

# Contributions to Dynamic Multicriteria Decision Making Models

Tiago C. Pais<sup>1</sup> Rita A. Ribeiro<sup>1</sup>

<sup>1</sup> UNINOVA, Campus FCT-Universidade Nova Lisboa,  
Caparica 2829-516, Portugal  
Email: {tpp, rar}@uninova.pt

**Abstract**— In this work we start by presenting a possible architecture for MultiCriteria Dynamic Decision Making (MCDDM) problems. Then we focus on contributions for the rating process because it involves two important aspects of any dynamic spatial-temporal decision problem: how to deal with uncertainty in dynamically changing input data and how consider different criteria importance, depending on criteria satisfaction and respective phase (or iteration) of the decision process. To explain and clarify the contributions we use an illustrative example of site selection for spacecraft landing on planets.

**Keywords**— multicriteria dynamic decision making, imprecision on input data, dynamic criteria weighting.

## 1 Introduction

The main objective of this work is to present some contributions for building general multicriteria dynamic decision making (MCDMM) models. We will focus on two important aspects of any dynamic spatial-temporal decision problem: a) how to deal with uncertainty in dynamically changing input data; b) and how to consider different criteria importance, depending on criteria satisfaction and on the decision process phase (iteration). These two aspects belong to the classical process of evaluating alternatives of MCDM [1] [2] [3].

In general, the aim of Multicriteria or Multi-attribute decision making problems is to find the best compromise solution from all feasible alternatives assessed by pre-defined criteria (or attributes). This type of problems is widespread in real life situations [2], [4]. There are many methods and approaches proposed in the literature to deal with this static decision process, from utility methods to scoring and ranking ones [2] [3]. However, when facing multicriteria dynamic decisions making problems (MCDDM), where feedback from step to step is required, there are few contributions in the literature (see for example [5] [6]).

To better explain our proposed contributions to deal with uncertainty in dynamically changing input data and criteria weights changes, during the temporal decision process, we use an simplified illustrative case study of a site selection problem for spacecraft landing on planets [7] [8].

## 2 MultiCriteria dynamic decision making (MCDDM)

Usually, a multicriteria decision model [1] includes 2 main tasks, rating the alternatives by aggregating their

classifications for each criterion and then ranking them. In this work we discuss a dynamic multicriteria decision model because we are addressing decision paradigms with several iterations, which need to take into account past/historic information, to reach a decision (or consensus).

According to Richardson and Pugh [9] system dynamics problems have two main features: “they are dynamic: they involve quantities which change over time”; “involves the notion of feedback.” This is exactly the perspective in this work: dynamically changing data and feedback from historical data.

A general multicriteria dynamic decision making (MCDDM) architecture is depicted in Fig. 1 (adapted from [7] and using concepts from [9]).

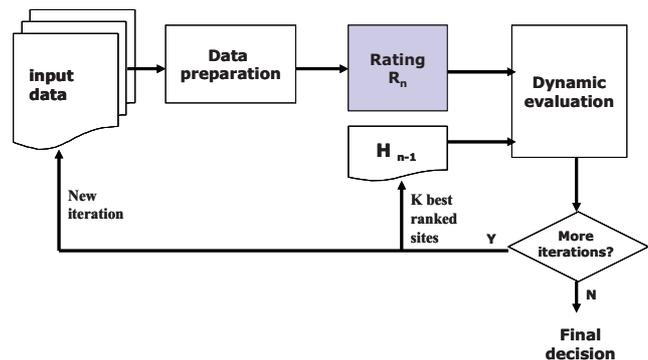


Figure 1: MCDDM architecture.

$R_n$  represents the set of alternatives rating values from iteration  $n$  and  $H_{n-1}$  represents the historic set of rated alternatives from the previous iteration. The feedback is the set of ranked alternatives of iteration  $n$  (updated historic set of each iteration).

