Flow Line Systems with Possibilistic Data: a System with Waiting Time in Line Uncertain

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Abstract— *This paper proposes to analyze two flow line systems in which we include possibilistic data -the priority-discipline is possibilistic instead of probabilistic- and measure the performances of the systems with the effectiveness measure "waiting time in queue". In a previous work we have analyzed and developed a queuing model with uncertain priority-discipline, using Zadeh's extension principle. Because of it in this paper the analysis of the model takes the work realized by Prade as a starting point to incorporate an uncertain priority-discipline in a flow line system by means of a possibilistic distribution. To verify the validity of the proposed method, we calculate the possibility distribution of the priority-discipline and the possibility distribution of the performance measure "waiting time in queue", in two flow line systems with different waiting lines: the determinist flow line system* $D/D/1$ *and the classic flow line system* $M/M/1/N$ *with finite capacity. Since the performance measure is expressed by membership function rather than by crisp value, the fuzziness of input information is conserved completely, and more information is provided for capacity planning in flow line systems.*

Keywords— Flow Line Systems, Fuzzy Numbers, Possibility Theory, Priority-Discipline

1 Introduction

In this work we present two flow line systems to which we add a possibilistic priority-discipline instead of a probabilistic priority-discipline. For the development of the models we will follow the methodology exposed by *Prade* [1] about possibilistic models of queuing systems. In the literature regarding the queuing models with uncertain data we have not found any article that uses and develops his method in order to incorporate uncertainty in the queuing models with priority-discipline (except for *Prade*'s work). For example, the works of Li and Lee [2], Negi and Lee [3], or more recently Chen [4] or Pardo and de la Fuente [5,6] among others, study the flow line systems with uncertain data according to the Zadeh's extension principle [7], and in the work of Pardo and de la Fuente [8] the optimization of the fuzzy queuing model with priority-discipline is also studied according to the Zadeh's extension principle.

2 Calculation of the membership function of possibilistic priority-discipline

The possibilistic priority-discipline indicates the possibility that has the unit that arrives to the system in the position *n* to be overtaken, displaced or served in a different position that *n* in the waiting line. To develop the proposed models in this work, we need to define a possibility distribution of the priority-discipline but the possibility distribution can be calculated from the different existing methods to construct possibility distributions from real data or from the opinion of the experts (for example in Dubois et al. [9]).

In this way, we define the following possibility distribution (we denote by μ_i^n) and it provides us with the possibility that has the unit that has arrived in the position *n* to be served in the position $n+i$. Logically, this is valid when there is a queue length of one or more units at the arrival of one unit to the system, depending on the case. Besides and because of simplicity, the possibility distribution of the priority-discipline will be considered independent of the unit *n*. For this reason, the possibility of the unit *n* to advance one or more positions in the queue is:

1. The possibility of being served in the position *n* is: $\mu_0^n = 1$.
2. The possibility of being served in the position $n-1$ is: $\mu_{-1}^n = \sigma_1 \left(\sigma_1 \le 1\right)$.
3. The possibility of being served in the position $n-2$ is: $\mu_{-2}^n = \sigma_2 \left(\sigma_2 \le 1\right)$. In this case we consider that the unit $n-1$ and the unit $n-2$ move back one position in the waiting line.
4. The possibility of being served in the position $n+i$ with $i=-3,-4,...$ is: $\mu_i^n = 0$. In this way one unit can not advance more than two positions in the waiting line.

From the following hypotheses, we calculate the possibility that has the waiting unit to be moved back one position in the waiting line because of the arrival of one unit with higher priority. Then:

1. There are two reasons for the unit *n* to move back one position in the waiting line:
   i) Because the unit $n+1$ overtakes the unit *n*. That happens because it has priority to advance one position in the queue or because it has priority to advance two positions in the queue. The

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possibility of the event "the unit $n+1$ overtakes the unit n" is the association of the two previous events and it is know that it has a certain rate according to the fuzzy logic (Zimmermann [10]):

$$\max\{\sigma_1, \sigma_2\} \quad (1)$$

ii) If the unit $n+1$ does not have higher priority to the unit n, but the unit $n+2$ has higher priority to the units n and $n+1$ and advances two positions, the units n and $n+1$ move back one position and that happens with a possibility grade of σ_2.

