Application of Zadeh's Impossibility Principle to Approximate Explanation

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Abstract: *We consider application of Zadeh's impossibility principle and extended logic FLe to approximate scientific explanation from the standpoint of the philosophy of science. First, traditional explanation models are sketched. Second, Zadeh's principle is considered in the broader context which includes reasoning and theory formation. Third, application ideas for using the impossibility principle to approximate explanation are provided. We focus on the role of approximate reasoning, approximate hypothesis assessment and approximate probability in explanation. It seems that approximate explanation can be usable when imprecise entities are involved in the conduct of inquiry.*

Keywords: *approximate explanation, approximate reasoning, impossibility principle.*

1 Introduction

Scientific explanations make our objects of research intelligible for us [28]. In practice this means that we aim to find the causes, functions or purposes of the phenomena under study. An explanation constitutes of two parts, the phenomenon or problem to be explained (explanandum) and our explanation for it (explanans). Explanations play an important role in theory formation and hypothesis assessment [12,17,18,28,29].

The study on the causes of the phenomena is a central issue in explanation. If we study the causes, Aristotle already suggested four types of causes, viz. material, formal, effective and final causes. Today, only two of these are usually applied in the conduct of inquiry. The natural sciences focus on the effective causes, whereas in the human sciences final causes are also considered [18].

Today, the mainstream seems to aim at precise explanation models, but thanks for the fuzzy systems and Zadeh's impossibility principle, it seems that we can also construct corresponding approximate models in the future.

Below we consider the prospects for applying the impossibility principle and other Zadeh's recent ideas to scientific explanation from the standpoint of the philosophy of science. Due to our philosophical approach, we mainly provide some general ideas for applying these methods. Lack of space puts also restrictions on our examination. Section 2 briefly presents the prevailing explanation models. In section 3 we integrate the impossibility principle with reasoning, theory formation, hypothesis testing and explanation. Section 4 considers aspects of approximate explanation, and section 5 concludes our examination.

2 Traditional explanation models

The pragmatic approach to explanation, which seems widely-used today, considers such phenomena as facts, regularities and occurrences. Our task is to provide the cause, origin, history, function or goal of a given phenomenon. In practice we provide such questions as "why?", "how?", "what is the purpose for?", "what is the reason for?" and "what is the function of?" [18,27,29].

The positivistic tradition, in turn, has emphasized syntactical and semantical aspects of explanation in which case concept analysis and logical structures are essential [18,27].

Another categorization of explanation stems from the causes. If we attempt to find effective causes of a phenomenon, we consider which agents, factors or events have initiated the phenomena under study. The well-known example of this idea is the subsumption model in the natural sciences [12,13,17].

The final causes, in turn, are those goals, purposes, intentions or motives of agents, in particular those of the human beings, which guide their behavior. Aristotle's and von Wright's practical syllogism models [29] apply this idea. The subsumption approach mainly provides why?- and how?-questions, and the latter tradition usually asks what is the purpose for or what is the reason for something.

We usually search for effective causes in the inanimated world, but in the animated world the problem arises whether we should also search for final causes. From the philosophical standpoint, the former approach mainly stems from the positivistic tradition and Marxism and the latter, which studies both of these causes, is known as the Geisteswissenschaft approach (hermeneutics, phenomenology etc.). In [29] these competitive approaches are referred to as the Galilean and Aristotelian traditions, respectively, and we use these terms below.

The explanation of the behavior of the human being has been a long-lasting controversy between these two traditions. The Galilean tradition assumes that our behavior is based on our stimulus-response functions and such feedback systems which are analogous to homeostats. In this framework we can even deal with *goal-directed* behavior of beings.

The Aristotelian tradition presupposes that the human beings are *goal-oriented* agents and thus we must also study their aims, intentions, motives and other underlying causes of behavior. Our goals can be both conscious and subconscious, and intentionality is usually related to the former category. A classical example within this tradition is that we should explain (inanimate) nature but understand history, i.e., historical research also takes into account the intentional factors of beings [2,7,15,26].