In summary, the main characteristics of processes/components, included in a MCDDM are:

a) Data preparation – this process deals with transformation of inputs for the decision model. It includes three main tasks: a.1) cleaning & filtering the data to extract the required criteria and the set of alternatives to be assessed (per iteration); a.2) normalization and representation of criteria for the decision model; a.3) Definition of weighting functions to express the relative importance for each criterion, within a phase or per iteration.

b) Rating process – This process refers to the classification of each alternative regarding all criteria, weighted with their relative importance. I.e. it refers to the aggregation of criteria values weighted with relative importance of criteria. Several methods exist in the literature [1] [2] [3], in our case

we used an improved WSM method, as explained in section 3.2..

c) Dynamic evaluation process – This process refers to the aggregation of historical information about alternatives with their current rating to obtain the set of ranked alternatives [7] for the current iteration (n). When there are no more iterations the decision process stops and the best alternative is the one with the highest rate. For this evaluation we propose to use full-reinforcement operators as described in [7]

d) Historical process – This process determines a subset of good alternatives, from the ranked alternatives, to be considered as feedback information from iteration to iteration. Since this process takes places after ranking, it means historical information for a specific site “remembers” its past behaviour plus the actual rating. There are different sizes (K) for this set of historical information, depending on different phases of the decision process. For our case study we consider three phases according to expert advice.

The topic addressed in detail, in this work, will be the rating process (b), which is highlighted in grey in Figure 1.

### 3 Case Study: Site selection for spacecraft landing on planets

The case study used for discussing the rating process of multicriteria dynamic decision making (MCDDM) models is based on a real project [7], which objective was to determine the best target site for landing on planets for spacecrafts. The case study MCDDM model was developed for the final descent of the spacecraft (around 2 Km from surface), when hazard maps can be obtained. Some values used in the description are just indicative due to reasons of confidentiality.

#### 3.1 Background

In this case study the criteria are hazard maps corresponding to images taken by the Spacecraft during the final descent (around 1000 m from surface) [8] [7]. We consider 7 input criteria, which correspond to the hazard maps obtained during the descent process: slope, texture, fuel, reachability, distance, shadow and scientific interest. The alternatives are the pixels of the combined search space (derived from merging 7 matrices of 512x512, corresponding to the input images/hazard maps), resulting from a data preparation process (not discussed in this work). The resulting set of alternatives is about 260.000 possible sites for each iteration (512\*512= 262,144). The number of iterations is about 40 and for each one there is a evaluation process (called ranking process) including a combination of the *k* best alternatives from iteration *n-1* (historic set) with the current rated ones at the *n* iteration (for details about this process see [7]). We only consider the last iteration as historical information for two reasons: a) time constrains- we have less than 1 minute to perform 40 iterations hence an efficient process is required; b) the historic set implicitly includes all past ratings because we aggregate previous information with current one, i.e we update the alternative evaluation at each iteration and pass this information to the next iteration.

As mentioned in the abstract, in this work we focus on the rating process of MCDMM because it involves two

important aspects of any dynamic spatial-temporal problem: how to deal with uncertainty in the input data of each iteration and how to consider different importance for each criteria, depending on the criteria satisfaction as well as on the phase of the decision process (iteration).

Before discussing the rating process we should explain that we assume 3 phases for this case study as follows. Phase 1 is executed until 1000 m of altitude and considers a historic of 250 set of sites, the rationale is that at higher attitudes there is a need for considering more alternatives (pixels) because the Spacecraft is still too far away from the surface and the information about the sites is still quite imprecise. Phase 2 is executed between the 1000 m and 300 m of altitude (parameters can be changed) and uses a historic set of 10 sites; since we are getting closer to the surface what we need is a small historic set (pixels) of “good” alternatives, ensuring sufficient distance between them (radius) to cover a reasonable search space. Phase 3 starts being executed at 300 m of altitude when the Spacecraft is quite close to the surface and the historic set considered was 5 sites.

Although it is out of scope to discuss the data preparation process for this case study we should mention that the results of this process are fuzzified variables (for references dealing with fuzzification see for example [4]), corresponding to each hazard map, adapted (i.e. re-calculated) at each iteration. Hence, a variable input includes a value and its corresponding membership value in the fuzzified variable.

