Image Analysis Applications of Morphological Operators based on Uninorms

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Abstract— This paper presents a continuation of the study on a mathematical morphology based on left-continuous conjunctive uninorms given in [1]. Experimental results are displayed using the morphological Top-Hat transformation, used to highlight certain components of the image, and on the reduction and elimination of noise using alternate filters that are generated with the operators of opening and closing associated with these morphological operators

Keywords— Mathematical morphology, Top-Hat, alternate filters, uninorms, representable uninorms, idempotent uninorm.

1 Introduction

The fuzzy mathematical morphology is a generalization of binary morphology [2] using techniques of fuzzy sets [3, 4, 5, 6]. The basic tools of mathematical morphology are the so-called morphological operations that are applied to an image A, which is modified through a structural element B, whose size and shape are chosen in order to study the structure of the image A. The basic morphological operations are: erosion, dilation, opening and closing. Other fuzzy extensions have been introduced by others authors, see fo example [7, 6].

The fuzzy operators used to build a fuzzy morphology are conjunctors and implications. Among them, the most commonly used are the t-norms and their residual implications. Recently conjunctive uninorms, as a particular case of conjunctors, have also been used in this area [8, 9, 1]. In these two last recent works the authors use two kinds of conjunctive uninorms in order to obtain a mathematical morphology with "good" properties, including duality between the morphological operators.

This work can be seen as a continuation of [1] where the authors made a comparative study of the results obtained using the morphology based on uninorms with those obtained using t-norms and using the classical umbra approach. In that work, it is checked out that uninorms detect the edges of the images better than other approaches. Also, in [1], properties of idempotence and generalized idempotence for opening and closing operators based on uninorms are shown. Thus, the aim of this paper is to extend this study to " Top-Hat" operators (residual between openings and closings, a morphological gradient) based on uninorms, as well as to the so-called alternate filters (alternate compositions of openings and closings). The Top-Hat is used to highlight certain components of the image, while the alternate filters are used in elimination and reduction of noise.

The residuals and morphological gradients are still under study and have applications in image analysis. Thus, in [10], it is proposed a directional transition detection algorithm based on morphological residues, considering linear structurant elements. In 2002, T. Chen et al. in [11], design on the basis of mathematical morphology, a new detector of soft edges in dark regions called "Pseudo Top-Hat" which provides, in the case of the classical image of the video cameraman (see Figure 1), a better performance than those achieved by other edge detection methods used to compare the results. In [12], morphological gradients based on openings and closings were used to detect edges in CT medical images altered by noise. The results were compared with other contours detectors, improving the obtained edge images. More recently, in [13, 14] gradient operators are applied again to edge detection and image segmentation, respectively. Thus, in [13], an edge detector is developed for obtaining thin edges in regions or images with low-contrast. Tested with multiple images, even some of them disturbed by noise type salt and pepper and Gaussian noise of mean zero and variance 0.02, the method performs very well in comparison with the other four methods with which they work. In [14] operators were designed based on multiscale morphological gradients, addressing the segmentation problem of images to Computed Axial Tomography (CAT). All these works show that the morphological gradients remain relevant and useful in the analysis and image processing.

The removal, reduction and smoothing of noise is one of the key tasks of analysis and image processing as a preliminary step to artificial vision. Mainly because many techniques of interpretation, measurement, segmentation, detection of structures, among others, fail or diminish its effectiveness in presence of noise. In [12, 13] the designed gradient operators are robust in the presence of noise. Many efforts have been devoted in the literature, and also are currently devoted, to the smoothing, reduction and elimination of noise, from both points of view: crisp and fuzzy. Some examples can be seen in [15, 16, 17]. More specifically, we can find a good job on fuzzy filters in [18] and more recently, in [19], where noise reduction is handled. The design of basic and fast filters remains a subject of interest. In this work it will be studied the feasibility of alternate filters, from opening and closing of the fuzzy morphology based on uninorms, in order to use them in the elimination and reduction of noise.