If we consider the nature of causes on the time axis, the distinction between the Galilean and Aristotelian explanations essentially means that in the former case the cause precedes and in the latter case it follows the effect. Another way to state this is that we use causal and teleological explanations, respectively.

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For example, if we state in the first day of the IFSA '09 Congress that Lotfi Zadeh is present because he was requested to deliver the keynote speech, we use the Galilean explanation. If, in turn, we state that Lotfi Zadeh is present in order to deliver the keynote speech, the Aristotelian approach is adopted. Naturally, we can often use these explanations simultaneously.

Hence, in this sense we can establish that

1. Requested for speech (causal explanans) → **Zadeh is present in the IFSA '09 (explanandum).**
2. **Zadeh is present in the IFSA '09 (explanandum)** ← In order to deliver speech (teleological explanans):

Other models for explanation are also available. For example, the genetic explanations aim to discover causes by examining the history or chronological order of events of a phenomenon. In the historical research and forensic sciences, inter alia, abduction is usually applied in this context.

The statistical and probabilistic explanations, in turn, are based on statistical results and probabilistic models, respectively. An example of the former type would be the reasoning that most of the participants in the IFSA '07 Congress were males because about 90 % of the researchers in the fuzzy community are males. In the latter case we could provide an explanation that if we are tossing a coin, about 50 % of the outcomes are heads because the frequency probability for this event is .5. In particular in the human sciences these explanation models are relevant because the human behavior is usually indeterministic by nature.

The traditional explanation models aim to operate with precise entities and traditional deduction, and thus certain problems can arise. First, they usually apply bivalent logic and this approach can yield overly rough models. Second, if mathematical expressions are used, they can be oversimplified or too idealized for modeling the corresponding real-world phenomena. Finally, this approach lacks an appropriate uniform language which can be used in the context of imprecise entities. Fuzzy systems, in turn, can provide some resolutions to these problems.

3 Impossibility principle and FLe

Recently, Zadeh has established the principles of the extended fuzzy logic, *FLe* (a.k.a. FL+), which is a combination of "traditional" provable and precisiated fuzzy logic, *FLp*, as well as a novel meta-level unprecisiated fuzzy logic, *Flu* [31,38]. He maintains that in the *FLp* the objects of discourse and analysis can be imprecise, uncertain, unreliable, incomplete or partially true, whereas the results of reasoning, deduction and computation are expected to be provably valid. In the *Flu*, in turn, membership functions and generalized constraints are not specified, and they are a matter of perception rather than measurement.

In our framework below this means that we can apply both traditional bivalent-based and novel approximate validity, definitions, axioms, theories and explanations, inter alia. In the former case we thus operate with precise theorems, classical deducibility, syllogisms and formal logic, whereas the latter is related to "f-entities" within informal and approximate reasoning and incomplete information (Fig. 1).

For example [31, p. 2],

"A simple example of a f-theorem in f-geometry is: f-medians of f-triangle are f-concurrent. This f-theorem can be f-proved by fuzzification of the familiar proof of the crisp version of the theorem."

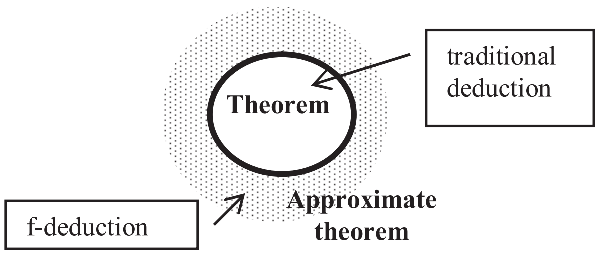

Figure 1. Precise and approximate theorem by applying the *FLe*.

F-validity and f-theoremhood in the *Flu* are examples of Zadeh's Impossibility Principle [31]. This principle informally states that in an environment of imprecision, uncertainty, incompleteness of information, conflicting goals and partiality of truth, p-validity is not, in general, an achievable objective.