Then, the possibility grade of the unit n to move one position back once it has arrived to the system is the association of the previous events, and according to the fuzzy logic (Zimmermann [10]):

$$\mu_1^n = \max\{\max\{\sigma_1, \sigma_2\}, \sigma_2\} = \max\{\sigma_1, \sigma_2\} \quad (2)$$

2. The unit n can move back two positions in the queue if it moves back twice consecutively, that means if the event "the unit n moves back one position" happens twice, what in fuzzy logic has a certain rate (Zimmermann [10]):

$$\mu_2^n = \min\{\mu_1^n, \mu_1^n\} = \mu_1^n = \max\{\sigma_1, \sigma_2\} \quad (3)$$

3. Likewise, the possibility grade of the unit to be moved back three or more positions in the waiting line can be calculated and in all of them we have:

$$\mu_i^n = \max\{\sigma_1, \sigma_2\} \quad \text{with} \quad i = 3, 4, ... \quad (4)$$

Then, the possibility grade of the unit n to move back in the waiting line, regardless of the number of positions is:

$$\mu_i^n = \max\{\sigma_1, \sigma_2\} \quad \text{with} \quad i = 1, 2, ... \quad (5)$$

The possibility distribution of the priority-discipline in the waiting line is:

$$\mu_i^n = \begin{cases} 0 & \text{if} \quad i = -3, -4, ... \\ \sigma_2 & \text{if} \quad i = -2 \\ \sigma_1 & \text{if} \quad i = -1 \\ 1 & \text{if} \quad i = 0 \\ \max\{\sigma_1, \sigma_2\} & \text{if} \quad i = 1, 2, ... \end{cases} \quad (6)$$

Once we know μ_i^n we obtain the "possibility distribution of the permanence time in line of the unit n" (denoted $\tilde{W}_q^n$), taking into account that if the unit n advances two positions, it does not have to wait in the waiting line to the service of the units $n-2$ and $n-1$. I it advances one position, we have to take away the service of the unit $n-1$. If it maintains its arrival position, its permanence time in queue does not change. And if it moves back in the queue it has to wait its permanence time and also so many service times as units that had overtaken it. Using Zadeh's notation to denote a fuzzy number (Zadeh [7]) (the possibility grade is previous to the symbol "/", the possible value is posterior, and the symbol "+" represents the association), if it is denoted by b the service time of one unit, the possibility distribution of the permanence time in queue of the unit n is:

$$\begin{aligned} \tilde{W}_q^n = {} & \mu_{-2}^n / \left(W_q^n - 2b\right) + \mu_{-1}^n / \left(W_q^n - b\right) + \\ & + \mu_0^n / W_q^n + \mu_1^n / \left(W_q^n + b\right) + \mu_2^n / \left(W_q^n + 2b\right) + ... \end{aligned} \quad (7)$$

Depending on the particular features of the flow line system to which is added this possibilistic priority-discipline, we obtain some results that differ among the different models. In the following sections we show the obtained theoretical results with two flow line systems with possibilistic priority-discipline.

3 Determinist flow line system $D/D/1$ with times among arrivals longer than the service time and possibilistic priority-discipline

In the determinist flow line system, the units arrive to the system with time among arrivals constant and are also served in constant times. It is supposed that the system is organised in a way that the arrivals happen in regular intervals and they are also regularly served in an only channel. The arrival source is considered unlimited and there is no limitation in the capacity of the system.

Saaty [11] analyses the determinist queuing system D/D/1 in transitory state. That means that the initial conditions (the number of units that there are in the system at the initial moment) affect the obtained results in the model. In addition to this, Saaty studies the system with times among arrivals longer than the service times. This condition supposes that the units take in arriving to the system more time that the system needs to serve them, that is why there will never be a waiting line when starting the system there are no units in it (initial condition $I = 0$ when $t = 0$, with I the number of initial units in the system). Each unit that arrives to the system is served before the arrival of the next unit. Due to this, Saaty studies the determinist model supposing that it is able to empty (the units in the system are not accumulated) under the initial existence conditions of I units in the system at the initial moment.

