On the Definition of Extended Norms and Co-norms to Aggregate Fuzzy Bipolar Conditions

Ludovic Liétard[1] Daniel Rocacher[2]

1 IRISA/IUT, rue Edouard Branly, BP 30219,
22302 Lannion Cedex, France
2 IRISA/ENSSAT, rue de Kérampont, BP 447,
22305 Lannion Cedex, France
Email: ludovic.lietard@univ-rennes1.fr, rocacher@enssat.fr

Abstract— *Fuzzy queries addressed to a relational database management system are based on the expression of user preferences which leads to obtain more appropriate answers. Fuzzy set theory provides powerful tools to define such conditions (fuzzy conditions). In this context, it is possible to consider fuzzy bipolar conditions which are made of two fuzzy sets to describe user's preferences: i) a fuzzy constraint to represent a mandatory requirement, ii) a fuzzy wish to represent an optional requirement. A Fuzzy constraint is mandatory in the sense that it* ***must*** *be fulfilled (and it can be used to discard elements). A fuzzy wish is optional since it cannot be used to discard elements (but its satisfaction brings a bonification). This paper is devoted to the aggregation of fuzzy bipolar conditions and to the definition of extended norms and co-norms.*

Keywords— *flexible querying, fuzzy bipolar condition, relational database.*

1 Introduction

When querying a relational database management system (DBMS), the expression of user preferences is a manner to integrate the personalized needs of the user. As a consequence, it leads to obtain more appropriate answers, since they can be ranked from the more relevant (the more preferred) to the less relevant (a set of discriminated answers being obtained). The problem of expressing and managing user preferences in the relational model of data has received more and more attention in the last few years [1, 2, 3, 4, 5, 6, 7] and we consider the study of bipolar conditions [8, 9, 10, 11, 12]. Such conditions are made of two components: a mandatory condition (also called a constraint) and an optional condition (also called a wish). More precisely, we consider fuzzy bipolar conditions where the constraint and the wish express user preferences and are defined by fuzzy sets.

The main interrogation systems proposed to integrate user preferences do not take into account such bipolar conditions. The SQLf language considers preferences defined by fuzzy sets and proposes a commensurable approach without bipolarity. It does not distinguish between constraints and wishes, each atomic fuzzy condition being a commensurable constraint. On the contrary, PreferenceSQL deals with a particular case of bipolarity where constraints are Boolean while wishes are not commensurable preferences. The preferences being not commensurable in PreferenceSQL, this proposition leads to a partial order of answers while SQLf language delivers a total order (the preferences being commensurable). One of the main differences [10] between these two approaches is that SQLf is set in the algebraic framework while it is not the case for PreferenceSQL. As a consequence, this article is a contribution to the extension of the relational algebra (and SQL) to fuzzy bipolar conditions. Our final aim is to propose an SQL-like query language where fuzzy bipolar conditions can be defined as an extension of fuzzy conditions. As a consequence, a total order of answers is expected.

Fuzzy bipolar conditions are introduced in section 2 and it is shown that a fuzzy condition defined by a fuzzy set is a particular case of a fuzzy bipolar condition. Section 3 is interested in extending logical operators to define more complex fuzzy bipolar conditions. These operators are soundness from an user's point of view but they cannot be considered as extended norms and co-norms. Section 4 proposes definition for an extended norm and an extended co-norm to aggregate fuzzy bipolar conditions.

2 Fuzzy bipolar conditions

Bipolarity can be defined as the human capability to evaluate situations in terms of pros and cons or positive and negative aspects. Depending of the relationship between these two aspects, Dubois and Prade [8, 9] distinguish between three categories of bipolarity, called types I, II and III.