#### 3.2 Rating process

The rating process deals with aggregation of all classifications obtained by each alternative in each criterion. During the three phases considered, we compute these values for each alternative (per iteration). The general formulation for the weighted sum method (WSM), used as rating process in our dynamic decision process (MCDDM) is:

$$r_i = \oplus (W(ac_{i1}) \otimes ac_{i1}, \dots, W(ac_{in}) \otimes ac_{in}) \quad (1)$$

Where,

$\oplus$  is the sum operator;

$\otimes$  is the product operator;

$ac_{ij}$  is the accuracy & confidence membership value of  $j^{th}$  criterion for site *i*. (section 3.3)., and

$$W(ac_{ij}) = \frac{L(ac_{ij})}{\sum_{k=1}^n L(ac_{ik})}$$

represents the normalized relative weight, considering a weighting function,  $L(ac_{ij})$  as described in section 3.4.,

The basic principle of the aggregation method WSM (weighted sum method) is that “The overall score of an alternative is computed as the weighted sum of the attribute values” [8]. In our case study we tested other methods of aggregation besides this one (e.g. WPM, Compromise ratio [2]), however WSM combined with the proposed weighting functions, showed the best trade-of between computational efficiency versus correctness of results.

3.3 Accuracy & confidence in input values

The accuracy and confidence parameters ( $a_{ij}$  and  $w_{cj}$ , respectively) will be taken into account in the decision model using the following expression:

$$ac_{ij} = w_{c_j} * (1 - \max_{x \in [a,b]} \{ |\mu(x) - \mu(x_{ij})| \}) * \mu(x_{ij}) \quad (2)$$

where:

- $w_{c_j}$  is the confidence associated to criterion  $j$ ;
- $x_{ij}$  is the value of  $j$ th criterion for site  $i$ ;
- $\mu(\cdot)$  is the membership degree in a fuzzy set;
- The interval  $[a,b]$  for  $x$  is defined by ( $a_{ij}$  is the accuracy associated to criterion  $j$  for site  $i$ ):

$$a = \begin{cases} \min(D), & \text{if } x_{ij} - a_{ij} < \min(D) \\ x_{ij} - a_{ij}, & \text{if } x_{ij} - a_{ij} \geq \min(D) \end{cases}$$

$$b = \begin{cases} x_{ij} + a_{ij}, & \text{if } x_{ij} + a_{ij} < \max(D) \\ \max(D), & \text{if } x_{ij} + a_{ij} \geq \max(D) \end{cases} \quad (3)$$

$D$  is the variable domain.

The accuracy is given as a percentage of criterion value and confidence belongs to the unit interval. For example, an accuracy of 90% for slope means that each slope value belongs to the interval  $[a, b]$  where  $a_{ij} = 0.9 \times x_{ij}$ . On the other hand, a 0.9 confidence value means that we have a confidence on the  $[a, b]$  interval of 0.9.

As can be observed, this formulation enables dealing with imprecision in the collected input hazard maps. Moreover, it allows dealing with two types of imprecision: lack of accuracy in the data and lack of confidence in the data. We believe this is a simple and easy way to tackle the problematic of how to handle imprecision in data in dynamic decision models. It is one of the contributions proposed in this work.

3.4 Weighting functions

The second contribution is the proposal is to compute the weights using the following linear weighting function [10], [11]:

$$L(ac_{ij}) = \alpha \frac{1 + \beta ac_{ij}}{1 + \beta}, \text{ where } 0 \leq \alpha \leq 1 \text{ and } 0 \leq \beta \leq 1 \quad (4)$$

Where:  $ac_{ij}$  is the accuracy and confidence membership value of  $j^{th}$  criterion for site  $i$ .

The logic of these weighting functions is that the satisfaction value of a criterion should influence its assigned relative importance. For example, if we are buying a car and the price is a “very important” criterion, if the car is quite expensive the final decision result should be less than the simple multiplication of weight\*satisfaction value. We will not use quadratic weighting functions because it penalizes values near 1, contrarily to the linear function that presents a more smooth behaviour, which better fits our model [10], [11].

We consider the relative importance of criteria has different morphologies for each criterion, depending on each of the three phases defined. The definition of these weighting functions morphologies is given by parameters  $\alpha$  and  $\beta$ . The  $\alpha$  parameter provides the semantics for the weighting functions as follows:

- Very Important (VI=1);
- Important (I=0.8);
- Average importance (A=0.6);
- Low importance (L=0.4);
- Very Low importance (VL=0.2).

The  $\beta$  parameter provides the slope for the weighting functions, which will depend on the criterion at hand, with the logic that higher  $\beta$  means higher slope. For our case study this parameter has the following values:

- High (H=1);
- Medium (M=0.6667);
- Low (L=0.3333);
- Null (N=0);

As mentioned above the relative importance of criteria changes according to the altitude of the spacecraft, i.e. depends on the closeness to planet surface. We defined different weights for each sub-phase we are in, as described in the next two sub-sections, but these importances’s can be modified by the decision maker anytime.