This work is organized as follows. In the next section we will introduce the fuzzy morphological operators based on left-continuous conjunctive uninorms and their basic properties. Section 3 analyzes the Top-Hat and filtering based on this morphology and shows some experimental results together with their interpretations. Finally, Section 4 is devoted to conclusions and future work.

ISBN: 978-989-95079-6-8

2 Fuzzy morphological operators and its properties

We assume known the basics facts on uninorms used in this work which, in any case, can be found in [20, 21, 22]. We will use the following notation: $\mathcal{I}$ is an implication, $\mathcal{C}$ a conjunction, $\mathcal{N}$ a strong negation, U a conjunctive uninorm with a neutral element e, $\mathcal{I}_U$ its residual implication, and finally A a gray-scale image and B a gray-scale structural element.

We recall the definitions of fuzzy morphological operators following the ideas of De Baets in [3]. The method consists in fuzzify the logical operations, i.e. the Boolean conjunction and the Boolean implication, to obtain fuzzy operators. An n-dimensional gray-scale image is modeled as an $I\!R^n \longrightarrow [0,1]$ function. The values of an image must be in $[0,1]$ in order to consider it as a fuzzy object. Then, we will proceed to explain this method from the following definitions and propositions.

Definition. The *fuzzy dilation* $D_{\mathcal{C}}(A,B)$ and *fuzzy erosion* $E_{\mathcal{I}}(A,B)$ of A by B are the gray-scale images defined by

$$D_{\mathcal{C}}(A,B)(y) = \sup_x \mathcal{C}(B(x-y), A(x)) \quad (1)$$

$$E_{\mathcal{I}}(A,B)(y) = \inf_x \mathcal{I}(B(x-y), A(x)). \quad (2)$$

Note that the reflection $-B$ of a fuzzy image B is defined by $-B(y) = B(-y)$, for all $y \in I\!R^n$. Given two images B_1, B_2, we will say that $B_1 \subseteq B_2$ when $B_1(y) \leq B_2(y)$ for all $y \in I\!R^n$.

Definition. The *fuzzy closing* $C_{\mathcal{C},\mathcal{I}}(A,B)$ and *fuzzy opening* $O_{\mathcal{C},\mathcal{I}}(A,B)$ of A by B are the gray-scale images defined by

$$C_{\mathcal{C},\mathcal{I}}(A,B)(y) = E_{\mathcal{I}}(D_{\mathcal{C}}(A,B),-B)(y) \quad (3)$$

$$O_{\mathcal{C},\mathcal{I}}(A,B)(y) = D_{\mathcal{C}}(E_{\mathcal{I}}(A,B),-B)(y). \quad (4)$$

In this paper, we use as conjunction two types of left-continuous conjunctive uninorms and as implication, their residual implications. Specifically these two types of uninorms are the following.

- The representable uninorms: Let $e \in]0,1[$ and let $h : [0,1] \longrightarrow [-\infty, \infty]$ be a strictly increasing, continuous function with $h(0) = -\infty$, $h(e) = 0$ y $h(1) = \infty$. Then $U_h(x,y) =$

  $$\begin{cases} h^{-1}(h(x)+h(y)), & \text{if } (x,y) \notin \{(1,0),(0,1)\}, \\ 0, & \text{in other case,} \end{cases}$$

  is a conjunctive representable uninorm with neutral element e, see [20], and its residual implication $\mathcal{I}_{U_h}$ is given by $\mathcal{I}_{U_h}(x,y) =$

  $$\begin{cases} h^{-1}(h(y)-h(x)), & \text{if } (x,y) \notin \{(0,0),(1,1)\}, \\ 1, & \text{in other case.} \end{cases}$$

  Moreover, U_h satisfies self duality (except at the points (0,1) and (1,0)) with respect to the strong negation $\mathcal{N}(x) = h^{-1}(-h(x))$, see [21].

