A Hybrid Meta-Heuristic to Solve the Portfolio Selection Problem

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Abstract— *We present a fuzzy version of the efficient portfolio selection problem which adds to the Markowitz's classical model the vagueness of the investor's preferences about the assumed risk. This is done by adapting some techniques previously applied by the authors in other logistic problems. We provide several procedures to solve it: an exact one and a hybrid meta-heuristic procedure more adequate for medium and large-sized problems. To show the usefullness of the model and the algorithms, we provide an example based on data from the Spanish index IBEX35.*

Keywords— Fuzzy programming, Heuristic strategies, Portfolio selection.

1 Introduction

The modern portfolio selection problem is a classical model for determining the optimal composition of a portfolio according to the preferences of an investor. The model is credited to Markowitz [17, 18], who is rightfully regarded as the founder of modern portfolio theory [6].

In a first approach, given n securities $S_1, S_2, \cdots, S_n$ in which we can invest non-negative quantities $x_1, x_2, \cdots, x_n$, a portfolio consists of a subset of securities in which all the capital should be invested satisfying investor's preferences. These preferences must take into account the risk that the investor is willing to assume, the expected return and, to a lesser extent, some diversification criteria.

One possibility is taking the return as objective function and including the risk as a constraint, i.e.

$$\begin{array}{lll} \text{Max.} & \text{Portfolio return} & \\ \text{s.t.} & \text{Risk of portfolio} \leq R & \\ & \displaystyle\sum_{i,=1}^{n} x_i = 1, & \\ & l_i \leq x_i \leq u_i, & i = 1, \dots, n. \\ & l_i \geq 0 & i = 1, \dots, n. \end{array} \quad (1)$$

where R is the maximum risk accepted by the investor, and the last constraints express that all the capital must be invested as well as diversification conditions.

However, we could also consider the dual version (in an economic sense) in which the risk is minimized taking the expected return as a constraint.

Obviously, the portfolio selection, like most financial problems, is related with uncertainty because it consists in taking a decision about future events. Therefore, we do not have at our disposal more than historical data which usually are managed with statistical methods [18, 6]. Moreover, it is not easy to model the investor's preferences. After the seminal work by Markowitz, attention has been given in the study of alternative models [13, 14]. Most of these models are based on probability distributions, which are used to characterize risk and return. For instance, in the Markowitz model, variance and mean were generally deemed satisfactory measures of risk and return, respectively [18, 21, 7, 12]. However, according to the chosen modellization of the expected risk and return, different models coexist to select the best portfolio. Some of them propose to describe the risk by means of mean-absolute deviation ([11], [24]), other authors propose linear programming [22] or multiobjective [15] models.

Another way of dealing with uncertainty is working with models based on soft computing. This can be done by means of vague goals for the expected return rate and risk [23, 16], or by using possibility distributions to model the returns. This fact permits the incorporation of expert knowledge by means of a possibility grade, to reflect the degree of similarity between the future state of stock markets and the state of previous periods.

In this paper, we present several ways of dealing with the uncertainty associated to the portfolio selection problem. Namely, we are going to show that the techniques used by the authors in logistic problems[2, 3, 4] can be generalized to deal with the risk and return fuzziness in the portfolio selection problem. These techniques allows us to include in our fuzzy model the vagueness related with the fact that the investor fixes arbitrarily the risk that he/she will assume. The uncertainty concerning to the valuation of risk and returns is modellized by means of the usual variance and mean.

To solve our model, meta-heuristic techniques are often needed because of the size of the problem or because a quick solution is required in order to reflect real-time cases. Therefore, we propose a hybrid meta-heuristic procedure.

The paper is organized as follows: The next section introduces the framework for portfolio selection used in this paper. Section 3 examines the portfolio selection with flexible constraints, incorporating this flexibility in a fuzzy model. In section 4, we present a hybrid meta-heuristic to solve the portfolio selection problem with flexible constraints. Finally, Section 5 outlines the most important conclusions.