3.4.1 Discussion of Criteria Weighting in case study

Phase 1 (altitude: > 1000m, <2000; historic set size 250)

This phase refers to altitudes between 1000m to 2000m and a historic size of 250 candidate alternatives. In Table 1 we show the proposed values for  $\alpha$  and  $\beta$  parameters for this phase, in Figure 2 we depict their respective plots, where Y-axis represents the weights and X-axis shows criteria satisfaction values within interval [0-1].

Table 1:  $\alpha$  &  $\beta$  parameters (Phase 1).

	Fuel Reach	Slope	Dist	Shad Text	ScIn		
$\alpha$	1	1	0.6	0.4	0.8	0.2	
$\beta$	0.25	0.667	0.111	0.333	0	0.333	0

For example, the rationale for “very important” is that although fuel, reachability and slope are all “very important” criteria for this phase, a lower satisfaction for criteria reachability should be quite penalized (e.g. X=0.25 corresponds to W=0.7, hence W(x)= 0.25\*0.7=> 0.175). Further,, fuel and slope also consider high penalties for lower satisfaction values, visible on their plot (Figure 2).

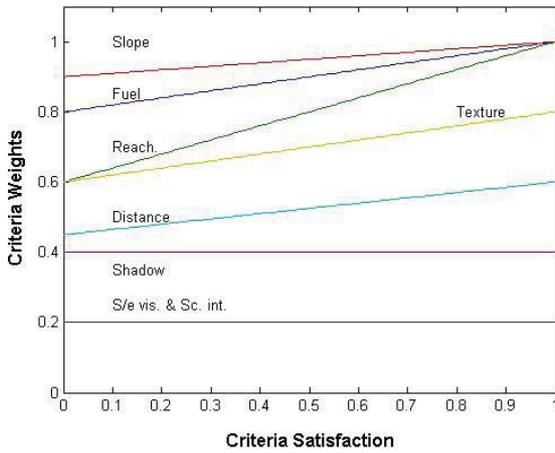


Figure 2: Weighting functions Phase I.

Phase II (altitude:  $\leq 1000m, >300m$ ; historic set size 10)  
 In Table 2 we depict the proposed values for  $\alpha$  and  $\beta$  parameters for this phase, in Figure 3 their respective plots.

Table 2:  $\alpha$  &  $\beta$  parameters (Phase II).

	Fuel	Reach	Slope	Dist	Shad	Text	ScIn
$\alpha$	0.6	1	1	0.6	0.4	0.8	0.2
$\beta$	0.5	0.667	0.111	0.333	0	0.333	0

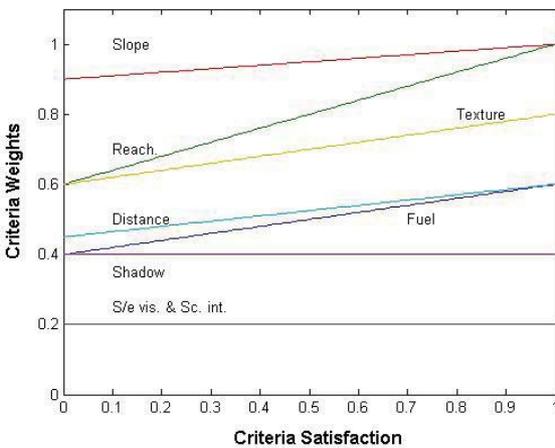


Figure 3: Weighting functions Phase II.

For example, in this phase the same rationale as in phase 1 applies for the two “very important” criteria, slope and reachability, although slope is less penalized for lower satisfaction of criteria (e.g. 0.8 weight for satisfaction 0.5 and weight 1 for satisfaction 1).

Phase III (altitude:  $\leq 300m$ ; historic set size 5)

In Table 3 we depict the proposed values for  $\alpha$  and  $\beta$  parameters for this phase, in Figure 4 their respective plots.

Table 3:  $\alpha$  &  $\beta$  parameters (Phase III).