- A specific type of idempotent uninorms: Let $\mathcal{N}$ be a strong negation. The function given by

  $$U^{\mathcal{N}}(x,y) = \begin{cases} \min(x,y), & \text{if } y \leq \mathcal{N}(x), \\ \max(x,y), & \text{in other case,} \end{cases}$$

  is a conjunctive idempotent uninorm. Its residual implication is given by (see [22])

  $$\mathcal{I}_{U^{\mathcal{N}}}(x,y) = \begin{cases} \min(\mathcal{N}(x),y), & \text{if } y < x, \\ \max(\mathcal{N}(x),y), & \text{if } y \geq x. \end{cases}$$

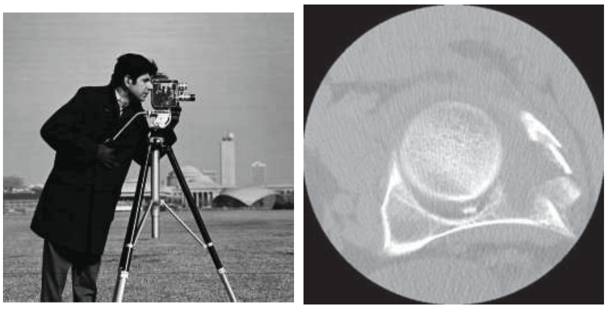

Figure 1: Original images used in experiments.

These two types of conjunctive uninorms guarantees most of the good algebraic and morphological properties associated with the morphological operators obtained from them ([1, 9, 21, 22]). Among them, we highlight those described below. In all properties, U is a left-continuous conjunctive uninorm with neutral element $e \in]0,1[$, $\mathcal{I}_U$ its residual implication, A is a gray-level image and B a gray-scale structural element.

- The fuzzy dilation D_U is increasing in both arguments, the fuzzy erosion $E_{\mathcal{I}}$ is increasing in their first argument and decreasing in their second one, the fuzzy closing $C_{U,\mathcal{I}_U}$ and the fuzzy opening $O_{U,\mathcal{I}_U}$ are both increasing in their first argument.
- If $B(0) = e$ the fuzzy dilation is extensive and the fuzzy erosion is anti-extensive $E_{\mathcal{I}_U}(A,B) \subseteq A \subseteq D_U(A,B)$.
- Moreover, the fuzzy closing is extensive and the fuzzy opening is anti-extensive: $O_{U,\mathcal{I}_U} \subseteq A \subseteq C_{U,\mathcal{I}_U}(A,B)$.
- The fuzzy closing and the fuzzy opening are idempotent, i.e.: $C_{U,\mathcal{I}_U}(C_{U,\mathcal{I}_U}(A,B),B) = C_{U,\mathcal{I}_U}(A,B)$, and $O_{U,\mathcal{I}_U}(O_{U,\mathcal{I}_U}(A,B),B) = O_{U,\mathcal{I}_U}(A,B)$.
- If $B(0) = e$, then $E_{\mathcal{I}_U}(A,B) \subseteq O_{U,\mathcal{I}_U}(A,B) \subseteq A \subseteq C_{U,\mathcal{I}_U}(A,B) \subseteq D_U(A,B)$.
- For the two previous conjunctives uninorms of type U_h and $U^{\mathcal{N}}$, the duality between fuzzy morphological operators is guaranteed.

3 Residuals and basic filters

In the following experiments, the idempotent uninorm $U^{\mathcal{N}}$ with $\mathcal{N}(x) = 1-x$ and the representable uninorm U_h with $h(x) = \ln\left(\frac{x}{1-x}\right)$ have been used. The obtained results are compared with the Łuckasiewicz t-norm and the classic development based on *umbra approach* (see [7]). In particular, the structural element used by the morphological operators is given by the following matrix:

$$B = e \cdot \begin{pmatrix} 0.86 & 0.86 & 0.86 \\ 0.86 & 1.00 & 0.86 \\ 0.86 & 0.86 & 0.86 \end{pmatrix}$$

ISBN: 978-989-95079-6-8

Figure 2: Left: closing. Right: opening. Top, an idempotent uninorm is used. Down, a representable uninorm is used.