The type I is called *symmetric univariate bipolarity*. It corresponds to the case where the scale to evaluate situations is bipolar which means defined by an ordered set (total order) with a neutral elements to separate positive and negative information (*symmetric bipolarity*). As a consequence a single value (*univariate*) from this scale expresses, at the same time the positive and negative aspects since the opposite evaluation is obtained by symmetry. An example of type I bipolarity is provided by probability measures, the neutral element being 0.5. The type II is called *symmetric bivariate unipolarity*. In this case, the scale to evaluate situations is unipolar which means that the neutral element is at one end of the scale and two dual values (symmetric bivariate) is necessary to judge a situation. Unlike the symmetric univariate bipolarity, it is possible for a situation to be evaluated positive and negative at the same time. An example of type II bipolarity is provided by possibility theory where a couple of dual unipolar indexes (possibility and possibility of opposite event) is used to evaluate a situation. The type III is called *asymmetric bipolarity*. In this case two bipolar scales are used to evaluate a situation, one expressing the positive, the other one the negative aspects. As a consequence, positive and negative evaluations are of different nature and can be provided by different kind of sources.

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A bipolar condition is made of two components: a *constraint* (a condition denoted C) whose negation defines not acceptable values (negative evaluation) and a *wish* (a condition denoted W) to define desired values (positive evaluation). The satisfaction to the constraint and the wish are based on two different bipolar scales, the two extremum being the total satisfaction 1 and the total non satisfaction 0. In addition, constraints and wishes are neither dual nor symmetric. As a consequence, a bipolar condition expresses a type III bipolarity.

This paper considers that a bipolar condition denoted (C, W) means “to satisfy C and, if possible, to satisfy W” and this meaning has two important consequences. Firstly, it implies that the satisfaction with respect to the constraint is mandatory while the satisfaction with respect to the wish is optional. Secondly, the set of wished values must be a subset of that of mandatory values ($W \subseteq C$). This property states that it is incoherent to wish non-acceptable values.

In the context of database querying with such conditions, tuples which do not satisfy the constraint are discarded (since this condition is mandatory), while tuples which do not satisfy the wish can be proposed to the user (the wish being optional). When C and W are Boolean conditions, the satisfaction with respect to the bipolar condition (C,W) is a couple (c, w) where c and w are values in {0, 1}. Tuples satisfying the constraint and the wish are returned in priority to the user. If such answers do not exist, the tuples satisfying only the constraint are delivered. When the two components express preferences and are defined by fuzzy sets, their satisfactions are sets in [0,1]. As a consequence, the satisfaction of a given element x with respect to a fuzzy bipolar condition is a couple of degrees ($\mu_C(x)$, $\mu_W(x)$), and their processing is more tricky. These two degrees are not commensurable (due to the chosen meaning for the fuzzy bipolar conditions) and, consequently, they cannot be aggregated to compute an overall degree of satisfaction. In addition, the constraint being mandatory, its satisfaction is firstly used to discriminate among answers. The satisfactions with respect to the wish being not mandatory, they can only be used to discriminate among answers having same evaluation with respect to the constraint. Thus, the wishes allow to differentiate between tuples which are equal with respect to their constraints and a *total order* is obtained on C and W (with (1, 1) as greatest element and (0, 0) as least element). In other words, it is possible to rank the tuples by using a *lexicographical* order on the constraints and the wishes:

a tuple t_i is preferred to a tuple t_j
if ($\mu_C(t_i) > \mu_C(t_j)$)
or (($\mu_C(t_i) = \mu_C(t_j)$) and ($\mu_W(t_i) > \mu_W(t_j)$)),

Obviously, a tuple t_i is similar to a tuple t_j (they have same level of preference) if and only if $\mu_C(t_i) = \mu_C(t_j)$ and $\mu_W(t_i) = \mu_W(t_j)$.

For the sake of simplicity, “t_i is preferred to t_j” is denoted $t_i > t_j$ while “t_i is similar to t_j” is denoted $t_i = t_j$.

Example 1. We consider relation Sales from Table 1 where attributes #Sale, Date, Town, Benefit are respectively the identification of a sale, the date and place where it has been concluded and the percentage of benefit for the salesman.