To know the evolution of this system and to calculate its performance measure, we have to analyse the behaviour of this flow line. The I units are found in the facilities when the system start to work and since the system is able to absorb the next arrivals (they are served very fast) then there will be an moment (denoted T) in which there will be no units in the system and each one arriving to the system will be served before the arrival of the next one. In T, the system reaches the steady state: the server stays idle until a new arrival, there are no units in the waiting line and the permanence time in queue is zero. In order to define the system completely, we need to calculate the number of units that arrive to the system before T (denoted A). They are the units that have to stay in the waiting line together with the initials.

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Regarding the performance measure "waiting time in queue of the unit *n*", W_q^n Saaty does not calculate it for the different values of *n*. That is why before incorporating a possibilistic priority-discipline, we will first calculate the variable *A* and the performance measure W_q^n. It will be so denoted:

a: Time among consecutive arrivals.

b: Service time.

I: Number of units in the system at the initial moment (including the unit that arrives to the system at the initial moment and the units that were already in it). The case $I=1$ means that at the arrival of the first unit, the system was empty.

The system reaches the steady state in *T* once the server has attended the initial units *I* and the new arrivals *A*. The arrival $A+1$ is the first one that finds the server free and goes directly to be attended. This arrival happens in $(A+1)a$. From here, the moment in that the first unit $A+1$ arrives to the system has to be longer than the time of service of the $I+A$ units, otherwise it is no possible that such arrival finds the system free. Then:

$$(A+1)a > (I+A)b \Leftrightarrow A > \frac{Ib-a}{a-b} \qquad (8)$$

The variable *A* has to be the first entire number that fulfils the previous expression, then:

$$A = \left[\frac{Ib-a}{a-b}\right]+1 = \left[\frac{b(I-1)}{a-b}\right] \qquad (9)$$

where $[x]$ represents the whole part of the number *x*, that is to say, the largest integer less than or equal to a number *x* (floor function).

The performance measure of the permanence time in queue of the unit *n*, W_q^n is:

$$W_q^n = \begin{cases} (n-1)b & \text{if} \quad n \le I \\ (n-1)b-(n-I)a & \text{if} \quad I < n \le I+A \\ 0 & \text{if} \quad n > I+A \end{cases} \qquad (10)$$

We obtain the result (10) from the following reasoning: If the arrival time to the system of the unit *n* is:

$$T_{ll}^n = \begin{cases} 0 & \text{if} \quad n \le I \\ (n-I)a & \text{if} \quad n > I \end{cases} \qquad (11)$$

and the time when the unit *n* is served is:

$$T_s^n = \begin{cases} (n-1)b & \text{if} \quad n \le I+A \\ (n-I)a & \text{if} \quad n > I+A \end{cases} \qquad (12)$$

then $W_q^n = T_s^n - T_{ll}^n$.

3.1 Example

Once we know the behaviour of the determinist flow line systems, following we show the incorporation of a possibilistic priority-discipline in the system $D/D/1$ with practice application, calculating the possibility distribution of the permanence time in queue for each unit that when arrive to the system have to stay in the waiting line for a flow line system with time among arrivals of 30 min., service time 20 min., 8 initial units and with the possibility distribution of the priority-discipline: $\mu_{-2}^n = 0.2$; $\mu_{-1}^n = 0.4$ and $\mu_0^n = 1$, from where we have that:

$$\mu_i^n = \max\{\sigma_1, \sigma_2\} = \max\{0.4, 0.2\} = 0.4, \; i = 1,2,...$$

First of all with the data of the example it will be $I=8$, $A=14$, and:

$$T_{ll}^n = \begin{cases} 0 & \text{if} \quad n \le 8 \\ (n-8)30 & \text{if} \quad n > 8 \end{cases} \qquad T_s^n = \begin{cases} (n-1)20 & \text{if} \quad n \le 22 \\ (n-8)30 & \text{if} \quad n > 22 \end{cases}$$

$$W_q^n = \begin{cases} (n-1)20 & \text{if} \quad n \le 8 \\ (n-1)20-(n-8)30 & \text{if} \quad 8 < n \le 22 \\ 0 & \text{if} \quad n > 22 \end{cases}$$