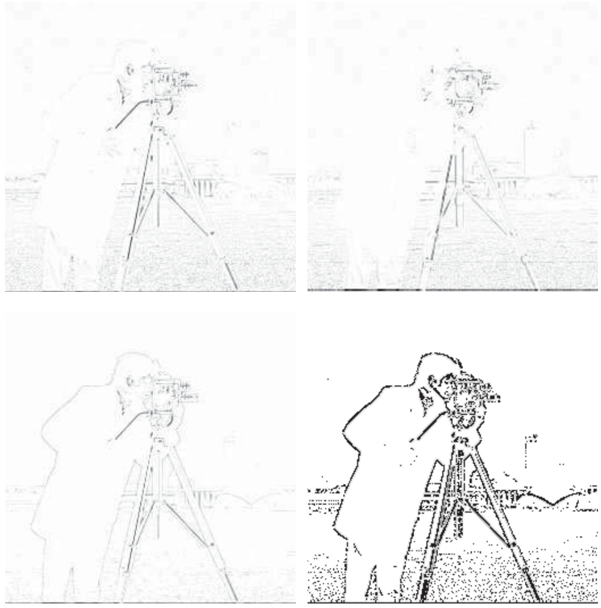

Figure 3: Left: Top-Hat. Right: Dual Top-Hat. Top: *umbra approach*. Down: Łuckasiewicz t-norm.

where e is the neutral element of the considered uninorm. The original images used in the experiments are shown in Figures 1 and 6 (they are represented by A in the fuzzy morphological operators). Two gray level original images are shown in Figure 1; one of them with size 256×256 is the classical image of a cameraman, and the other one of size 512×512 is the medical image of a hip.

The residual of two morphological operations or transformations is their difference. The first residual that can be defined in mathematical morphology is the morphological gradient, which is used as an edge detector and as a first approximation to a morphological segmentation. The morphological gradient, known in morphology as the Beucher gradient, is the difference between a dilation and an erosion, a dilation and the original image or the difference between the original image and its erosion. When we use the dilation and the erosion of a fuzzy set A by the structural element B, where the uninorm U is considered as a conjunctor and $\mathcal{I}_U$ as the residual implicator, the gradients are given by:

$$\nabla^-_{U,\mathcal{I}_U}(A,B) = A \setminus E_{\mathcal{I}_U}(A,B), \quad (5)$$

$$\nabla^+_{U,\mathcal{I}_U}(A,B) = D_U(A,B) \setminus A, \quad (6)$$

$$\nabla_{U,\mathcal{I}_U}(A,B) = D_U(A,B) \setminus E_{\mathcal{I}_U}(A,B), \quad (7)$$

where expression (5) represents the gradient by erosion, (6) the gradient by dilation and (7) the symmetrical gradient. From the point of view of the fuzzy inclusion, $\nabla^-_{U,\mathcal{I}_U}(A,B) \subseteq \nabla_{U,\mathcal{I}_U}(A,B)$ and $\nabla_{U,\mathcal{I}_U}(A,B) \supseteq \nabla^+_{U,\mathcal{I}_U}(A,B)$ are satisfied.

The goal of the gradients is the detection of the edges and/or the perimeter of the objects of the image. The correct choice of the structural element and the gradient will depend on the geometry of the objects. The application of the symmetrical gradient based on uninorms to the edge detection can be seen in [1], where the obtained results are better than those obtained by the gradients using t-norms or the classic approach.