A salesman wants to know the town where a sale has occurred with a *high* benefit and, if possible, a benefit *around 100%.* A *high* benefit is a mandatory condition while condition *around 100%* is optional. These two fuzzy conditions are given by Figure 1 and define the fuzzy bipolar condition (C, W) where C is "*high* benefit" and W is "benefit *around 100%*".

Table 1 : Relation Sales.

#Sale	Date	Town	Benefit
#1	01-05-07	Paris	82%
#2	04-06-07	Paris	85%
#3	08-09-07	Londres	59%
#4	15-06-07	Nice	94%
#5	17-07-07	Londres	97%
#6	08-09-07	Madrid	56%

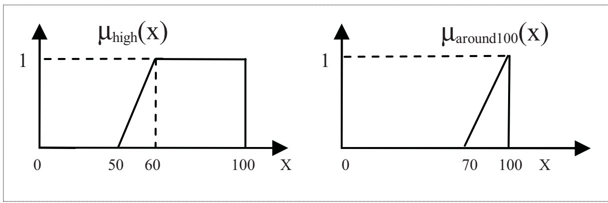

Figure 1 : *high* benefit and benefit *around 100%.*

The satisfactions with respect to the constraint C and the wish W are given by Table 2.

Table 2 : Satisfactions with respect to the fuzzy bipolar condition : "*high* benefit" and if possible "*around 100%*".

#Sale	(*high* benefit, benefit *around 100%*) (μ_C(#Sale), μ_W(#Sale)),
#1	(1, 0.4)
#2	(1, 0.5)
#3	(0.9, 0)
#4	(1, 0.8)
#5	(1, 0.9)
#6	(0.6, 0)

The lexicographical order gives :

#5 > #4 > #2 > #1 > #3 > #6.

Sale #5 is the best sale with respect to the bipolar condition (C, W) because, among the best sales according to the constraint (at degree 1), it is the best one according to the wish. ◆

The property of constraints and wishes expressed on a same universe is that the set of wished values must be a subset of that of mandatory values ($W \subseteq C$). In case of fuzzy sets, this property of inclusion is rewritten :

for each value x in X, $\mu_W(x) \leq \mu_C(x)$,

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such that X is the domain set where the fuzzy bipolar condition applies. Such a property can be checked on the fuzzy bipolar condition provided by example 1. This property is natural and is always implicit in the formulation of fuzzy bipolar conditions. As a consequence, a condition formulated as "a *cheap* and, if possible, a *low* consuming car" should be rewritten : "a *cheap* car and, if possible, a *cheap* and *low* consuming car".

An important particular case arises when a fuzzy bipolar condition is a fuzzy predicate (a pure constraint). Since a fuzzy predicate C can be rewritten "C and, if possible, C", a pure constraint is defined by a fuzzy bipolar condition where the constraint equals the wish ((C, C)). As a consequence, a fuzzy bipolar condition is a generalization of a fuzzy condition.

3 Logical operators and fuzzy bipolar conditions

In this section, we consider several fuzzy bipolar conditions and their aggregation using conjunction and disjunction operators.

A first idea to define the conjunction of two fuzzy bipolar conditions (C_1, W_1) and (C_2, W_2) is to compute the minimum of both the satisfactions to the constraints and to the wishes. In this case "(C_1, W_1) and (C_2, W_2)" is the fuzzy bipolar condition "to satisfy (C_1 and C_2) and if possible to satisfy (W_1 and W_2)". Since the minimum is a triangular norm, this operation can be generalized to define parameterized operators $O_{op1,op2}$ using various norms and conorms :

$$(C_1,W_1)\ O_{op1,\ op2}\ (C_2,W_2) \equiv (op_1(C_1,\ C_2),\ op_2(W_1,W_2)),\ (1)$$

where op_1 and op_2 are either a norm or a co-norm. Depending on the choice for op_1 and op_2, different meanings can be obtained for the operator (as shown by Table 3).