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2 Portfolio selection

We could consider the problem of an investor who intends to invest some money in securities in such a way that the rate of return is maximized and a given level of risk cannot be surpassed. If we have n securities, r_i is the return of the i-th security and x_i is the proportion of total investment funds devoted to the i-th security, then we have the constraint

$$\sum_{i=1}^{n} x_i = 1.$$

The risk of investment can be measured by the variance $x^t S x$, where $S = [s_{ij}]$ is the variance-covariance matrix. If R is the maximum risk accepted by the investor, we have the constraint

$$\sum_{i,j=1}^{n} s_{ij} x_i x_j \leq R.$$

Then, one of the simplest models for the portfolio selection problem is:

$$\begin{aligned} \text{Max.} \quad & \sum_{i=1}^{n} r_i x_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} x_i = 1, \\ & \sum_{i,j=1}^{n} s_{ij} x_i x_j \leq R, \\ & x_i \geq 0, \;\; i = 1, \ldots, n. \end{aligned} \tag{2}$$

However, choosing the best investment options for a portfolio is a difficult task due to economic environment uncertainty and the problem of suitably reflecting decision maker desires in the model. Both stochastic and fuzzy programming provide different ways of handling the first kind of uncertainty [9]. This paper presents a fuzzy approach for the second kind, in which the portfolio problem includes the subjective criteria of the decision maker to determine the level of risk that he or she is able to support and the level of satisfaction to be assigned to a possible increase in return.

3 Flexible Portfolio selection

The portfolio selection problem has two data concerning decision maker preferences, namely the capital to be invested and the risk to be assumed. The investor can be assumed to know with certainty the capital that he/she would like to consider, and in fact, in the model this quantity has been normalized to the unit. However, determining the risk to be assumed could be more flexible. As a result, it is worth incorporating this flexibility in a fuzzy model.

The main idea is to consider partially feasible solutions involving slightly greater risk than that fixed by the decision maker, and to study the possibilities that they offer in order to improve the expected return. When compared with the logistic models (actually with the p-median case [2, 3, 4]), this problem happens to be more complicated, because the p-median problem is linear whereas the risk constraint in the portfolio model is quadratic. Moreover, in the p-median case, a small reduction in covered demand affected optimal cost in a simple linear way, whereas the way in which the maximum expected return depends on the accepted risk is rather more complicated.

A fuzzy set $\tilde{S}$ of partially feasible solutions is defined so that portfolio selection belongs to $\tilde{S}$ with a degree of membership that depends on how much it exceeds the risk R_0 fixed by the investor. On the other hand, a second fuzzy set $\tilde{G}$ is defined, whose membership function reflects the improvement of the return provided by a partially feasible solution with respect to the optimal crisp return z^*. In practice, we consider piecewise linear membership functions

$$\mu_{\tilde{S}}(x) = \begin{cases} 1 & \text{if } r \leq R_0, \\ 1 - \frac{r - R_0}{p_f} & \text{if } R_0 < r < R_0 + p_f, \\ 0 & \text{if } r \geq R_0 + p_f, \end{cases}$$

$$\mu_{\tilde{G}}(x) = \begin{cases} 0 & \text{if } z \leq z^*, \\ \frac{z - z^*}{p_g} & \text{if } z^* < z < z^* + p_g, \\ 1 & \text{if } z \geq z^* + p_g, \end{cases}$$

where r and z are the risk and the return provided by the portfolio x (which is assumed to satisfy the constraints of (2), except the second one); the parameter p_f is the maximum increment in the risk that the decision maker can accept, and p_g is the increment of the return that the decision maker would consider completely satisfactory. From this, we can define a global degree of satisfaction

$$\lambda(x) = \min\{\mu_{\tilde{G}}(x), \mu_{\tilde{S}}(x)\},$$

which is the membership degree to the fuzzy intersection of $\tilde{S} \cap \tilde{G}$. The fuzzy portfolio model becomes

$$\begin{aligned} \text{Max.} \quad & \lambda(x) \\ \text{s.t.} \quad & x \in \tilde{S}. \end{aligned} \tag{3}$$

In order to solve it, the optimal solution of (2) will be calculated for each risk level R. Therefore, we solve explicitly the Kuhn-Tucker conditions for the problem. To carry out the computations in a generic framework, we start making a change of variables to diagonalize the risk matrix. As we are interested in small variations of R, the variables x_i that are zero in the optimal crisp portfolio can be removed and hence assume that non-negative conditions are not active. We can also assume that the risk constraint is active.