The opening and closing defined in the previous section are the most elementary morphological filters, which are called *basic filters*. As we have seen in the previous section, the opening is an antiextensive morphological filter and the closing is an extensive morphological filter. So, in a first step, they can be used to remove non desired objects in an image. We can observe that the opening of a gray level image by a structural element removes the light zones of less size than the element and makes the light objects darker. The morphological closing helps to remove dark structures of less size than the structural element, toning down the dark objects. The size and shape of the used structurant element in the opening must agree with the image structures that we want to remove. Sometimes, structurant element of great size will remove the non desired shapes of an image, but also they can remove others shapes and the rest of the structures can be affected. Reduced sizes will be optimal when the images have small details. The effect of the closing and the opening on the cameraman's image can be observed in Figure 2, using the idempotent uninorm (top) and the representable uninorm (down).

The opening and the closing are morphological transformations and they are sensitive to compute the associated residuals, which are called *Top-Hat* transformations. The Top-Hat transformation, initiated by F. Meyer in 1977 (see [2]), finds structures of the images that have been removed by the opening or closing filter. If we choose the appropriated size and shape of the structural element, it is possible to filter and to remove some elements in the original image. The difference operation between the original map and the filtered map increases considerably the contrast of the removed zones.

The Top-Hat transformations are defined as a residual between the identity and the opening, which is called opening Top-Hat or white Top-Hat, or between the closing and the identity, which is called dual Top-Hat or black Top-Hat. They

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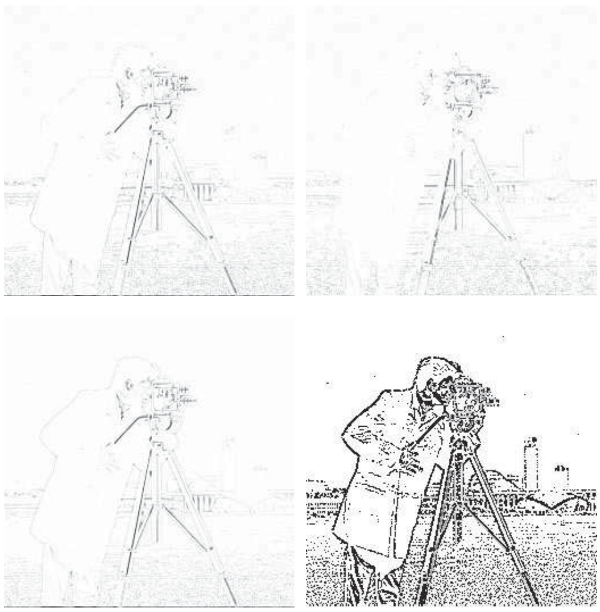

Figure 4: Left: Top-Hat. Right: Dual Top-Hat. Top, idempotent uninorm. Down, representable uninorm.

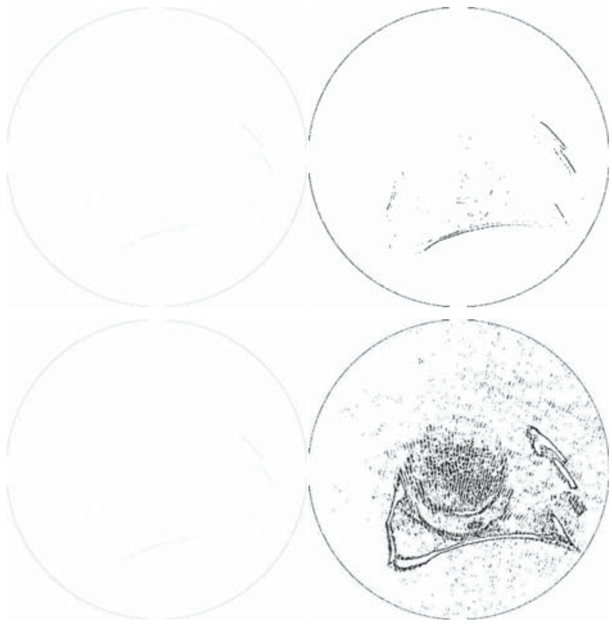

Figure 5: Left: Top-Hat. Right: Dual Top-Hat. Top, Łuckasiewicz t-norm. Down, representable uninorm.

