Time Series Analysis and Prediction Based on Fuzzy Rules and the Fuzzy Transform

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Abstract— *A new methodology for analysis and forecasting of time series is proposed. It directly employs two techniques: the fuzzy transform and the perception-based logical deduction. Due to the usage of both of them and due to the innovative approach consisting in a construction of several independent models, the methodology is successfully applicable for robust long time predictions.*

Keywords— Time series, fuzzy transform, perception-based logical deduction, fuzzy partition.

1 Introduction

We propose a new methodology for forecasting time series which is based on combination of two techniques: fuzzy transform and perception-based logical deduction. The proposed methodology consists of two phases: analysis of a time series and its forecast. In both phases, the above mentioned techniques play an essential role.

The organization of the work is as follows. After preliminaries and recalling the techniques in Section 2, we continue with a detailed description of the proposed approach in Section 3. Then we demonstrate the results in Section 4.

2 Preliminaries

Let us briefly recall the main tools employed in the suggested approach, the *fuzzy transform* (F-transform) [1] and the *perception-based logical deduction* [2, 3], in particular.

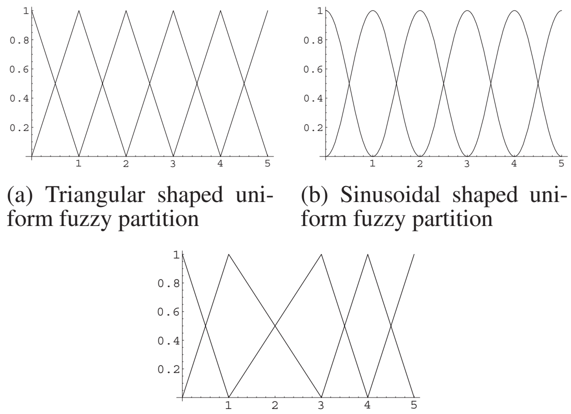
2.1 Fuzzy (F)-Transform

Fuzzy transform, in detailed described in [1], consists in two transformations. The first one, sometimes called the direct one, maps a continuous function defined on a fixed real interval $[a, b]$ into an n-dimensional vector. The inverse one maps the vector back to the space of continuous functions.

The F-transform is defined with respect to *basic functions* forming a fuzzy partition.

Definition 2.1 Let $c_1 < \cdots < c_n$ be fixed nodes within $[a, b]$, such that $c_1 = a, c_n = b$ and $n \geq 2$. We say that fuzzy sets $A_1, \ldots, A_n \in \mathcal{F}([a, b])$ are *basic functions* forming a fuzzy partition of $[a, b]$ if they fulfill the following conditions for $i = 1, \ldots, n$:

1. $A_i(c_i) = 1$;
2. $A_i(x) = 0$ for $x \notin (c_{i-1}, c_{i+1})$ where for the uniformity of the denotation we put $x_0 = x_1 = a$ and $x_{n+1} = x_n = b$;



(a) Triangular shaped uniform fuzzy partition

(b) Sinusoidal shaped uniform fuzzy partition

(c) Triangular shaped non-uniform fuzzy partition with a symmetry

Figure 1: Graphic presentation of distinct fuzzy partitions.

3. A_i is continuous
4. A_i strictly increases on $[c_{i-1}, c_i]$ and strictly decreases on $[c_i, c_{i+1}]$;
5. for all $x \in [a, b]$

$$\sum_{i=1}^{n} A_i(x) = 1. \tag{1}$$

Usually, the *uniform fuzzy partition* is used i.e. n equidistant nodes $c_i = c_{i-1} + h$ are fixed. Let us remark that the shape of the basic functions is not predetermined and it can be chosen based on further requirements. For some examples of fuzzy partitions fulfilling the Definition 2.1, see Fig. 1.

Definition 2.2 Let a fuzzy partition of $[a, b]$ be given by basic functions $A_1, \ldots, A_n, n \geq 2$ and let $f : [a, b] \to \mathbb{R}$ be an arbitrary continuous function. The n-tuple of real numbers $[F_1, \ldots, F_n]$ given by

$$F_i = \frac{\int_a^b f(x)A_i(x)dx}{\int_a^b A_i(x)dx}, \; i = 1, \ldots, n \tag{2}$$

is the *direct fuzzy transform (F-transform)* of f with respect to the given fuzzy partition. $F_1, \ldots, F_n$ are the *components* of the F-transform of f.