Table 3 : Different meanings for $O_{op1,op2}$.

	op_1 is a norm	op_1 is a co-norm
op_2 is a norm	$(C_1,W_1)\ O_{op1,op2}\ (C_2,W_2)$ ≡ (C_1 and C_2) and if possible (W_1 and W_2)	$(C_1,W_1)\ O_{op1,\ op2}\ (C_2,W_2)$ ≡ (C_1 or C_2) and if possible (W_1 and W_2)
op_2 is a co-norm	$(C_1,W_1)\ O_{op1,op2}\ (C_2,\ W_2)$ ≡ (C_1 and C_2) and if possible (W_1 or W_2)	$(C_1,W_1)\ O_{op1,op2}\ (C_2,W_2)$ ≡ (C_1 or C_2) and if possible (W_1 or W_2)

Example 2. A client looks for a trip in a travel agency. His request is expressed as: "a *cheap* journey, if possible with a *small* flight time, and an hotel in a *small* town, if possible located *in the countryside*". This query is a selection R:Θ where relation R describes trips proposed by the agency and condition Θ is the conjunction between two fuzzy bipolar conditions "(C_1,W_1) and (C_2,W_2)". Constraint C_1 is the fuzzy condition "a *cheap* journey", W_1 is the fuzzy wish "a *cheap* journey with a *small* flight time" and C_2 is the fuzzy constraint "hotel in a *small* town" while W_2 is the fuzzy wish "hotel in a *small* town and *in the countryside*". Each one of these two atomic bipolar fuzzy conditions is defined on several attributes (on the Cartesian product of their respective domains). The different satisfactions with respect to the constraints and wishes are given by Table 4. Obviously, the result depends on the chosen operator to represent the conjunction. We propose the two different conjunction operators $O_{min,\ min}$ and $O_{min,\ max}$.

Table 4 : Satisfactions with respect to (C_1,W_1) and (C_2,W_2)

#Trip	C_1 "a *cheap* journey"	W_1 "a *cheap* journey and a *small* flight time"	C_2 "*small* town"	W_2 "a *small* town and *in the countryside*"
#1	0.9	0.7	1	0.1
#2	1	0.5	0.9	0.5
#3	0.9	0.3	1	1
#4	0.7	0.6	0.6	0.5
#5	0.8	0.2	0.6	0.5

When the operator $O_{min,\ min}$ is chosen, the meaning of the query becomes:

"a *cheap* journey with an hotel in a *small* town, if possible with a *small* flight time ***and*** an hotel located *in the countryside*".

The results are given by Table 5.

Table 5 : Results provided by $O_{min,\ min}$

#Trip	μ_C(#Trip)	μ_W(#Trip)
#2	0.9	0.5
#3	0.9	0.3
#1	0.9	0.1
#4	0.6	0.5
#5	0.6	0.2

We obtain the order : #2 > #3 > #1 > #4 > #5.

When the operator $O_{min,\ max}$ is chosen, the meaning of the query becomes:

"a *cheap* journey with an hotel in a *small* town, if possible with a *small* flight time ***or*** an hotel located *in the countryside*".

The results are given by Table 6:

Table 6 : Results provided by $O_{min,\ max}$

#Trip	μ_C(#Trip)	μ_W(#Trip)
#3	0.9	1
#1	0.9	0.7
#2	0.9	0.5
#4	0.6	0.6
#5	0.6	0.5

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We obtain the order : #3 > #1 > #2 > #4 > #5.

The obtained order is different from the ones provided by $O_{min,\ min}$ (the meanings of the operator being different). This example shows that the two different operators considered here lead to two different results since they have two different meanings ($O_{min,\ min}$ means "*all* the constraints and *all* the wishes", while $O_{min,\ max}$ means "*all* the constraints and *at least one* wish"). ♦

Last example has shown that, when $O_{min,\ min}$ and $O_{min,\ max}$ are used, the fundamental property of constraints and wishes ($W \subseteq C$) is lost (since the result are based on ad-hoc operators). However, we think that this loss is not an issue since it reflects the user's attitude with respect to the aggregation of wishes (either all, either at least one). However, it stresses the need for other operators (as extended norms and co-norms).

