})} \cdot \lambda_A & \text{if} \quad d_{ki} - \delta_{ki} \le s_{ki} \le d_{ki}^A[r] \\ \lambda_A + \dfrac{s_{ki} - d_{ki}^A[r]}{d_{ki} - d_{ki}^A[r]} \cdot (1 - \lambda_A) & \text{if} \quad d_{ki}^A[r] < s_{ki} \le d_{ki} \\ 1 & \text{if} \quad d_{ki} < s_{ki} \end{cases} \tag{6}$$

$k = 1,\ldots,K;\ i = 1,\ldots,N$

For calculating a compromise solution of the multi-objective system

$$\begin{array}{l}(\mu_Z(\mathbf{x},\mathbf{w}), \mu_{x11}(x_{11}), \ldots, \mu_{xKN}(x_{KN}), \\ \quad \mu_{s11}(s_{11}), \ldots, \mu_{sKN}(s_{KN})) \end{array} \rightarrow \text{Max}$$

subject to (7)

$$x_{ki} \le p_{ki} + \pi_{ki}, \quad k = 1,\ldots,K;\ i = 1,\ldots,N$$

$$s_{ki} = x_{ki} - \sum_{\substack{j=1 \\ j \ne i}}^{N} y_{kij} + \sum_{\substack{j=1 \\ j \ne i}}^{N} y_{kji} \ge d_{ki} - \delta_{ki}, \quad k = 1,\ldots,K;\ i = 1,\ldots,N$$

$$\sum_{k=1}^{K} y_{kij} - M\, w_{ij} \le 0, \quad i,j = 1,\ldots,m$$

$$x_{ij}, y_{kij}, w_{ij} \ge 0, \quad k = 1\ldots,K, \quad i,j = 1,\ldots,N$$

we use the compromise objective function

$$\begin{array}{l}\mu = \text{Min}\ (\mu_Z(\mathbf{x},\mathbf{w}), \mu_{x11}(x_{11}), \ldots, \mu_{xKN}(x_{KN}), \\ \quad \mu_{s11}(s_{11}), \ldots, \mu_{sKN}(s_{KN}))\end{array} \tag{8}$$

Then, we get a pareto-optimal compromise solution of (7) by solving the crisp mathematical programming system (9).

$$\lambda \rightarrow \text{Max}$$

subject to (9)

$$\lambda \le \mu_Z(\mathbf{x},\mathbf{w})$$

$$\lambda \le \mu_{xki}(x_{ki}), \quad k = 1,\ldots,K;\ i = 1,\ldots,N$$

$$\lambda \le \mu_{ski}(s_{ki}), \quad k = 1,\ldots,K;\ i = 1,\ldots,N$$

$$x_{ki} \le p_{ki} + \pi_{ki}, \quad k = 1,\ldots,K;\ i = 1,\ldots,N$$

$$s_{ki} = x_{ki} - \sum_{\substack{j=1 \\ j \ne i}}^{N} y_{kij} + \sum_{\substack{j=1 \\ j \ne i}}^{N} y_{kji} \ge d_{ki} + \delta_{ki}, \quad k = 1,\ldots,K;\ i = 1,\ldots,N$$

$$\sum_{k=1}^{K} y_{kij} - M\, w_{ij} \le 0, \quad i,j = 1,\ldots,m$$

$$x_{ij}, y_{kij}, w_{ij} \ge 0, \quad k = 1\ldots,K, \quad i,j = 1,\ldots,N$$

As the management is only interested in a solution that satisfies all aspiration levels, it is sufficient to solve the following crisp linear LP-system, where $r = 1$; for details see [1; 2].

$$\lambda \rightarrow \text{Max}$$

subject to (10)

$$(z^A[r] - \underline{z})\lambda + (1 - \lambda_A)\left(\sum_{i=1}^{N}\sum_{k=1}^{K} C_{ki} x_{ki} + \sum_{i=1}^{N}\sum_{j=1}^{K} T_{ji} w_{ji}\right) \le z^A[r] - \lambda_A \underline{z}$$

$$(p_{ki}^A[r] - p_{ki})\lambda + (1 - \lambda_A) x_{ki} \le p_{ki}^A[r] - \lambda_A p_{ki}, \quad k = 1,\ldots,K;\ i = 1,\ldots,N$$

$$(d_{ki} - d_{ki}^A[r])\lambda - (1 - \lambda_A)\left(x_{ki} - \sum_{\substack{j=1 \\ j \ne i}}^{N} y_{kij} + \sum_{\substack{j=1 \\ j \ne i}}^{N} y_{kji}\right) \le \lambda_A d_{ki} - d_{ki}^A[r], \quad k = 1,\ldots,K;\ i = 1,\ldots,N$$

$$\sum_{k=1}^{K} y_{kij} - M\, w_{ij} \le 0, \quad i,j = 1,\ldots,m$$

$$\lambda, x_{ij}, y_{kij}, w_{ij} \ge 0, \quad k = 1\ldots,K, \quad i,j = 1,\ldots,N$$

Obviously, a solution of the system (10) with $\lambda \ge \lambda_A$ is a pareto-optimal solution of the systems (7) and (1) and fulfils all aspiration levels $z^A[1], p_{ki}^A[1], d_{ki}^A[1]$.