ISBN: 978-989-95079-6-8

In practice, function f is usually not given analytically but we are at least provided by some data obtained, e.g., by some measurements. In this case, Definition 2.2 can be modified in such a way that finite summations replace definite integrals in formula (2).

Let us be given n basic functions forming a fuzzy partition of $[a, b]$ and let the function f be given at $T > n$ fixed nodes $x_1, \ldots, x_T \in [a, b]$. We say that the set of nodes $\{x_1, \ldots, x_T\}$ is *sufficiently dense with respect to the fuzzy partition* if

$$(\forall i)(\exists t) \quad A_i(x_t) > 0. \tag{3}$$

Definition 2.3 Let a fuzzy partition of $[a, b]$ be given by basic functions $A_1, \ldots, A_n, n \geq 2$ and let $f : [a, b] \to \mathbb{R}$ be a function known at a set $\{x_1, \ldots, x_T\}$ of sufficiently dense nodes with respect to the given fuzzy partition. The n-tuple of real numbers $[F_1, \ldots, F_n]$ given by

$$F_i = \frac{\sum_{t=1}^{T} f(x_t)A_i(x_t)}{\sum_{t=1}^{T} A_i(x_t)}, \ i = 1, \ldots, n \tag{4}$$

is the *(discrete) direct F-transform* of f with respect to the given fuzzy partition. $F_1, \ldots, F_n$ are the *components* of the F-transform of f.

Since this paper deals with the application of the F-transform to the analysis and prediction of time series, i.e., discrete problem, there is no danger of confusion and we may easily, for the sake of simplicity, omit the word “discrete” and simply talk about the “F-transform” in the rest of the paper.

The F-transform of f with respect to $A_1, \ldots, A_n$ will be denoted by $F_n[f] = [F1, \ldots, Fn]$. It has been proved [1] that the components of the F-transform are the weighted mean values of an original function where the weights are given by the basic functions.

The original function f can be approximately reconstructed (with the help of the inversion formula) from its fuzzy transform $F_n[f]$. The function represented by the inversion formula is called the *inverse F-transform*.

Definition 2.4 Let $F_n[f]$ be the direct F-transform of f with respect to $A_1, \ldots, A_n \in \mathcal{F}([a, b])$. Then the function $f_{F,n}$ given on $[a, b]$ as follows

$$f_{F,n}(x) = \sum_{i=1}^{n} F_i A_i(x), \tag{5}$$

is called the *inverse F-transform* of f.

The inverse F-transform is a continuous function on $[a, b]$ no matter whether we have started from analytically given continuous f or we have been given only function values on a sufficiently dense set. Anyhow, for the time series problem, we consider the inverse F-transform at the same points where the original function is given.

2.2 Perception-based Logical Deduction

In this subsection, we briefly recall the main idea of the *perception-based logical deduction* (PbLD). For details, let us refer to [2, 3].

The PbLD is a method of deducing conclusions on the basis of a *linguistic description* which is a set of m fuzzy/linguistic rules:

IF x_1 is $\mathcal{A}_{11}$ **AND** $\cdots$ **AND** x_q is $\mathcal{A}_{1q}$ **THEN** z is $\mathcal{B}_1$
...
AND
...
IF x_1 is $\mathcal{A}_{m1}$ **AND** $\cdots$ **AND** x_q is $\mathcal{A}_{mq}$ **THEN** z is $\mathcal{B}_m$ (6)

where $\mathcal{A}_{11}, \ldots, \mathcal{A}_{nq}, \mathcal{B}_1, \mathcal{B}_m$ are specific linguistic expressions called *evaluative expressions* which have the form

$$\langle \text{ linguistic hedge } \rangle \ \{ \textit{small, medium, big} \}.$$

Note that extension of $\mathcal{A}$ is a functional value of its intension in a point w. These extensions are specific fuzzy sets having the form depicted in Fig.2. Let us shortly mention, that formal theory of evaluative linguistic expression is a deep and firm theory and it would be out of range of the conference paper to recall it in a more detailed way to readers. Therefore, we refer to its detailed description and justification published in [4].