are defined by:

$$\rho_{U,\mathcal{I}_U}(A,B) = A \setminus O_{U,\mathcal{I}_U}(A,B) \text{ (Top-Hat)}$$
$$\rho^d_{U,\mathcal{I}_U}(A,B) = C_{U,\mathcal{I}_U}(A,B) \setminus A \text{ (dual Top-Hat)}$$

The Top-Hat enhances the light objects that have been removed by the opening and it is used to extract contrasted components from the background, while the dual Top-Hat extracts the dark components that have been removed by the closing. Usually, the Top-Hat removes the soft trends, making thus an enhanced contrast. The detection of light objects is improved with an opening Top-Hat, in the same way with the dual Top-Hat does with the dark objects. The size of the structurant element is a deciding factor in order to apply the Top-Hat transformations.

The Top-Hat is idempotent (but it is not increasing) and it is antiextensive. The dual Top-Hat is neither increasing nor idempotent.

The obtained edges using the Top-Hat and the dual Top-Hat can be seen in Figures 3 and 4 for the cameraman image. The collected information using the idempotent uninorm is similar to the classical approach, and in the case of the representable uninorm, we can obtain (with the dual approach) more information than in the rest of the cases. Both top-hats are shown in Figure 5 for the hip image and it can be observed that the representable uninorm shows again more information.

It has been indicated that opening and closing are the basic filters of the fuzzy mathematical morphology. In order to build new mophological filters ([2]), we start with the basic filters and using composition or combination with other operations. New filters as sequential filters, center operators, contrast filters, etc. can be implemented. The basic applications of the morphological filters are the noise reduction and the selected extraction of image objects. Both applications are important in artificial vision since interpretation or measure techniques will fail in presence of noise, whereas the structure and the object selection are fundamental in segmentation processes.

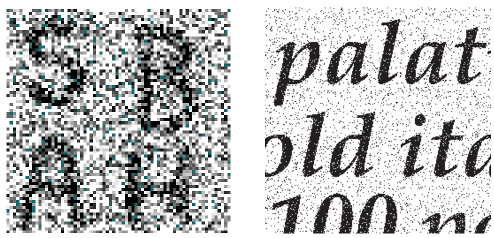

Figure 6: Original images with noise.

The first built filters from opening and closing are the so called *alternate filters*. Let $\xi(A,B)$ and $\psi(A,B)$ be the opening and closing, respectively, of a fuzzy object A by the structural element B using uninorm U as a conjuctor and $\mathcal{I}_U$ as a residual implicator. Using these two filters, four idempotent and growing filters can be generated: $\xi\,\psi$, $\psi\,\xi$, $\psi\,\xi\,\psi$ and $\xi\,\psi\,\xi$. These filters have similar properties to the classical morphological filters. Due to the idempotent property, the composition of more of three operators doesn't provide a new filter, except that we change the structural element.

In Figure 6, two binary images with added noise are shown. In the left image of size 70×74 the 60% of the points have been substituted by random graylevel noise[1]. In the right image of size 256×256, a gaussian noise has been added. Figure 7 shows the results obtained when we apply, from left to right, the alternate filters ψ, $\xi\,\psi$ and $\psi\,\xi\,\psi$, respectively. From top to bottom, the obtained results are shown using the different approaches. We can see that the uninorms remove the noise of the image in such a way that the *original* image is discernible. The same structure is shown in Figure 8.

The filtered image using the filter $\psi\,\xi$ and using the idempotent uninorm (very similar to the obtained image using the representable uninorm), Łuckasiewicz t-norm and *umbra approach* are shown in Figure 9. The filtered image with the

[1]This image has been downloaded of the web page of J. A. Sethian, http://math.berkeley.edu/˜sethian/.