	Fuel	Reach	Slope	Dist	Shad	Text	ScIn
$\alpha$	0.2	1	1	0.2	0.4	0.8	0.2
$\beta$	0	0.667	0.111	0	0	0.333	0

For example, in this phase scientific interest and shadow do not play an important role because the alternatives are focused on a small area. Further, it almost does not require fuel due to the surface proximity. Hence their weights are small and constant, 0.2 for fuel & scientific interest, and 0.4 for shadow (all flat functions not dependent on satisfaction of criterion).

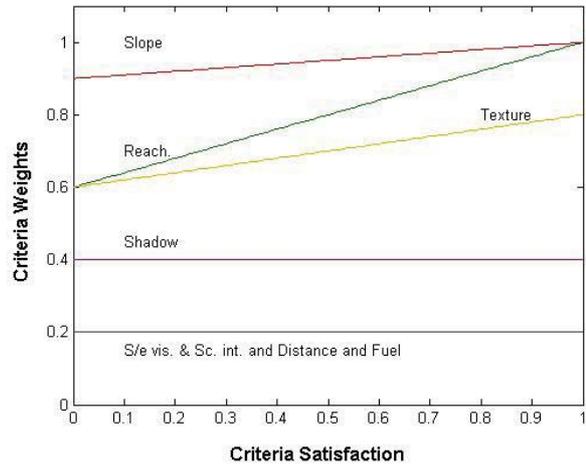


Figure 4: Weighting functions Phase III.

In summary, the relative importance proposed for each criterion followed the semantic rational:

- Slope relative importance (weight) in all phases is considered to be “very important”;
- Reachability relative importance (weight) in all phases is considered to be “very important”;
- Texture relative importance (weight) in all phases is considered to be “important”;
- Shadow relative importance (weight) in all phases is considered to be “low”;
- Distance relative importance (weight) in the first 2 phases is considered to be “average” and then it will change to “low” and “very low”;
- Scientific interest (Sc int.) relative importance (weight) in all phases is considered to be “very low”;
- Sun/Earth visibility (S/e vis.) relative importance (weight) in all phases is considered to be “very low”;
- Fuel relative importance (weight) in phase 1 is considered to be “very important”;
- Fuel relative importance (weight) in phase 2 is considered to be “medium”;
- Fuel weight in phase 3 is considered to be “very low”.

The rational above is expressed by the  $\alpha$  parameter, while the  $\beta$  parameter provides more or less penalties for lower satisfaction values in the criteria, as depicted in the figures for each phase. A simplification of the weighting functions could be to define “a priori” a slope for each importance i.e. avoids having to define different  $\beta$  parameters for each weighting functions. This option simplifies the weight assignment process but it is less accurate.

An important point to highlight is that the parameter values proposed for the phases (or iterations) can be tuned for any other dynamic decision application.

This section concludes the second proposed contribution of this work, i.e. contributions to deal with different criteria weights for steps of the dynamic decision process.

#### 4 Discussion

In Table 4 are depicted the best three alternatives on the last iteration of each phase using default weights (described in previous section). For instance, the best alternative on the end of the 2<sup>nd</sup> phase (iteration 25) has 3D coordinates: (86 , -27 , 12) and rating value of 0.7807.

We can observe that from 16th to the 25th iteration the best site is completely different. However, the 2nd and 3rd best sites seem to be on the same region on both iterations (50 meters south). Furthermore, the best and 2nd best sites from iteration 25 changed positions with each other on the 40th iteration.

Table 4 – Rating values and 3D coordinates of the best sites on the last iteration of each phase (default weights).

Iter.	3D coord (meters)	R <sub>n</sub>
16 (1 <sup>st</sup> Phase)	(51 , 29 , 35)	0.7833
	(65 , -0.4 , 33)	0.7801
	(94 , 26 , 35)	0.7798
25 (2 <sup>nd</sup> Phase)	(86 , -27 , 12)	0.7807
	(66 , -50 , 10)	0.7783
	(95 , -26 , 12)	0.7772
40 (3 <sup>rd</sup> Phase)	(79 , -53 , -6)	0.9955
	(86 , -53 , -6)	0.5119
	-	-

In table 5 we present the results regarding the best sites on the last iteration of the 2<sup>nd</sup> and 3<sup>rd</sup> phase, only this time changing the Distance criterion weight to Very High on both phases. For illustrative purposes we considered the distance towards the centre of image for computing the Distance Criterion.