4 Extended norms and co-norms

This section is interested in defining operators acting as extended norms and co-norms for fuzzy bipolar conditions. Subsection 4.1 shows that it is not possible to define an extended norm (and an extended co-norm) using any operator $O_{op1,\ op2}$ where op_1 and op_2 are either a norm or a co-norm (cf. section 3). Section 4.2 defines a couple of extended norm and co-norm (and their associated negation).

4.1 Extended norms, co-norms and $O_{op1,\ op2}$

The different properties bring by norms and co-norms are necessary conditions to keep the algebraic framework of computations (or at least to achieve computations in an human-consistent framework). The operator $O_{min,min}$ is a rather rational candidate for an extended norm but next example shows that it does not respect the monotonicity property of norms and it concludes that this loss is very damageable.

Example 3. The property of monotonicity of a given norm $\wedge$ is defined by:

$$a \geq b \text{ and } c \geq d \Rightarrow (a \wedge c) \geq (b \wedge d)$$

The following counter example shows it is not respected by $O_{min,min}$:

Let x and y be two elements and two fuzzy bipolar conditions FBC1 and FBC2. The respective satisfaction of x and y to these two fuzzy bipolar conditions are shown by Figure 2.

	FBC1	FBC2
x	(0.7, 0.7)	(0.9, 0.4)
y	(0.7, 0.6)	(0.8, 0.5)

Figure 2 : Satisfactions of x and y to FBC1, FBC2.

The property of monotonicity does not hold in this case since:

$$(0.7, 0.7) \geq (0.7, 0.6) \text{ and } (0.9, 0.4) \geq (0.8, 0.5)$$

while :

$$(0.7, 0.7)\ O_{min,min}\ (0.9, 0.4)$$
$$< (0.7, 0.6)\ O_{min,min}\ (0.8, 0.5).$$

This behaviour is not acceptable for a conjunction since x is preferred to y when considering criteria FBC1 and when considering criteria FBC2, but x is not preferred to y when considering the conjunction "FBC1 and FBC2".♦

More generally, an extended norm (or co-norm) cannot be defined by any operator $O_{op1,op2}$ as demonstrated hereafter (the demonstration is only provided for the norm and it can be easily adapted for a co-norm).

Demonstration. We assume an extended norm defined as $O_{op1,op2}$:

$$(C_1,W_1)\ O_{op1,\ op2}\ (C_2,W_2) \equiv (op_1(C_1, C_2), op_2(W_1,W_2)),$$

where op_1 and op_2 are either a norm or a co-norm. The demonstration is based on two steps. At first, two properties which are necessarily satisfied by such an operator are pointed out. Secondly, it is shown that these properties are not compatible with an extended norm since the property of monotonicity does not hold.

First step: Since a fuzzy condition (a fuzzy set) is a particular case of a fuzzy bipolar condition, an extended norm for fuzzy bipolar conditions should revert to a norm in case of a fuzzy set. A fuzzy set being defined as a fuzzy bipolar condition where the constraint equals the wish, the application of an extended norm $O_{op1,op2}$ can be rewritten (in case of fuzzy sets):

$$(C_1,C_1)\ O_{op1,\ op2}\ (C_2,C_2) \equiv (op_1(C_1, C_2), op_2(C_1,C_2)),$$

In addition $(op_1(C_1, C_2), op_2(C_1,C_2))$ is nothing but the expression of a norm $\wedge$ between C_1 and C_2 . Since $\wedge(C_1, C_2)$ can be represented by $(\wedge(C_1, C_2), \wedge(C_1,C_2))$ we immediately get two properties:

P1) op1 equals op2,
P2) op1 is a norm (denoted $\wedge$).