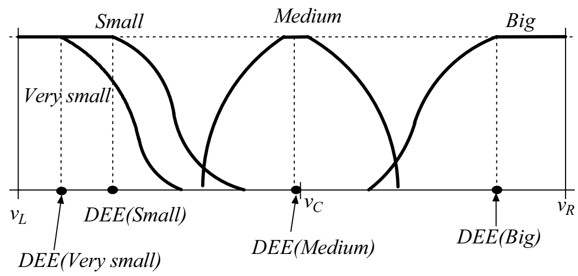


Figure 2: Fuzzy sets representing typical evaluative linguistic expressions.

In the frame of the above outlined theory, that the rules in (6) are taken as genuine conditional expressions of natural language that are interpreted accordingly, see [5], i.e. intension and extensions of them are also defined. The extension of each rule is a special fuzzy relation computed using a t-norm (usually minimum) and Łukasiewicz (residual) implication.

By *perception* we understand an evaluative expression assigned to the given value in the given context. The choice of perception is not arbitrary and besides the context, it also depends on the topic of the specified linguistic expression. The PbLD method uses a sophisticated algorithm for choosing the best fitting rule(s). The most proper crisp output is obtained using a DEE (Defuzzification of Evaluative Expressions) method [2].

2.3 Time Series

Let

$$\{x_t | t = 1, \ldots, T\} \subset \mathbb{R}, \quad T \geq 3 \tag{7}$$

be a given time series. The task is to analyze and forecast the time series, i.e., to determine values

$$\{x_t | t = T + 1, \ldots, T + k\} \subset \mathbb{R}, \quad k \geq 1. \tag{8}$$

There are two standard approaches to forecast a time series. The first one uses autoregressive and moving averages models of the so called *Box-Jenkins methodology* [6]. This approach,

ISBN: 978-989-95079-6-8

no matter how useful and successful in forecasts it may be, besides stationarity and seasonality provides us with a non-transparent and non-interpretable analysis. Therefore, it is not appropriate for an extension by means and tools directly using the fuzzy sets theory, where the interpretability and transparency play a crucial role.

Remark 2.5 A study presenting Takagi-Sugeno rules [7] in the view of the autocorrelation Box-Jenkins methodology has been published [8]. Nevertheless, the Takagi-Sugeno rules are rather regression-based than linguistic-based in comparison with (6). Analogously, distinct neuro-fuzzy approaches, which are on the border of neural networks, Takagi-Sugeno models and evolving fuzzy systems, are very often and successfully used [9, 10]. But well tuned Gaussian fuzzy sets with a centroid at node 5.6989 and a width parameter equal to 2.8893 (see [10]) constructed as a product of an employed optimization technique are undoubtably far from the interpretability of systems using models of fragments of natural language. Therefore, these approaches, no matter how effective and powerful, are closer to standard regression methods.

The second approach consists in the so called *decomposition* [11] of a time series where each element x_t of the time series is decomposed into the following components

$$x_t = Tr_t + S_t + C_t + E_t \tag{9}$$

where Tr_t, S_t, C_t, E_t are the t-th *trend, seasonal, cyclic* and *error* components, respectively.

Trend and seasonal components may be analyzed. The cyclic component is a bit problematic. The name "cyclic" comes from the economical cycles which are not regular and they are dependent on many outer factors to be analyzed from the past. The error component is a random noise which basically cannot be forecasted and therefore is omitted from our further considerations.

After these omissions, for further investigation, usually the following simplified decomposed model is considered

$$x_t = Tr_t + S_t. \tag{10}$$

Remark 2.6 Formulas (9) and (10) describe so called additive decomposition. Using multiplications instead of summations, we would get the multiplicative decomposition. Since the treatment is analogous, for the sake of simplicity we restrict our focus on the additive ones but the further methods and results are similar.

3 Proposed approach

Let us introduce our suggested approach directly employing the techniques briefly recalled in Section 2.

3.1 Trend and Seasonal Analysis

Traditional approach to the trend analysis assumes the trend to be an a priori given function, e.g., linear, polynomial, exponential or a kind of a saturation function such as sigmoidal function, for instance. This approach simplifies the analysis, which consists in a regressive determination of parameters of the predetermined function, and even the prediction, which consists in a simple prolongation – i.e. in an evaluation of the determined trend function at nodes $T+1, \dots, T+k$.