Moreover, if λ is greater than λ_A, the management can increase some of the aspiration levels to $z^A[2], p_{ki}^A[2], d_{ki}^A[2]$ and can calculate a new pareto-optimal solution by means of the revised system (10), and so on. When the management is satisfied with the non-integer solution, it is time for looking for an integer solution. Due to the soft constraints several integer solution will exist in the neighborhood of the non-integer solution.

4 Numerical example

The manufacturer HAM produces four products P1, P2, P3 and P4 in three product plants in **E**ssen, **K**rakow and **L**yon.

Table 1: Production costs in € per unit

	P1	**P2**	**P3**	**P4**
E	200	400	330	500
K	180	300	280	
L	220		300	570

Table 2: Transportation costs in € per truck

	E	**K**	**L**
E		1000	600
K	1000		1200
L	600	1200	

Table 3: Secure production capacities p_{ki}

	p_{1i}	p_{2i}	p_{3i}	p_{4i}
E	600	450	250	250
K	350	200	250	
L	250		400	150

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Table 4: Maximal production capacities $p_{ki}+\pi_{ki}$

	$p_{1i}+\pi_{1i}$	$p_{2i}+\pi_{2i}$	$p_{3i}+\pi_{3i}$	$p_{4i}+\pi_{4i}$
E	750	500	300	300
K	500	300	300	
L	300		500	200

Table 5: Minimal demands $d_{ki}-\delta_{ki}$

	$d_{1i}-\delta_{1i}$	$d_{2i}-\delta_{2i}$	$d_{3i}-\delta_{3i}$	$d_{4i}-\delta_{4i}$
E	650	300	300	150
K	450	100	200	80
L	150	150	250	100

Table 6: Maximal demands d_{ki}

	d_{1i}	d_{2i}	d_{3i}	d_{4i}
E	750	400	350	200
K	350	150	200	120
L	200	200	400	130

Using the LP-systems (2) and (3), we get the minimal total cost $\underline{z}$ = 864,100 € and the maximal cost $\overline{z}$ = 1,091,700 €. With all these information the management of HAM specifies for the total cost the aspiration level z[1] = 950,000 € and for the outputs and the demands the following aspiration levels.

Table 7: Aspiration levels $p_{ki}[1]$

output	$p_{1i}[1]$	$p_{2i}[1]$	$p_{3i}[1]$	$p_{4i}[1]$
E	720	480	280	280
K	450	270	280	
L	280		470	180

Table 8: Aspiration levels $d_{ki}[1]$

demand	$d_{1i}[1]$	$d_{2i}[1]$	$d_{3i}[1]$	$d_{4i}[1]$
E	670	350	330	180
K	280	120	200	100
L	180	170	300	110

As $\lambda = 0.53 > \lambda_A = 0.5$, the solution of the corresponding LP-system (10) fulfills all aspiration levels. E. g. the total cost of this plan is 945,082.50 €.

Moreover this result indicates that it is possible to improve the aspiration levels. For simplification we assume that the management is only interested in lower cost and decides to reduce the total cost to z[2] = 940,000 €.

Even the solution of the revised system (10) fulfils all aspiration levels. We have $\lambda = 0.5077$, z = 938,712.20 € and the following production and transportation plan.

Table 9: Production and transportation plan

x1e	x1k	x1l	x2e	x2k	x3e
671,05	280,92	180,26	372,24	269,08	250,79
x3k	x3l	x4e	x4l	wek	wel
279,61	301,32	279,61	111,18	4,97	1,08
wke	wkl	wle	wlk	y1ek	y1el
3,98	7,43	0,00	0,05	0,00	0,00
y1ke	y1kl	y1le	y1lk	y2ek	y2el
0,00	0,00	0,00	0,00	0,00	21,58
y2ke	y2kl	y3ek	y3el	y3ke	y3kl
0,00	148,68	0,00	0,00	79,61	0,00
y3le	y3lk	y4ek	y4el	y4le	y4lk
0,00	0,00	99,34	0,00	0,00	0,92

As we have soft constraints it is very simple to derive an integer plan that fulfils all aspiration levels and actually leads to lower total cost z= 937,100 €.

Table 10: Integer production and transportation plan

x1e	x1k	x1l	x2e	x2k	x3e
670	280	180	370	270	250
x3k	x3l	x4e	x4l	wek	wel
280	300	280	110	5	1
wke	wkl	wle	wlk	y1ek	y1el
4	8	0	0	0	0
y1ke	y1kl	y1le	y1lk	y2ek	y2el
0	0	0	0	0	20
y2ke	y2kl	y3ek	y3el	y3ke	y3kl
0	150	0	0	80	0
y3le	y3lk	y4ek	y4el	y4le	y4lk
0	0	100	0	0	0

5 Conclusions

Fuzzy mathematical programming systems offer the possibility to model real problems as precisely as a decision maker is able to describe them. In doing so non-satisfying modeling can be avoided. Another advantage of fuzzy models is the fact that (mixed) integer programming problems can be solved very easily because the boundaries are not crisp but fuzzy and points with integer variables in the neighborhood of an optimal solution are feasible in general.

In this paper we have assumed that only the right-hand sides of some constraints are not known exactly. The model can be extended to the case that additionally coefficients of the objective function or of the constraints are described in form of fuzzy intervals. Moreover, it is possible to allow several objective functions. All these systems can adequately be solved by means of the inter-active algorithm FULPAL, see [1; 2; 3].

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