The fuzzy modus ponens is the well-known application of this principle in semantic validity, i.e., we can replace the traditional syllogism [8,11,14,19]

A
If A, then B
Thus, B

with its approximate counterpart

A'
If A, then B
Thus B'

in which A≈A' and B≈B'. In the latter case we can thus draw approximate conclusions, i.e., if A' resembles the antecedent A, the conclusion B' resembles the consequent B. In other words, the approximate conclusion is in the proximity to its true counterpart. F-validity is the corner stone in the *FLe* and it is also essential in approximate theory formation and explanation.

In [21,22,23], Zadeh's *FLe* is considered from the standpoint of reasoning and theory formation and it was assumed that the approximation stems essentially from the former. We assume below that if our statements are true, it means that they are identical with their true counterparts. The non-true statements, in turn, are more or less distinct from their true counterparts, or in other words, their distances between their true counterparts are non-zero.

For example, given that John is young (true counterpart), we can reason that the statement

"John is young"

is true. On the other hand, "John is fairly young" is non-true because the predicates "is young" and "is fairly young" are dissimilar.

In fact, we can thus operate with a f-similarity relation of the *Flu,* and then the truth values are specified according

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to this operation. The higher this similarity, the more true the statement in consideration [23].

Then it seems plausible that, for example, the truth value of the statement

"John is fairly young", provided that he is young

is fairly true [23].

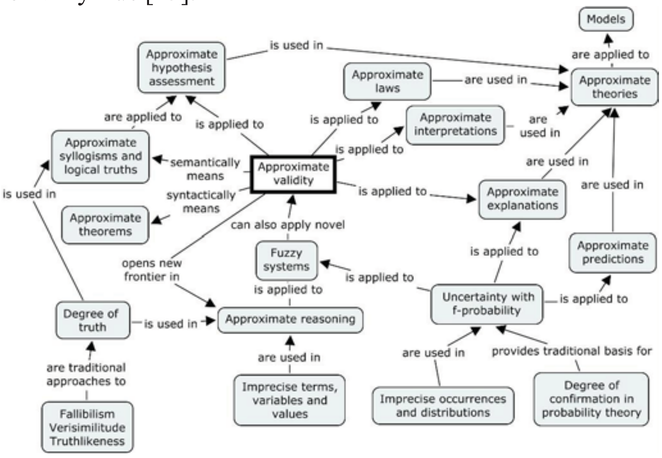

Figure 2. The role of Zadeh's *FLe* in the conduct of inquiry.

We thus assume that approximate theorems are in the neighborhood of their true counterparts. We can also assume that in theory formation approximate laws and theories are in the proximity to their true counterparts and approximate explanations are close to the corresponding true explanations. Figure 2 depicts these ideas at a general level when we establish that our examination on the *FLe* is mainly based approximate validity.

In the case of theory formation, for example, the constituent or "law" of one physical theory states that the boiling point of water is 100 ºC at sea level, but we can also apply it as an approximate law in the low altitudes in general, i.e., we can state that the boiling point of water is *approximately 100 ºC close to* sea level [19,22].

In addition, within the traditional approach to the theory formation and model construction we only use their approximations and idealizations on some occasions. In the former case the degree of truth of a theory or model often increases when we replace the approximate theory, law or model with its less approximate counterpart [18,19]. Hence, in this respect the law

the boiling point of water is 100 ºC at sea level

is a more true than

the boiling point of water is 100 ºC.

Idealizations, in turn, are typical when mathematical or statistical models are used. For example, in statistics such ideal curves as the Gaussian curve are used when the empirical data sets, the distributions of which are only more or less similar to their mathematical counterparts, are examined. Thus, we actually draw statistical conclusions on approximate grounds (c.f. also section 4).

Popper has already considered the foregoing aspects in his theory of verisimilitude from the bivalent standpoint [24,25]. He presupposes that we aim to formulate true theories (cognitivism) but in practice our theories can be non-true. If we perform successful research, our theories will approach their true counterparts and thus they are always corrigible by nature. The similar idea is applied in Peirce's fallibilism [17,18], and in general, in scientific realism (Peirce, Lenin, Popper, Hempel). Niiniluoto [20] has considered approximate theories and explanations within his theory of truthlikeness, but he has also applied bivalent logic in this context. In fact, if we replace bivalent logic with *FLe*, our approach is fairly close to his ideas [23].