Second step: Due to the previous result, any $O_{op1,op2}$ operator defining an extended norm is rewritten $O_{\wedge,\wedge}$ where $\wedge$ is a norm. As a consequence, the property of monotonicity does not hold since:

$$(0, 1) \geq (0, 1)$$
$$\text{and } (1, 0) \geq (0, 1)$$

while:

$$(0, 1)\ O_{\wedge,\wedge}\ (1, 0) < (0, 1)\ O_{\wedge,\wedge}\ (0, 1)$$

(since $(0, 1)\ O_{\wedge,\wedge}\ (1, 0) = (0 \wedge 1, 1 \wedge 0) = (0, 0)$
and $(0, 1)\ O_{\wedge,\wedge}\ (0, 1) = (0 \wedge 0, 1 \wedge 1) = (0, 1)$). ♦

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This subsection has shown that it is necessary to define operators satisfying the properties of norms and co-norms (we recall that the total order considered here is the lexicographical order with (1, 1) as greatest element and (0, 0) as least element).

4.2 Definition for a couple of extended norm and co-norm

An extended norm $\wedge$ and an extended co-norm $\vee$ can be defined by the minimum and maximum with respect to the lexicographical order (which provides a total order):

$$(x, y) \wedge (x', y') = \min((x, y), (x', y')), \quad (2)$$

$$(x, y) \vee (x', y') = \max((x, y), (x', y')). \quad (3)$$

Due to the total order, it is easy to prove that $\wedge$ and $\vee$ are respectively an extended norm and an extended co-norm.

Similarly, a negation operator $\urcorner$ can be defined by:

$$\urcorner(x, y) = (1-x, 1-y). \quad (4)$$

Next proof shows that the De Morgan laws are preserved:

$$\urcorner(x, y) \vee \urcorner(x', y') = \urcorner[(x, y) \wedge (x', y')]$$

$$\urcorner(x, y) \wedge \urcorner(x', y') = \urcorner[(x, y) \vee (x', y')].$$

Proof. We consider $(x, y) \wedge (x', y') = \min((x, y), (x', y')) = (x, y)$. The properties to demonstrate obviously hold when (x = x') and (y = y'). When $(x \neq x')$ or $(y \neq y')$, we get $(x < x')$ or $((x = x')$ and $(y < y'))$. The complementation to one gives : $(1-x > 1-x')$ or $((1-x = 1-x')$ and $(1-y > 1-y'))$. As a consequence $(1 - x, 1 -y) \vee (1-x', 1-y') = (1-x, 1-y)$ which can be rewritten $\urcorner(x, y) \vee \urcorner(x', y') = \urcorner(x, y)$. Then we get $\urcorner(x, y) \vee \urcorner(x', y') = \urcorner(x, y) = \urcorner[(x, y) \wedge (x', y')]$. The first law is then demonstrated and the second one is valid .♦

Example 4. We consider the framework and the query of example 2 ("a *cheap* journey, if possible with a *small* flight time, and an hotel in a *small* town, if possible located *in the countryside*). Constraint C_1 is the fuzzy condition "a *cheap* journey", W_1 is the fuzzy wish "a *cheap* journey with a *small* flight time" and C_2 is the fuzzy constraint "hotel in a *small* town" while W_2 is the fuzzy wish "hotel in a *small* town and *in the countryside*".

When the norm $\wedge$ is chosen, the conjunction returns the worst case between (C_1,W_1) and (C_2,W_2) and the meaning of the query is a pure conjunction (the worst evaluation being retained). We get the results provided by Table 7.

Table 7 : Results provided by norm $\wedge$

#Trip	μ_C(#Trip)	μ_W(#Trip)
#1	0.9	0.7
#2	0.9	0.5
#3	0.9	0.3
#4	0.6	0.5
#5	0.6	0.5

We obtain the order : #1 > #2 > #3 > #4 = #5.