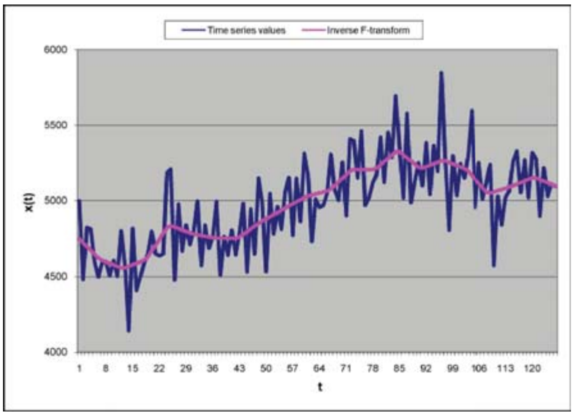


Figure 3: Time series with its inverse F-transform. Standard trend analysis would be inappropriate due to irregular cyclic changes.

But such an approach is not always the most proper one, see Fig. 3. The problem is that such an approach where we fix a trend function for the whole domain is too restrictive. The trend may change during the measurements of a time series especially in case of a long time series. For such cases, complicated adaptive trend changing models are constructed. As a typical example, we may especially in these days of the financial crisis recall the equity indexes where we may hardly simply prolong the trends while very often after robust growth radical and dramatic falls come followed by stagnation and then again by growths, due to the cyclic influences. Here, the prolongation might be very often the worst thing we may apply in predictions.

We propose the F-transform method for the trend description. The F-transform does not fix any shape of the curve, but it possess powerful approximation, noise reduction and computational properties [12].

The time series x_t may be viewed as a function x on the interval $[0, T]$ which we are not given analytically, but we are provided by measurements $x(t) = x_t$ at nodes $t = 1, \dots, T$. Let us build a uniform fuzzy partition according to Definition 2.1 such that each basic function $A_2, \dots, A_{n-1}$ "cover" the number of nodes equal to the number of nodes belonging to a season. For example, in case of a time series on the monthly basis, each basic function covers 12 point except basic functions A_1, A_n which cover the first and the last 6 nodes, respectively. So, the set of nodes is sufficiently dense with respect to the fuzzy partition. From now on, we will explicitly refer to such a time series on the monthly basis since everything may be easily generalized for a different seasonality.

Remark 3.1 Usually, we are provided by some additional information about a time series, for example that it is a GDP growth per month. In cases, when such information is not at disposal or does not lead to a sufficient knowledge to determine the seasonality (diesel engine pressure etc. [13]), the perception-based logical deduction with appropriate rules may be successfully used. For details, we refer the reader to [14].

Let $F_n[x] = [X_1, \dots, X_n]$ be the F-transform of the function x w.r.t. the given fuzzy partition and let $x_{F,n}$ be its inverse F-transform. The inverse F-transform will serve us as a model

ISBN: 978-989-95079-6-8

of the trend component (including cyclic changes). It does not fix a priori some shape of the function. Therefore, we may easily from (9) with omitted error component determine the seasonal components S_t

$$S_t = x_t - x_{F,n}(t) \tag{11}$$

where $x_{F,n}(t) = Tr_t + C_t$.

3.2 Trend Forecast

The suggested approach to the trend analysis implicitly treats also possible cyclic component influences without complicated adaptive trend changing mechanisms. This is a significant difference to the classical approach, where we first model the trend only and then determine the seasonal components which are influenced by the cyclic irregular changes. Our approach treats the problem the other way around and the trend model $x_{F,n}$ primarily serves us to get pure seasonal components without the cyclic influences.

On the other hand, we cannot easily forecast such a trend model by the prolongation, i.e. by the evaluation of the predetermined fixed trend function at nodes $t = T+1, \ldots, T+k$. Due to the drawback of such a traditional approach, it is not disadvantage but vice-versa, as explained below.