4 Approximate explanation

As was stated above, our examination is based on the idea of approximate reasoning in the *FLe*. Thus, in an approximate explanation model the explanans is in the neighborhood of its true counterpart. In practice we now assess the degree of similarity between our explanans and its true counterpart. The higher the degree of similarity, the higher the degree of truth for our explanans (Fig. 3).

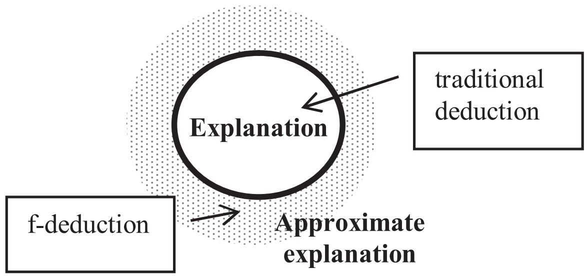

Figure 3. Precise and approximate explanation by applying the *FLe*.

Hence, we have to consider again which logical tools are appropriate to us. As above, within simple statements we assess the similarity between the predicates, i.e.,

truth(John is fairly young), provided that he is young,

is based on the similarity between "young" and "fairly young".

The more complex statements can also be examined within the *FLe* because we can apply the fuzzy similarity measure to compound and quantified statements [31-38]. Then, for example, we should be able to assess such truth values as

truth(John is fairly young and very decent),

provided that he is young and decent.

Another example is the assessment of such quantified expressions as

truth(all Swedes are fairly tall),

provided that most Swedes are tall.

In the former case some concrete resolutions are already available in the *FLe*, whereas fuzzy quantification still awaits usable truth evaluation results. Approximate quantification is problematic because then we operate at two levels, with approximate quantifiers and (compound) statements. Application of the extension principle and Zadeh's idea on

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f-transformation [38] can be a possible resolution for these, and one method is suggested in [23]. Naturally, these approaches are context-dependent by nature. Hence, more concrete results are expected for assessing the truth values and validity within the *FLe*.

In the probability and statistical explanation models mentioned above we can apply corresponding approximate theories. For example, if in the former case we are tossing the coin in the real world, only approximately 50 % will be heads. Hence, if we ask why about 50 % of these outcomes are heads, we can provide an approximate explanation that it is due to this approximate frequency probability. In other words, our approximate probability statements are close to their true counterparts.

The same goes with the statistical explanations. If we explain why most of the attendants in the IFSA '07 Congress were males, we rely on the approximate statistical result that about 90 % of persons in the fuzzy community are males.

We have to bear in mind that both probabilistic and statistical explanation models above are distinct from the forecasting models. In the former case we know the conclusion and we derive it from the relevant premises, whereas in forecasting the conclusion is unknown for us. Naturally, the systematic power of a theory presupposes both of these features.

In the foregoing models fuzzy probability approach usually means approximate probability variables and values of these variables, i.e., statements of the type

the probability that John is young is very high,

and this area has already been studied quite much in the literature [4,5,6,9,31,37]. In Zadeh's *FLe* we can also apply such approximate probability distributions as approximate normal distributions by using the f-transformation. In this case even such precise variable values as

probability(John's age is 20 years)

yield approximate probabilities. With imprecise values (e.g., John is fairly old) we obtain even more imprecise outputs (Fig. 4).

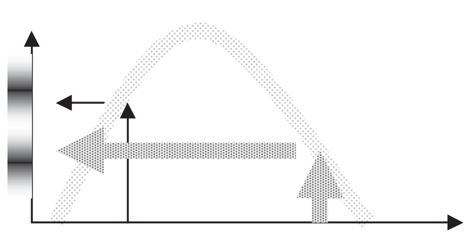

Figure 4. Approximate probability distributions yield approximate outputs even from precise inputs.

The foregoing examples of probability are mainly based on the physicalistic cases, i.e., on the facts and data of the real world. However, Zadeh's theories seem also applicable to epistemic probability in which case these outcomes base on our inference and beliefs [1,30]. In practice we consider the relationship between the hypotheses and their evidence, and we thus assess the degrees of truth, confirmation or belief of hypotheses according to the available relevant evidence.