Here again, the obtained order is different from the ones provided by $O_{min,\ max}$ and $O_{min,\ min}$ (the meanings of the operator being different, cf example 3). In addition, one may remark that the fundamental property of fuzzy wish and constraint ($W \subseteq C$) holds in this case (since μ_W(#Trip) $\leq$ μ_C(#Trip) for any trip). ♦

5 Particular case of fuzzy conditions

This subsection shows the behaviours of the operators defined in the previous sections in the particular case where the fuzzy bipolar conditions are pure constraints (W = C).

When dealing with pure constraints, the extended norm $\wedge$ and co-norm $\vee$ are nothing but the norm min and the co-norm max of fuzzy sets.

Proof. We consider $(x, x) \wedge (y, y) = (x, x)$. It means (x, x) is the minimum when the lexicographical order is used which can be rewritten $(x < y)$ or $(x = y)$ and then:

$$(x, x) \wedge (y, y) = (x, x) = (\min(x, y), \min(x, y)).$$

The expression (min(x, y), min(x, y)) is the representation (in terms of constraints and wishes) of the minimum of the two degrees x and y which demonstrate that $(x, x) \wedge (y, y)$ represents the norm min. A similar proof holds for $\vee$ and the co-norm max.♦

The extended norm $\wedge$ and co-norm $\vee$ can also be defined by the parameterized operator $O_{op1,op2}$ where op_1 equals op_2 and is either the minimum or the maximum (it is not the case in general since $\wedge$ and $\vee$ cannot be expressed by $O_{op1,op2}$, as example $(0.6, 0.8) \wedge (0.5, 1) = (0.5, 1)$ while $(0.6, 0.8)\ O_{min,\ min}\ (0.5, 1) = (0.5, 0.8)$).

Proof. We consider $(x, x) \wedge (y, y) = (x, x)$ and $(x, x) \vee (y, y) = (y, y)$. It means $(x < y)$ or $(x = y)$ and then :

$$(x, x) \wedge (y, y) = (x, x) = (\min(x, y), \min(x, y))$$
$$= (x, x)\ O_{min,\ min}\ (y, y),$$

$$(x, x) \vee (y, y) = (y, y) = (\max(x, y), \max(x, y))$$
$$= (x, x)\ O_{max,\ max}\ (y, y).\ ♦$$

More generally, when op_1 equals op_2 the parameterized operator $O_{op1,op2}$ is either a norm or a co-norm between two fuzzy sets (the one chosen to implement op_1 and op_2).

The main question is about the meaning of $O_{op1,op2}$ when op_1 and op_2 differs. First of all, for a sake of simplicity, we assume that op_1 is a norm. Since the lexicographical order is used, it is possible to claim that $O_{op1,op2}$ allows for a refinement of norm op_1 since results are firstly ranked on the degree provided by op1, op_2 being only used to discriminate among elements having same degrees for the norm op_1. Similarly, when op1 is a co-norm, the lexicographical order provides a refinement of this co-norm.

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Example 5. We consider four items x,y,z and t which are valuated using a conjunction between the satisfactions of two fuzzy bipolar conditions FBC1 and FBC2 (cf. Table 8).

Table 8 : Satisfactions with respect to FBC1 and FBC2

item	FBC1	FBC2
x	(0.7, 0.7)	(0.8,0.8)
y	(0.5, 0.5)	(1, 1)
z	(0.5, 0.5)	(0.8, 0.8)
t	(0.3, 0.3)	(1, 1)

The satisfactions with respect to the conjunction are given by Table 9.