Standard linear algebra theory ensures us that we can decompose

$$S = A^t D A, \tag{4}$$

where the matrix D is diagonal and A is regular. Then the change of variables $y = Ax$ transforms the problem to

$$\begin{aligned} \text{Max.} \quad & \sum_{i=1}^{n} r'_i y_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} b_i y_i = 1, \\ & \sum_{i=1}^{n} d_i y_i^2 = R, \\ & A^{-1} y \geq 0, \end{aligned} \tag{5}$$

where $r' = r^t A^{-1}$, $b = (1, \ldots, 1) A^{-1}$.

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As nonnegative conditions are inactive, the optimal solution must satisfy

$$\sum_{i=1}^{n} b_i y_i = 1, \quad \sum_{i=1}^{n} d_i y_i^2 = R, \quad r_i' - b_i\lambda - 2d_i y_i \mu = 0,$$

where λ and μ are the Lagrange multipliers of the capital and risk constraints respectively. Hence:

$$y_i = \frac{r_i' - b_i\lambda}{2d_i\mu},$$

and, from the first constraint, we get:

$$\sum_{i=1}^{n} \frac{b_i r_i'}{2d_i} - \sum_{i=1}^{n} \frac{b_i^2}{2d_i}\lambda = \mu.$$

Call

$$K = \sum_{i=1}^{n} \frac{b_i r_i'}{2d_i}, \qquad L = \sum_{i=1}^{n} \frac{b_i^2}{2d_i},$$

so that $\mu = K - L\lambda$. Hence:

$$y_i(\lambda) = \frac{r_i' - b_i\lambda}{2d_i(K - L\lambda)}.$$

The second constraint gives us:

$$\sum_{i=1}^{n} \frac{(r_i' - b_i\lambda)^2}{4d_i} = R(K - L\lambda)^2$$

or

$$\sum_{i=1}^{n} \frac{r_i'^2}{4d_i} - \sum_{i=1}^{n} \frac{r_i' b_i}{2d_i}\lambda + \sum_{i=1}^{n} \frac{b_i^2}{4d_i}\lambda^2 = \\ = R(K^2 - 2KL\lambda + L^2\lambda^2),$$

$$\sum_{i=1}^{n} \frac{r_i'^2}{4d_i} - K\lambda + \frac{L}{2}\lambda^2 = R(K^2 - 2KL\lambda + L^2\lambda^2).$$

If

$$M = \sum_{i=1}^{n} \frac{r_i'^2}{4d_i} \tag{6}$$

we get

$$(RL^2 - L/2)\lambda^2 + (K - 2KLR)\lambda + RK^2 - M = 0.$$

This is a second degree equation on λ with discriminant

$$\Delta(R) = K^2 - 2ML + (4ML^2 - 2K^2L)R.$$

Hence,

$$\lambda(R) = \frac{2KLR - K \pm \sqrt{\Delta(R)}}{2L^2R - L},$$

and so

$$\mu(R) = K - L\lambda(R) = \frac{\pm\sqrt{\Delta(R)}}{1 - 2LR}.$$

The sign must be chosen so that $\mu(R) \geq 0$ (the Kuhn-Tucker sign condition). Once the right sign is chosen, this function $\lambda(R)$ allows us to calculate $y_i(R)$ and, from this, we get the optimal portfolio $x_i(R)$ for each R such that $x_i(R) > 0$. This reduces the range of R to a certain interval that can be computed. When we reach one end point of this interval, we need to start again with the problem corresponding to the new set of positive variables.