The gradation of truth was already studied above. As regards the hypothesis testing in general, due to the lack of space we only focus on the degree of confirmation in the context of the explanatory hypotheses and the hypothetico-deductive method [12,13,17,18].

In this hypothesis assessment we usually apply three methods. First, Mill's method of difference [17,18], i.e.,

> *"If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former; the circumstance in which alone the two instances differ, is the effect, or the cause, or an indispensable part of the cause, of the phenomenon."*

Second, the disjunctive syllogism

A or B
not A
thus, B.

Third, the modus tollens syllogism

if A, then B
not B
thus, not A.

Mill's principle and the disjunctive syllogism provide us with the general roadmap for excluding the false hypotheses and within the hypothetico-deductive method this task is carried out in practice by applying the modus tollens syllogism.

Hence, given the hypothesis A and its observable or testable logical consequence B, the justifiability of B is assessed according to our knowledge, experiments and observations. If these facts are inconsistent with B, the modus tollens leads us to the conclusion that our hypothesis is false. According to the disjunctive syllogism, in turn, this hypothesis can be rejected or excluded.

If, on the other hand, our experiments and observations correspond with B, the traditional modus tollens will not provide us with any resolution. Hence, in the latter case we have to replace deduction with induction, and we can use such expressions as "our hypothesis is confirmed" in this context [17,18,24,25]. Sufficient degree of confirmation, in turn, will lead to the truth or acceptance of the hypothesis (c.f. below).

In the traditional conduct of inquiry this method thus uses the hypotheses, which stem from the researcher's context of discovery and inventions, deduces tests and experiments from these hypotheses and finally either rejects or confirms the hypotheses according to the known facts and empirical evidence. However, this approach does not provide any resolution to the problem how we can invent or discover novel hypotheses.

Statistical tests also apply the disjunctive method and physicalistic probability when we consider the acceptance of the null and alternative hypotheses according to the tests of significance. We accept the null hypothesis if the value of our test variable does not deviate too much from the "usual" case. Hence, in fact we consider the so-called type 1 error, i.e., we attempt to find justified reasons to reject the null hypotheses. In practice this error is estimated with the p-

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value in which case we consider the rejection of the null hypotheses if the p-value is sufficiently small (e.g., $p < .05$), i.e., the p-value is our risk to make an erroneous decision if we reject the null hypothesis.

In practice, we have three alternatives in this case [10,16], viz.

1. the null hypothesis is true but our data set is exceptional
2. the null hypothesis is false and our data set is typical
3. the null hypothesis is false and our data set is exceptional

In the conventional case the statistical hypotheses are mutually exclusive, and thus the rejection of one means the acceptance of the other. Hence, according to the disjunctive method, our reasoning boils down to the metarules

1. if the p-value is not small, accept the null hypothesis
2. if the p-value is small, reject the null hypothesis (and accept the alternative hypothesis)

The statistical decision making is thus in this respect based on approximate reasoning and probability. Zadeh's *FLe* can make this reasoning more formal and informative if we operated with the degrees of acceptance and rejection in this context. Then, we could apply such metarule as

> the smaller the p-value, the lower the risk to reject the null hypothesis (the higher the degree of rejection for the null hypothesis)

With the *FLe* we could also take into account better the borderline of these two, viz. the area in which we hesitate over these alternatives. This would mean the fuzzy rules which we have already stated above, i.e.,

1. If the p-value is not small, then accept the null hypothesis.
2. If the p-value is small, then reject the null hypothesis.

With these rules the corresponding fuzzy system could also operate fluently with the borderline cases of p. However, more concrete tools for considering this problem are still expected within the *FLe*.

As regards the modus tollens syllogism in hypothesis assessment, we already considered its bivalent version above. If we apply its approximate version, we can also use linguistic and approximate constituents. Thus, given that A ≈ A' and B ≈ B', these syllogisms are such as [23]

> if A, then B
> B'
> thus, A'

In this case the truth values of the premises may also be between true and false. This syllogism is analogous to the bivalent case when A' and B' are the antonyms of A and B, respectively, because given that the implication and B' are true, B is false, and thus A must be false (and A' is thus true).