Before calculating the possibility distribution of the permanence time in queue of the client *n* and given the features of the system, we have to make the following remarks:

- Because of the evolution of this system there is a unit *N*, such that $I < N \le I+A$ $(8 < N \le 22)$, which fulfils that every unit $n > N$ is served before the arrival of the next unit and it is not possible for the unit $n+1$ to overtake the unit *n*. That means, if it is denoted by T_s^N at the same moment that the unit *N* is served and by T_{ll}^{N+1} the arrival time of the unit $N+1$, then *N* is the first unit that fulfils: $T_s^N < T_{ll}^{N+1}$, so there is no possibility for the position *n* to be moved back in the waiting line, then $\mu_j^n = 0$, $j = 1,2,...$ and $n > N$, with:

  $$T_s^N = (N-1)20 < (N+1-8)30 = T_{ll}^{N+1}$$

  From here, we obtain $N > 19$, then in the example it is $N = 20$. In this way we obtain:

  1. If $n < 20$ is $\mu_j^n = 0.4$ with $j = 1,2,...,20-n$ and $\mu_j^n = 0$ with $j = 20-n+1,....$
  2. If $n \ge 20$ is $\mu_j^n = 0$ with $j = 1,2,...$
  3. And if from the unit 20 it is not possible to move back in the waiting line, then it is not possible to overtake in the line from the unit 21 and the following ones. Then, if $n \ge 21$ is $\mu_{-2}^n = \mu_{-1}^n = 0$.

- The possibilities μ_{-2}^n and μ_{-1}^n when $n \le 20$ depend on the unit *n*:

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1. If $n=1$, that is, the first unit in the system, then: $\mu_{-2}^{1}=\mu_{-1}^{1}=0$.

2. If $n=2$, then: $\mu_{-2}^{2}=0$ and $\mu_{-1}^{2}=0.4$, given that the second unit arrives at the same time that the first one and there is a possibility that it advances in the waiting line.

3. If $3\le n\le I=8$, they are units that arrive to the system in the moment zero and there is a possibility that all of them change their position advancing one or two positions in the waiting line, then: $\mu_{-2}^{n}=0.2$ and $\mu_{-1}^{n}=0.4$

4. If $9\le n\le 20$, because of the features of that system we know that the waiting line diminishes for any unit that enters, that means that it is possible for the last arrivals that the level of the queue is lower than 2 or 1 unit and they can not advance one or two positions respectively. To determine what units can make its priority effective, we follow the next reasoning: We denote N_1 the unit with priority to advance two positions in the waiting line, in order to make this priority effective, the arrival time of the unit N_1 must be lower or the same that the service time of the unit N_1-2 (the unit previous to N_1 in two positions) so it can occupy its position, then $T_{ll}^{N_1}\le T_{s}^{N_1-2}$, and from here we get $N_1\le 18$. And we denote N_2 the unit with priority to advance one position in the waiting line, this priority can be likewise effective if the arrival time of the unit N_2 is lower or the same that the service time of the unit N_2-1 (the unit previous to N_2 one position), then $T_{ll}^{N_2}\le T_{s}^{N_2-1}$, from where it is $N_2\le 20$. Then:

   i) If $9\le n\le 18$, then $\mu_{-2}^{n}=0.2$ and $\mu_{-1}^{n}=0.4$

   ii) If $n=19$ or $n=20$, then $\mu_{-2}^{n}=0$ and $\mu_{-1}^{n}=0.4$

With all this we calculate the possibility distribution of the permanence time in queue of the unit *n*:

$$\begin{aligned}\tilde{W}_q^n &= \mu_{-2}^n/\left(W_q^n-40\right)+\mu_{-1}^n/\left(W_q^n-20\right)+\\&+1/W_q^n+\mu_1^n/\left(W_q^n+20\right)+\mu_2^n/\left(W_q^n+40\right)+\\&+\dots+\mu_{20-n}^n/\left(W_q^n+\left(20-n\right)20\right)\end{aligned}$$