We can also compute the return

$$F(R) = \sum_{i=1}^{n} r_i' y_i(R).$$

Since

$$y_i(\lambda) = \frac{r_i' - b_i\lambda}{2d_i\mu} = \\ \mp = \frac{2LR-1}{2d_i\sqrt{\Delta(R)}}\left(r_i' - b_i\frac{2KLR-K\pm\sqrt{\Delta(R)}}{2L^2R-L}\right), \quad 1 \leq i \leq n,$$

we have

$$F(R) = \\ \mp\frac{2LR-1}{2\sqrt{\Delta(R)}}\sum_{i=1}^{n}\left(\frac{r_i'^2}{d_i} - \frac{r_i' b_i}{d_i}\frac{2KLR-K\pm\sqrt{\Delta(R)}}{2L^2R-L}\right).$$

This expression allows us to calculate the degree of improvement of the goal $\mu_g(R)$ of the best portfolio with risk R, whereas its degree of feasibility $\mu_f(R)$ is trivially computed.

Now we can determine the risk R^* such that $\mu_f(R^*) = \mu_g(R^*)$, which is easily shown to be the risk of the portfolio maximizing λ. The portfolio $x(R^*)$ corresponding to $y(R^*)$ by the change of variables is the optimal solution of (3).

4 A hybrid meta-heuristic to solve the Portfolio Selection problem

When the size of the problem increases, traditional methods become useless as they would need too much time, due to the combinatorial explosion in the solution space. Genetic Algorithms [19, 5] are meta-heuristic methods that have already shown their kindness to solve optimization problems.

In this section we describe a hybrid meta-heuristic for the portfolio selection problem with flexible constraints, called `GAFUZ-PF` (Genetic algorithm + simulated Annealing + FUZzy PortFolio). This meta-heuristic uses a hybrid scheme, mixing the ideas of Simulated Annealing technique with the classic Genetic Algorithm. Note that we need the optimal crisp return z^* of the portfolio selection problem (2) because $\mu_{\tilde{G}}(x)$ depends on it. Hence, solving the crisp problem must be considered as a part of the process of solving the fuzzy problem (this means to solve two NP-hard problems). In order to realize this idea, we use three different populations on three stages of the genetic algorithm. First of all, it uses a population to achieve feasible solutions to the problem (with a risk less than or equal to R fixed) which will be part of the population of the algorithm. Once feasible solutions are found, in the second stage, the algorithm uses other population that evolves searching the best return. The third stage uses another population, which looks for the best fitness $\lambda(x)$.

Simulated Annealing [1, 10] is a meta-heuristic used for optimization problems. This heuristic replaces the current solution by a random "nearby" solution, chosen with a probability that depends on the difference between the corresponding

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function values and a global parameter T (called the temperature), that is gradually decreased during the process. The use of this idea with Genetic Algorithm is suggested by Mitchell [19], using it with the individual selection function. We apply Simulated Annealing to mutation function. Having a mutation function with a big mutation rate helps us to keep diversity in the population, but it introduces distortion on the population when the algorithm approaches to the optimum. So, it seems a good idea to use Simulated Annealing with a temperature depending on the number of generations and the fitness of the solution in order to make possible a big mutation rate at the beginning, and reduce it while increasing the number of generations and the fitness. Now we specify our options and we give a general outline of our approach. Figure 1 shows the scheme of `GAFUZ-PF` hybrid meta-heuristic.