We follow the idea of [15] and for the trend forecast, we employ the perception-based logical deduction. As antecedent variables, we consider the F-transform components of the given time series X_i, $i = 1, \ldots, n$ as well as their differences of the first and the second order

$$\begin{aligned} \Delta X_i &= X_i - X_{i-1}, & i &= 2, \ldots, n \\ \Delta^2 X_i &= \Delta X_i - \Delta X_{i-1}, & i &= 3, \ldots, n \end{aligned} \tag{12}$$

respectively.

So, we will deal, e.g., with fuzzy rules such as the following one

$$\textbf{IF } \Delta X_{i-1} \text{ is } \mathcal{A}_{\Delta i-1} \textbf{ AND } X_i \text{ is } \mathcal{A}_i \textbf{THEN } \Delta X_{i+1} \text{ is } \mathcal{B}. \tag{13}$$

The differences of the F-transform components expressing the time series trend (and the cycle) of distinct orders are capable to describe the dynamics of the time series better than the F-transform components itself. Fuzzy rules such as (13) may describe logical dependencies of the trend changes (hidden cycle influences) which is highly desirable and suggested in comparison with the standard prolongation of the trend observed in the past. The advantage of the transparently interpretable form of the rules using fragments of natural language is doubtless. This might be helpful in better understanding of functionalities and motive factors determining the changes in a process yielding the analyzed times series.

Let us mention that fuzzy rules such as (13) are automatically generated by the *linguistic learning* algorithm [16] implemented in the software package LFLC 2000 [17] from the F-transform components of the time series and their differences.

Remark 3.2 Although the fuzzy rules are also generated from the past, the suggested approach may be able to learn, describe and successfully predict the future of equity indexes mentioned as a motivation example at the beginning of Subsection 3.1. Of course, if a similar progress has been observed and measured in the past. Prolongation of a trend function is not capable of this task.

3.3 Independent Models

Fuzzy rules and the perception-based logical deduction may help us to forecast next F-transform components $X_{n+1}, \ldots, X_{n+\ell}$ of the time series from which we easily determine trend components of the time series as the values of the inverse F-transform $x_{F,n+\ell}(T+1), \ldots, x_{F,n+\ell}(T+k)$, where $\ell < k$.

The problem may appear when the forecast is supposed to be for a too long term. In case if we want to predict the time series x_t on a monthly basis for the next 18 months, we first have to forecast X_{n+1}. For this, we use components $X_1, \ldots, X_n$ and differences $\Delta X_2, \ldots, \Delta X_n$, $\Delta^2 X_3, \ldots, \Delta^2 X_n$. Then, using the same fuzzy rules and antecedent variables $X_1, \ldots, X_{n+1}$, $\Delta X_2, \ldots, \Delta X_{n+1}$ and $\Delta^2 X_3, \ldots, \Delta^2 X_{n+1}$ we may forecast X_{n+2}. And finally, we are able to forecast X_{n+3} based on $X_1, \ldots, X_{n+2}$, $\Delta X_2, \ldots, \Delta X_{n+2}$ and $\Delta^2 X_3, \ldots, \Delta^2 X_{n+2}$.

Clearly, there is a danger of the forecast error propagation since we forecast from forecasted values. The longer-term prediction, the higher is the damage.

To overcome this problem, we suggest to build a finite number of independent trend forecasting models using the technique described in Subsection 3.2. For the sake of simplicity, let us consider ΔX_{i-1} and X_i as the antecedent variables. Then set of rules (13) will serve us as a model for the prediction the next F-transform component X_{n+1}.

Similarly, a set of rules

$$\textbf{IF } \Delta X_{i-1} \text{ is } \mathcal{A}_{\Delta i-1} \textbf{ AND } X_i \text{ is } \mathcal{A}_i \textbf{ THEN } \Delta X_{i+2} \text{ is } \mathcal{B} \tag{14}$$

generated again by the linguistic learning algorithm from the differences of the F-transform components, will serve us as a model for the prediction the next but one F-transform component X_{n+2}. And analogously, we may build the third model

$$\textbf{IF } \Delta X_{i-1} \text{ is } \mathcal{A}_{\Delta i-1} \textbf{ AND } X_i \text{ is } \mathcal{A}_i \textbf{ THEN } \Delta X_{i+3} \text{ is } \mathcal{B} \tag{15}$$

and further models, if necessary because of a longer-term forecast than just to next 18 values $x_{T+1}, \ldots, x_{T+18}$.