Table 9 : Satisfactions with respect to the conjunction

item	FBC1 $O_{min, min}$ FBC2	FBC1 $O_{min, max}$ FBC2
x	(0.7, 0.7)	(0.7,0.8)
y	(0.5, 0.5)	(0.5, 1)
z	(0.5, 0.5)	(0.5, 0.8)
t	(0.3, 0.3)	(0.3, 1)

When the minimum ($O_{min, min}$) is considered we get x > y = z > t, while $O_{min, max}$ gives x > y > z > t. The operator $O_{min, max}$ allows to discriminate elements y and z which are undistinguishable when $O_{min, min}$ is considered. ♦

6 Conclusion

This paper has dealt with the evaluation of fuzzy bipolar conditions into fuzzy queries addressed to a regular relational database (where the stored data are precisely known). Such conditions are made of two components: a mandatory condition (also called a constraint) and an optional condition (also called a wish). More precisely, we have considered fuzzy bipolar conditions where the constraint and the wish express user preferences and are defined by fuzzy sets. In particular, we have shown that a fuzzy bipolar condition is a generalization of a fuzzy condition (defined by a fuzzy set) and several operators to aggregate fuzzy bipolar conditions have been defined.

This paper has pointed out the importance to define an extended norm and an extended co-norm for fuzzy bipolar conditions. It proposes a couple of extended norm and co-norm which is a generalization of the couple (min, max) of fuzzy sets. In the particular case where fuzzy bipolar conditions are fuzzy conditions, some proposed operators leads to a refinement of norm and conorm as it is the case for the leximin and discrimin orders [13].

References

[1] Agrawal A. and Wimmers E.L. "A Framework for Expressing and Combining Preferences". *In Proc. of the 2000 ACM SIGMOD International Conference on Management of Data,* Dallas, USA, pp. 297–306, 2000.

[2] Bordogna G., Pasi G. "Linguistic aggregation operators of selection criteria in fuzzy information retrieval" In *Int. Jour. of Intelligent Systems*, vol. 10(2), pp.233–248, 1995.

[3] Borzsnyi, S. Kossmann D. and Stoker K. "The Skyline Operator". In Proc. of the 17th International Conference on Data Engineering, (ICDE01), Heidelberg, Germany, pp. 421–430, 2001.

[4] Bosc P., Pivert O. "SQLf: a relational database language for fuzzy querying". In *IEEE Transactions on Fuzzy Systems*, vol(3) pp.1–17, 1995.

[5] Chomicki J. "Preference Formulas in Relational Queries". In ACM Transactions on Database Systems, (TODS'03), 28(4):1–39, 2003.

[6] Kießling W. "Foundations of Preferences in Database Systems". In Proc. of the 28th International Conference on Very Large Data bases, (VLDB), Hong Kong, China, pp. 331–322, 2002.

[7] Kießling W. and Kostler G. "Preference SQL - Design, Implementation, Experiences". In Proc. of the 28th International Conference on Very Large Databases, (VLDB), Hong Kong, China, pp. 990–1001, 2002.

[8] Dubois D. and Prade H. "Handling Bipolar Queries in Fuzzy Information Processing". In Handbook of Research on Fuzzy Information Processing in Databases, J. Galindo (Ed.), 97–114, 2008.

[9] Dubois D. and Prade H. "An Introduction to Bipolar Representations of Information and Preference". In International Journal of Intelligent Systems, vol. 23, 866-877, 2008.

[10] Liétard L., Rocacher D. and Tbahriti S.-E., Preferences and Bipolarity in Query Language, International Conference of the North American Fuzzy Information Processing Society (NAFIPS 2008), New-York, USA.

[11] Zadrozny S. "Bipolar Queries Revisited". In Modeling Decisions for Artificial Intelligence, V. Torra et al. (Eds.), 387-398, 2005.

[12] Zadrozny S. and Kacprzyk J. "Bipolar Queries and Queries with Preferences". In Proceedings of the 17th International Conference on Database and Expert Systems Applications (DEXA'06), 415-419, 2006.

[13] Dubois D., Fargier H. and Prade H. "Beyond min aggregation in multi-criteria decision: (ordered) weighted Min, Discrimin, Leximin". In The Ordered Weighted Averaging Operators - Theory and Applications. R.R. Yager, J. Kacprzyk (Eds.), Kluwer Academic Publ., Boston, 181–192, 1997.

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