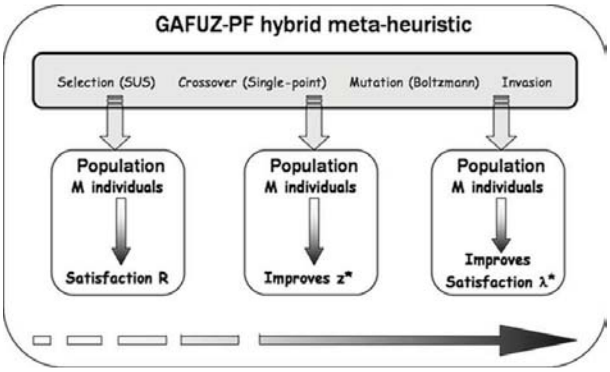

Figure 1: Scheme of hybrid meta-heuristic

4.1 Encoding, Fitness function and Population size

Each element of the population of the algorithm represents a solution to the described portfolio problem. The population encoding is made having an array of n-elements, where each gene represents the invest on one asset. The size of the population is fixed to 50 individuals, as it is known to be the best average size to avoid a too slow algorithm [19]. Furthermore, we use three different populations against time: the first one evolves searching for individuals to satisfy the second constraint of the problem 2. When it founds a suitable solution, it changes to the second stage, where it improves the return of the portfolio, taking account of the flexible constraint of R. Once the algorithm gets a state with $\lambda(x) > 0$, it changes the population and evolves taking into account p_f and p_g. The fitness function is the objective function of the problem (3).

The algorithm scheme is the classical one, with selection, crossover, mutation, invasion and having the number of generations as stop criteria.

4.2 Selecting the individuals

In order to select individuals for the crossover operator, we use the Stochastical Universal Sampling (SUS), as it has been shown to be better than Roulette Wheel [5]. Based on their fitness, SUS places the individuals on a Roulete and spuns it once with n selectors equally separated to select n individuals.

4.3 Crossover: generating new individuals

The crossover operator is the single-point crossover [19] adapted to the problem constraints: there is a fixed part from one parent that goes directly to the son; the genes from the second part (from the second parent) are added to the son (choosing them randomly) while the sum of its genes is ≤ 1.

4.4 Mutation

When we talk about mutation, there are some parameters used. The mutation rate (MUR) refers to the percentage of population individuals that will be mutated. On the other hand, we have the number of genes that will change (NR). As a mutation scheme we use Boltzmann mutation which is based on Simulated Annealing [5], with the following annealing function:

$$r = 1 - e^{-1/(n*max(\lambda(x)))}$$

It defines the mutation rate across the algorithm, reducing it as the algorithm evolves in order not to disturb the good solutions achieved when the algorithm is finishing. This rate specifies the percentage of individuals that would be mutated, since the number of mutations per individual is 1, as it is known to be the best choice [20]. A mutation consists in substract a random value from a gene and add this amount to other gene, both of them choose randomly.

4.5 Invasion

Finally, invasion is another genetic operator that helps to keep diversity in the population. It introduces a number of new randomly generated individuals, which replace some individuals at random in the population (except the best one). This helps to introduce new genes in the population, like mutation does. The invasion rate (IR) selected is 1% .

4.6 Stop criteria

The detention criterion is the number of generations (NG).

4.7 General outline of the `GAFUZ-PF` meta-heuristic

Now we have shown the configuration of the algorithm, we will see its general outline in algorithm 1.

4.8 Experiments

To test `GAFUZ-PF` hybrid meta-heuristic we have used various test problems. The PC used for the executions has the following features: Intel Pentium IV 3.00 GHz 2048 MB RAM.

- We have considered the returns on 20 assets from the Spanish index IBEX35. The set of assets included in the experiment represents Acesa (ACE), Arcelor (ACR), ACS, Altadis (ALT), BBVA, Bankinter (BKT), Dragados (DRC), Endesa (ELE), FCC, Iberdrola (IBE), Metrovacesa (MVC), NH Hoteles (NHH), Banco Popular (POP), Repsol (REP), SCH (SAN), Telefónica (TEF), Unión Fenosa (UNF), Vallermoso (VAL), Acerinox (ACX), Acciona (ANA) data respectively. We have considered the observations of the Wednesday prices as an estimate of the weekly prices. Hence, the return on the j- th asset during the k-th week is defined as $r_{kj} = (p_{(k+1)} - p_{kj})/p_{kj}$, where p_{kj} is the price of the j-th asset on the Wednesday of the k-th week. The used data base covers the period from January 1998 to March 2003. Table 1 contains the obtained results for two different risk levels R