ISBN: 978-989-95079-6-8

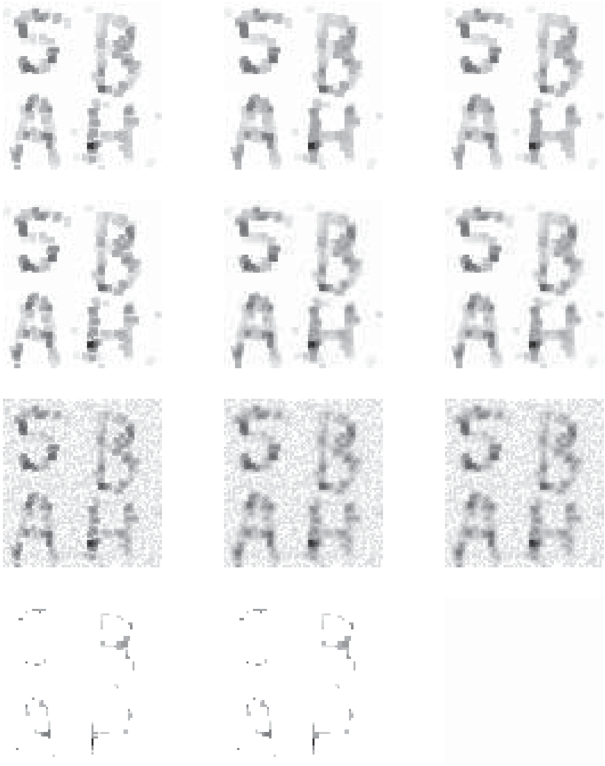

Figure 7: From left to right: filtered image using ξ, $\xi\,\psi$ and $\psi\,\xi\,\psi$, respectively. Top to bottom: idempotent uninorm, representable uninorm, Łuckasiewicz t-norm and *umbra approach*.

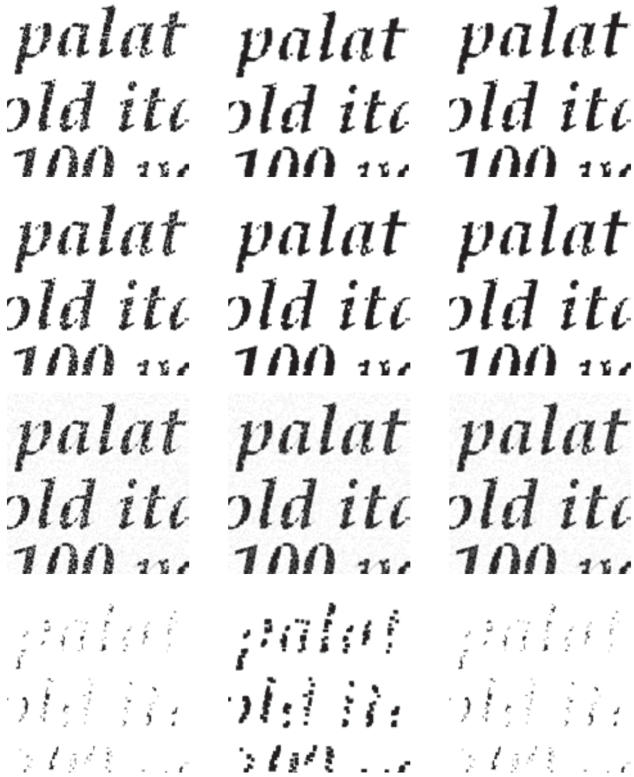

Figure 8: From left to right: filtered image using ξ, $\xi\,\psi$ and $\psi\,\xi\,\psi$, respectively. From top to bottom: idempotent uninorm, representable uninorm, Łuckasiewicz t-norm and *umbra approach*.

structure $\psi\,\xi\,\psi$ is very similar, except in the classical approach where the image gets smudged and in the case of the t-norm, the noise is softer.

A chest CT image of size 425×334 with "salt and pepper" added noise is shown in Figure 10 using 0.02 as a parameter. The results obtained using filter $\psi\,\xi\,\psi$ are shown in Figure 11. In the top left image, the Łuckasiewicz t-norm has been used and most of the noise remains. In the right image, the filtered image using the *umbra approach* is shown, a lot of "salt" noise is magnified. In the bottom images, the results obtained using uninorms are shown. The results using an idempotent uninorm are shown in the left image and the results using a representable uninorm are shown in the right image. The function generator of the representable uninorm is $h(x) = \frac{x-0.5}{x(1-x)}$. In the cases using uninorms the results are better than the ones using t-norms and umbra approach, because the noise is practically removed.