Following is detailed this possibility distribution for some values of *n*:

$$\begin{aligned}\tilde{W}_q^1 &= 1/0+0.4/20+\dots+0.4/380\\ \tilde{W}_q^2 &= 0.4/0+1/20+0.4/40+\dots+0.4/380\\ \tilde{W}_q^8 &= 0.2/100+0.4/120+1/140+0.4/160+\dots+0.4/380\\ \tilde{W}_q^{15} &= 0.2/30+0.4/50+1/70+0.4/90+\dots+0.4/170\\ \tilde{W}_q^{20} &= 0.4/0+1/20\\ \tilde{W}_q^{21} &= 1/10\\ \tilde{W}_q^{n} &= 1/0 \quad \text{if} \quad n\ge 22\end{aligned}$$

Just as the possibilistic priority-discipline has been adapted to the determinist flow line system $D/D/1$, with time among arrivals longer than the service time, it can be incorporated to other determinist queuing systems only taking into account the features of the system and the evolution of the waiting line.

4 Flow line systems $M/M/1/N$ with possibilistic priority-discipline

The classic flow line system $M/M/1/N$ with finite capacity is considered, with times among arrivals and service time distributed according to an exponential of parameters λ and μ respectively, equals to all the units and regardless the kind of priority. The system has an unlimited enter source and the capacity is *N* units in the system.

The system is analysed in steady state so we know the probabilities of the system to be empty [12]:

$$P_0=\left(\sum_{n=0}^{N}\left(\frac{\lambda}{\mu}\right)^n\right)^{-1} \tag{13}$$

and that the units *n* are found in the system:

$$P_n=P_0\left(\frac{\lambda}{\mu}\right)^n \text{ with } n=1,2,\dots,N \tag{14}$$

and we also know that the average permanence time in queue of each unit is:

$$W_q=\frac{L_q}{\lambda\left(1-P_N\right)} \tag{15}$$

where:

$$L_q=P_0\frac{\lambda}{\mu}\left(\sum_{n=2}^{N}\left(n-1\right)\left(\frac{\lambda}{\mu}\right)^{n-1}\right) \tag{16}$$

is average length of the units in queue.

To incorporate the possibilistic priority-discipline before indicated we have to take into account that in the stochastic flow line systems we do not know for sure the moment when the unit arrives to the system, which the length of the queue is when the unit arrives and if, after that, one ore more units can arrive and make that the previous unit move back in the queue from its arrival position. Then, we should calculate the average possibility to advance one or two

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positions in line, the average possibility to remain in the arrival position or the average possibility of moving back in the queue:

1. When the unit *n* arrives to the system, if it has priority to overtake 2 units and besides there is a waiting line with 2 or more units in the queue (3 units or more in the system), the priority-discipline will be made effective. The possibility of overtaken two units is μ_{-2}^{n} and the probability that 3 or more units are found in the system is $P_{n\geq 3} = 1 - P_0 - P_1 - P_2$. For this reason the average possibility of one unit advancing two positions in the queue (denoted p_{-2}) happens when both happen at the same time, and using the definition of Zadeh [13] of the probability of a fuzzy event is:

$$p_{-2} = \mu_{-2}^{n} \cdot P_{n\geq 3} \tag{17}$$

2. With a similar reasoning to the previous one, we conclude that the average possibility of one unit advancing one position is:

$$p_{-1} = \mu_{-1}^{n} \cdot P_{n\geq 2} \tag{18}$$

   with $P_{n\geq 2} = 1 - P_0 - P_1$

3. The possibility of remaining in its position in the waiting line always happens, regardless the number of units that there are in the system at the moment that the unit *n* arrives, then:

$$p_0 = \mu_0^{n} = 1 \tag{19}$$

4. The average possibility of moving back in the queue, p_i with $i = 1, 2, ..., N-1$ happens when the unit has to wait to be attended since another unit can arrive meanwhile with higher priority. One unit has to wait if the system is busy when it arrives to it and this happens with probability $P_{n>0} = 1 - P_0$, then:

$$p_i = \mu_i^{n} \cdot P_{n>0} \tag{20}$$

Once we know the average possibility of advancing or moving back in the waiting line, the possibility distribution of the average permanence time in queue is:

$$\begin{aligned}\tilde{W}_q &= p_{-2} / \left(W_q - 2/\mu\right) + p_{-1} / \left(W_q - 1/\mu\right) + \\ &\quad + p_0 / W_q + p_1 / \left(W_q + 1/\mu\right) + p_2 / \left(W_q + 2/\mu\right) + ...\end{aligned} \tag{21}$$

4.1 Example

To conclude, we illustrate with an example the incorporation of a possibilistic priority-discipline calculating the possibility distribution of the average permanence time in queue for a flow line system $M/M/1/10$ of parameters $\lambda = 6$ and $\mu = 8$ units per hour, and $N = 10$ units. The possibility distribution of the waiting discipline is: $\mu_{-2}^{n} = 0.4$, $\mu_{-1}^{n} = 0.6$ and $\mu_0^{n} = 1$. From here: $\mu_i^{n} = \max\{0.4, 0.6\} = 0.6$, with $i = 1, 2, ..., 9$.

With this data we have: $P_0 = 0.261$, $P_1 = 0.1958$ and $P_2 = 0.1468$, then $P_{n\geq 3} = 0.3964$, $P_{n\geq 2} = 0.5432$ and $P_{n>0} = 0.739$. The average possibility of advancing or moving back in the waiting line is: $p_{-2} = 0.1586$, $p_{-1} = 0.3259$, $p_0 = 1$ and $p_i = 0.4434$ with $i = 1, 2, ..., 9$ The average permanence time in the waiting line of one unit is: $W_q = 0.3004$. And the possibility distribution of the average permanence time in the queue is:

$$\begin{aligned}\tilde{W}_q &= 0.1586 / 0.0504 + 0.3259 / 0.1754 + \\ &\quad + 1 / 0.3004 + 0.4434 / 0.4254 + \\ &\quad + 0.4434 / 0.5504 + ...\end{aligned}$$

The possibility distribution $\tilde{W}_q$ is represented in the Fig. 1.

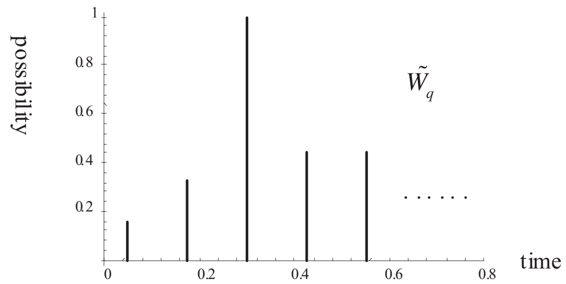

Figure 1: Possibility distribution $\tilde{W}_q$

The possibilistic priority-discipline can be adapted to other stochastic flow line system only by following a similar procedure to the one carried out in the last example.

5 Conclusions

In this work, taking the work started by *Prade* [1] as a starting point and incorporating it a possibilistic priority-discipline, we have given a more realistic representation of the flow line systems. For this we have defined a possibility distribution for the waiting-discipline but it can be calculated from real data or from the opinion of the experts. To illustrate the proposed method and the obtained theoretical results we have carried out two applications: one, to incorporate the possibilistic priority-discipline in a pure determinist flow line system and the other one to incorporate the uncertain priority-discipline in a classic flow line model. In both applications we have developed the technique to incorporate the priority-discipline given the particular features of the system and on the other hand, we have exposed an example that explains the proposed methodology.

Even so, the flow line systems with uncertain waiting line presented and developed in this work provide the decision maker with more complete and informative results and also with a better knowledge about the behaviour of the system due to the results that are possibility distributions that

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include all the uncertainty contained in the system. It is also a simple methodology that can be adapted to the different flow line systems with uncertain priority-discipline. For this reason we think that the proposed flow line system with uncertain data can have more applications than the classic one.

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