Within the *FLe* the other extreme could be that B' is false in which case B is true and A is thus anything from false to true (and A' is also anything from false to true). In practice this could mean that our hypothesis, A, may be true at least to some extent.

For example,

> If John is young, then he is still a schoolboy.
> John is retired.
> Thus, John is old and reject the hypothesis "John is young".

> If John is young, then he is still a schoolboy.
> John is still a schoolboy.
> Thus, the degree of truth for John being young can be non-zero.

It seems that the approximate version of the modus tollens is more versatile because it is also usable when the second premise is non-false. Then we can at least approximately apply the metarule that the more convincing the evidence for the hypothesis, the higher the degree of truth of our hypothesis. In addition, the more various experiments and observations support our hypotheses, the more true hypothesis is obtained. We can also use the concepts "degree of confirmation" or "degree of acceptance / rejection" in this context if necessary.

Within the *FLe* we can also assume that the implication in the approximate modus tollens syllogism is non-true. For example, if this implication is only fairly true, we can establish that even the false B does not necessarily lead to mere false hypotheses. Equally the truth value of B close to true may already lead to conclusion that the truth value of our hypothesis is anything between false and true. In general, we may thus assume that the non-true implications cause more "dispersion", granulation or imprecision to our conclusions, and loose reasoning links of this type are typical in the human sciences in which we usually operate with noisy data and the complex interrelationships between the variables.

We may also apply the foregoing hypothesis testing to interpretation, which is a widely-used technique in the qualitative research. In this context we attempt to provide a "true" interpretation according to our preliminary interpretation hypothesis (*foreknowledge* in hermeneutics). This procedure of assessing hypotheses is often more subjective by nature than in the quantitative research. However, the *FLe* also seems appropriate to qualitative hypotheses assessment because this inquiry is usually linguistic and approximate by nature [2,3,23].

Zadeh has also considered the idea of the second-order probability under the *FLe* [31,37]. In this context we examine such statements as

> The probability that "the probability of John being very young is fairly high" is high.

This subject matter would also extend a new frontier within both the fuzzy systems and the probability theory, but it also awaits much further studies.

Approximate scientific explanation is related to reasoning, theory formation and hypothesis testing. These, in turn, base much on approximate and probabilistic reasoning. Hence, this object of research is very wide. Zadeh's *FLe* seems to provide promising tools for resolving many of the foregoing problems, but much further studies are still expected in this area.

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5 Conclusions

We have provided some guidelines how Zadeh's impossibility principle and the extended logic *FLe* could be applied to scientific explanation. We have also examined reasoning, hypothesis assessment and theory formation because these are closely related to explanation.

First traditional reasoning, theory formation and explanation, which use precise entities and bivalent logic in particular in the quantitative research, were considered.

Second, we dealt with Zadeh's *FLe* and his other recent theories, and we examined how we could design usable approximate theories and explanations which essentially base on approximate reasoning and probability.

In particular, we provided examples on our approximation approach to scientific laws, theory formation, statistical and probabilistic reasoning, hypothesis assessment and two well-known syllogisms. These ideas, in turn, were relevant when approximate explanation was considered.

It seems that the foregoing methods are usable in approximate explanation and in this context we could apply the general principle that approximate explanations are in the neighborhood of their true counterparts. This aim is essentially achieved by using approximate reasoning.

Zadeh's novel ideas seem promising in particular in the qualitative research because then we operate with imperfect information, and in this context we still lack a uniform and usable logico-linguistic system. It also seems possible to apply his ideas to the quantitative research, in particular if we attempt to mimic human reasoning in a computer environment.

We must bear in mind that several of the foregoing issues are still controversial even in the philosophy of science, and the same problem will likely concern Zadeh's and Author's ideas above.

However, due to the lack of space, this paper only provides at metalevel some general guidelines for approximate scientific explanation and related topics from the standpoint of philosophy of science, and hence this relevant subject matter still awaits much further studies in the future.

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