These type of filters have been used in the industrial control of pieces with partial occlusions or even in edge detection with noise elimination.

4 Conclusions and future work

In this work we have shown how the alternate filters based on opening and closing morphological operators using uninorms can remove the noise of an image preserving their structure. The obtained results improve the obtained results using t-norms and *umbra approach*. Also, we have obtained images very similar to the original ones without noise. So, the alternate filters based on uninorms are suitable to remove different

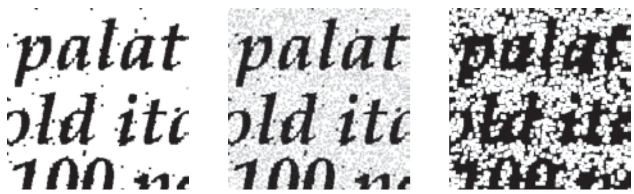

Figure 9: From left to right: filtered image using $\psi\,\xi$, an idempotent uninorm, Łuckasiewicz t-norm and *umbra approach*.

types of noise. The future work will consist on the construction of new filters. Also, we have shown the usefullness of the opening and closing filters in the edge detection. The future work includes the study of the effect of the choice of the structural element too. We have used isotropic structural elements in the experiments. That is, they have the same effect in all directions where they can be applied. We think that in some images, the use of anisotropic structural elements would improve the results.

We also want to apply the previous techniques to a more extended set of gray level images, even color images.

Acknowledgment

This work has been supported by the Government Spanish Grant MTM2006-05540, with FEDER support.

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ISBN: 978-989-95079-6-8

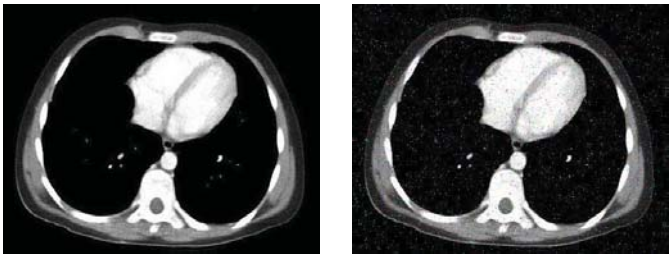

Figure 10: Left: Original image. Right: image with “salt and pepper” added noise.

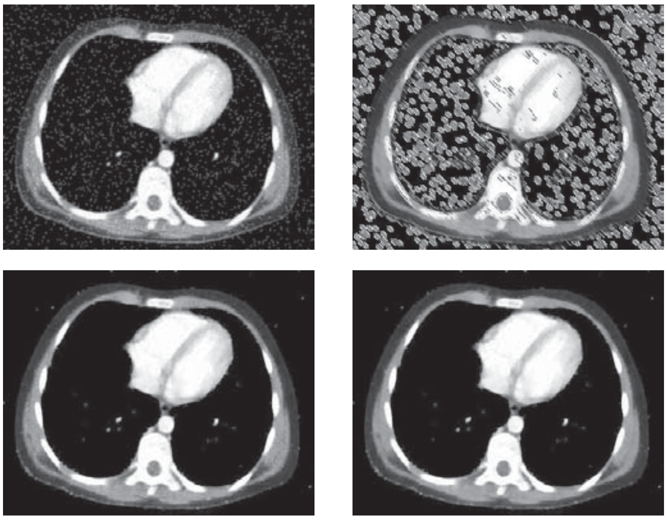

Figure 11: From left to right and from top to bottom, filtered image using $\psi\,\xi\,\psi$, and Łuckasiewicz t-norm, *umbra approach*, idempotent uninorm and representable uninorm.

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ISBN: 978-989-95079-6-8