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Algorithm 1 - `GAFUZ-PF`: Hybrid meta-heuristic for the portfolio selection

```
GAFUZ-PF( )
Require: r_i, s_ij, R, p_f, p_g
Ensure: z*, x̄*, R*, λ(x), z, x̄, R
  begin
  Generate initial population with 50 individuals at random.
  Calculate the fitness of each individual.
  while Number of generations < NG do
    SELECT 50 individuals
    if ∄ individual | risk of the individual ≤ R then
      select using risk
    else
      if the sum of individual fitness is = 0 then
        select using z*
      else
        select using fitness
      end if
    end if
    CROSS the 50 individuals to obtain 50 new individuals
    Replace 49 individuals of the population with the new
    individuals, keeping the best (elitism)
    MUR=Boltzmann(max.(λ(x)), generation_number)
    MUTATE M × MUR individuals, changing NR genes
    Replace 50 × IR individuals with random generated individuals
    Calculate the new individuals' fitness, return and risk
  end while
  end
```

with the same parameters p_f and p_g. The first column contains the assets apearing either in the crisp or the fuzzy solution. The other four columns contain the crisp and the fuzzy solution in both cases. The first one has a medium satisfaction level ($\lambda = 0.45$) whereas the second one has a very low one ($\lambda = 0.14$). This means that, in the first case, the fuzzy solution is an alternative solution that the investor should take into account, whereas, in the second one, the crisp solution cannot be substantially improved. The last two rows of the table contain the crisp and fuzzy returns and risks for the two cases. These risks measure also the efficiency of our metaheuristic: the optimal crisp solution should have a risk equal to the given one, and the near-optimal crisp solutions given by our metaheuristic have a risk level differing by a 0.54% and 0.19%, respectively, from the optimal one. The time used for the algorithm to obtain the solution to this problem is 1.61 seconds on average.

	$R = 0.000494$		$R = 0.000839$	
	$p_f = 0.0002,\ p_g = 0.002$		$p_f = 0.0002,\ p_g = 0.002$	
	Crisp	Fuzzy	Crisp	Fuzzy
ACE	0.316	0.200	0	0
ACR	0.011	0	0	0
ALT	0.168	0.199	0.207	0.162
DRC	0.019	0.205	0.353	0.461
IBE	0.165	0.014	0	0
MVC	0.050	0.102	0.157	0.098
NHH	0.011	0.033	0.056	0.065
POP	0.62	0	0	0
REP	0.031	0	0.001	0
UNF	0	0.077	0.0395	0.052
VAL	0.036	0	0	0
ACX	0.053	0.069	0.051	0.017
ANA	0.078	0.100	0.139	0.145
λ	0	0.454078	0	0.145928
Return	0.00202704	0.0029352	0.00353188	0.00382374
Risk	0.000491306	0.000603184	0.00083643	0.0010098

Table 1: Obtained solutions by `GAFUZ-PF`

5 Conclusions

In this paper we have presented the problem of portfolio selection, which is a difficult problem to solve due to economic environment uncertainty and the problem of suitably reflecting decision maker desires in the model. We present an approach where the problem of fuzzy portfolio includes the subjective criteria of the decision maker to determine the level of risk that he or she is able to support and the level of satisfaction to be assigned to a possible increase in return.

We have proposed an exact method to solve the fuzzy model of the portfolio problem with the main idea of finding partially feasible solutions involving slightly greater risk than that fixed by the decision maker, and to study the possibilities that they offer in order to improve the expected return.

We have also proposed a hybrid meta-heuristic to solve the fuzzy model for problems with medium or big size where the traditional methods become useless as they would need too much time due to the combinatorial explosion in the solution space. The proposed meta-heuristic uses a hybrid scheme, which combines ideas from the Simulated Annealing technique and genetic algorithms. To test the proposed meta-heuristic we have used several test problems based on dates of IBEX35, the best-known index of Spanish Stock Markets. The results allow us to verify that with the small problems the solutions are very similar to those obtained with the exact model. Moreover, our proposal allows us to work with big problems (large time series, intra-day data, etc.) and the time taken to get the results is about 1.6 seconds, achieving the goal we wanted to get with that meta-heuristic.

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