The suggested approach fully avoids the problem of the error accumulation due to the “forecasting from forecasted”.

3.4 Seasonal Component Forecast

Forecasted F-transform components $X_{n+1}, \ldots, X_{n+\ell}$ determine the trend and cyclic components of the time series predicted for the future which are given as the values of the inverse F-transform $x_{F,n+\ell}(T+1), \ldots, x_{F,n+\ell}(T+k)$. To forecast the seasonal components and to compose them with the latter values is the last step in the time series prediction procedure.

The seasonal components are forecasted as follows. Let

$$\mathbf{S}_\xi = [S_{p\cdot(\xi-1)}, S_{p\cdot(\xi-1)+1}, S_{p\cdot(\xi-1)+2}, \ldots, S_{p\cdot(\xi-1)+p-1}] \tag{16}$$

be the ξ-th vector of the seasonal components (11) where p denotes the period of the seasonality, i.e., in our case of a time series on a monthly basis $p = 12$ and the $\mathbf{S}_\xi$ is the vector of the January, February etc. to December measurements of the ξ-th year.

Stationarity assumption on the seasonal component of the time series is considered, i.e., we assume that $\mathbf{S}_\xi$ is a linear

ISBN: 978-989-95079-6-8

combination of previous θ vectors $\mathbf{S}_{\xi-\theta}, \ldots, \mathbf{S}_{\xi-1}$. It means that we generate the following system of equations

$$\mathbf{S}_{\xi} = \sum_{j=1}^{\theta} d_j \cdot \mathbf{S}_{\xi-j}, \quad \xi > \theta \tag{17}$$

to which we search for the optimal solution with respect to coefficients $d_1, \ldots, d_\theta$. The computed coefficients are then used to determine the $\mathbf{S}_\xi$

Let us mention that the stationarity is a standard assumption which might be easily checked, see [6, 18].

Composition of both forecasts, of the forecasted F-transform components and the seasonal components is the last step to get the overall time series forecast. It is done inversely to the original decomposition which was either additive or multiplicative.

3.5 Optimization

There are some unknown parameters in the whole procedure which are to be determined individually for every single time series. Basically, it is the list of antecedent variables for the prediction of the F-transform components, the number of the antecedent variables and the parameter θ.

The time series is divided into the *learning set* and the *validation set* in such a way that the latter one is given by the last values of the times series of the length equal to the expected forecast. It means, the learning set is given by $\{x_1, \ldots, x_{T-k-1}\}$ and we cut $\{x_{T-k}, \ldots, x_T\}$ off the time series to determine the validation.

There is a software [19], developed at the *Institute for Research and Applications of Fuzzy Modeling*, where the parameters θ and the maximal number of antecedent variables are set up by a user. Then the software computes all possible combinations of antecedent variables up to the maximal number and combines it with seasonal components determined based on the optimal solution of (17) for all possible θ, again up to the pre-specified maximal one. For this computations, only the learning set is used.

All computed models are used to forecast $\{x'_{T-k}, \ldots, x'_T\}$ and these forecasts are compared with the validation set $\{x_{T-k}, \ldots, x_T\}$. As a suggestion to a user, all models are ordered according to a pre-specified error criterion which may be generally arbitrarily. A user may then employ any of the optimized and tuned models for the forecast of $x_{T+1}, \ldots, x_{T+k}$.

4 Demonstration

We applied our approach to analyze and forecast 111 time series on a monthly basis from the *Artificial Neural Network & Computational Intelligence Forecasting Competition NN3*. All the time series were "real-world" and chosen from different fields such as economy, meteorology, industry etc. while no such an information about the meaning or an origin of any time series was provided to participants.

There were 580 participants who registered to the competition and downloaded the time series but only 25 finally afforded to submit their results which illustrates the complicacy of the task to deal with such a huge and heterogenous set of time series. This fact is even supported by the average 15.44% error result (measured by SMAPE - Symmetric Mean Absolute Percent Error) obtained by commercial statistical software *ForecastPro* for the whole set of time series. Let us note that the *ForecastPro* software was used only as a benchmark together with five other benchmarks. Let us also remark that it employs all the existing standard techniques from the Box-Jenkins methodology while participants were limited by a single method of a non-statistical nature.

Our approach (without the construction of independent models at that times) took the 12th position with the overall average error 18.81%. In an unofficially evaluated subset of time series with a seasonal character, our approach took the 3rd rank among all participants and even beaten four of the six statistical benchmarks. The average error in this category measured by SMAPE was 12.7%.

The construction of independent models may even improve the results. For an illustration, let us turn the attention to the time series NN3_106 from the competition, see Fig. 4.

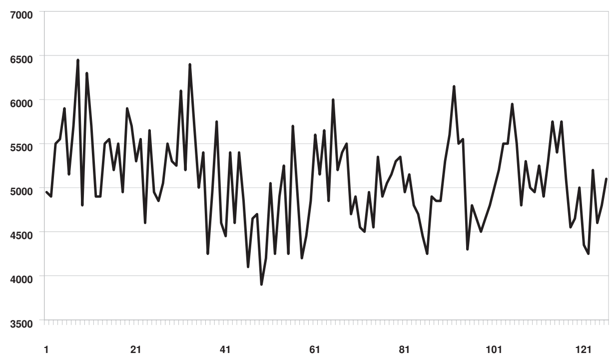


Figure 4: Time series NN3_106 from the NN3 competition.

It is a 126 months long time series with a hardly determinable trend - a typical representative of an appropriate time series for our approach. We cut off last 18 months to create a validation set and we use the first 114 months as a learning set to determine our model. The model may be used to forecast the last 18 months and then compared with the original time series values forming the validation set. There is the black line displaying the validation set of the original time series in Fig. 5.

The dotted grey line in the same figure denotes the forecast made by our methodology with a single trend prediction model optimized as described in Subsection 3.5. The generated model consisted of 15 fuzzy rules with the following 3 input variables $X_i, \Delta^2 X_i, \Delta^2 X_{i-1}$ and the output variable ΔX_{i+1}. The SMAPE error was 5.70%.

If we generate 3 independent models – all of them with the following 3 input variables $X_i, \Delta^2 X_i, \Delta^2 X_{i-1}$, every one of them predicting a different F-transform component – $\Delta X_{i+1}, \Delta X_{i+2}, \Delta X_{i+3}$, respectively, we improve the results. Due to the F-transform, the above mentioned 3 models predicting independently 3 first order differences of the F-transform components are fully sufficient to forecast values

ISBN: 978-989-95079-6-8

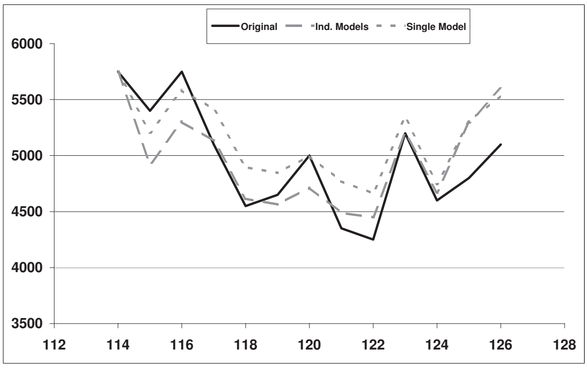


Figure 5: Time series NN3_106 (black line) and its two forecasts, one based on our methodology with a single model (dotted grey line), one forecast employs our methodology with 3 independent models (dashed grey line).

of 18 months ahead. The improve results are depicted by the grey dashed line on Fig. 5 and yielded error 4.77%.

5 Conclusions

We have proposed a novel fuzzy approach to analysis and prediction of time series. It directly uses two techniques, the F-transform and the perception-based logical deduction. Their combination helps to avoid the classical problem with prolongation of a pre-specified curve. Moreover, due to the interpretability of fuzzy rules used by the perception-based logical deduction, a user may get into a deeper understanding not only of the automatically generated model but also to the whole process whose measured samples yielded the analyzed time series.

Furthermore, due to the F-transform and the possibility to construct several independent trend forecasting models using fuzzy rules, we may easily produce long-time forecasts without the influence of the propagated and accumulated error due to the forecasting from forecasted values.

Acknowledgment

This work has been supported by project DAR 1M0572 of the MŠMT ČR.

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