Comparing the results of Table 5 with Table 4, we can observe that at iteration 25<sup>th</sup> the first result is exactly the same and the 2<sup>nd</sup> and 3<sup>rd</sup> too, but on a different order. The best site maintained is position very close to the centre of the image and the distance criterion is now considered to be very high. By the same reason, site (95 , -26 , 12) went to the 2<sup>nd</sup> position because it is must closer to the image centre than site (66 , -50 , 10)..

On the other hand, the 2 best sites on the 40<sup>th</sup> iteration are completely different on both Tables. The sites on Table 4 are not close to the centre of the image and when the Distance criterion is set to very high the model tends to rate higher sites closer to the centre of the image. Hence, the best sites on Table 5 are closer to centre of the image (see Figure 5).

It should be noted that the experiences discussed here were adaptations of tests done within a research project [7] [8]. Furthermore, here we do not discuss the results rating

values, because we are not addressing the aggregation methods used to evaluate the alternatives.

Table 5 – Rating and 3D coordinates of 3 best sites on last iterations of 2<sup>nd</sup> and 3<sup>rd</sup> phases (weight for Distance Criterion =Very high).

Iter.	3D coord (meters)	R <sub>n</sub>
25 (2 <sup>nd</sup> Phase)	(86 , -27 , 12)	0.7991
	(95 , -26 , 12)	0.7938
	(66 , -50 , 10)	0.7932
40 (3 <sup>rd</sup> Phase)	(88 , -31 , -3)	0.9979
	(78 , -30 , -3)	0.9977
	-	-

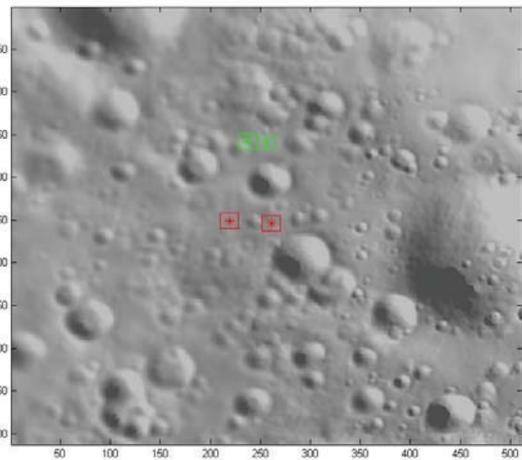


Figure 5 – Image of the surface on iteration 40. Green stars best sites from Table 4 and red stars from Table 5.

In Figure 5 we show the results, just discussed for the last iteration (40). It is quite notorious the influence of changing criterion weights for Distance, from average to very-high, on the last phase. Raising the importance of that criterion “forces” the final decision choice to be closer (shortest distance) to the initial target (center of image), even though the surface is more “bumpy” than the choices from Phase II.

#### 5 Conclusions

In this work we presented a general architecture to cope with dynamic multi-attribute decision problems. We focused our attention on two spatial-temporal aspects of the rating process, specifically proposing contributions for dealing with uncertainty in dynamically changing input data and also for dealing with dynamic changes in criteria importance. Potential applications of this work could be on medical diagnose decision support and/ or fault, detection and isolation (FDI) problems.

#### Acknowledgment

The authors wish to thank Yannick Devassoux from Astrium EADS for his assistance with understanding the case study domain. This work was partially funded by EADS Astrium contract 4572019617.

## References

- [1] Ribeiro, R.A., Fuzzy multiple attribute decision making: A review and new preference elicitation techniques. *Fuzzy Sets and Systems* 78(2): 155-181, 1996
- [2] E. Triantaphyllou, *Multi criteria decision making method: a comparative study*, Kluwer academic publishers, Applied optimization series, Vol. 44, 2002.
- [3] Chen, S.-J. and C.-L. Hwang. Fuzzy Multiple Attribute Decision Making, *Lecture Notes in Economics and Mathematical Systems*, Springer-Verlag, 1992.
- [4] Ross, T. (2004). *Fuzzy Logic with Engineering Applications*, John Wiley & Sons.
- [5] Townsend, J. T. & Busmeyer, J. Dynamic representation of decision making. In: R. Port and T. Van Gelder (Eds.), *Mind in Motion: Dynamics, Behavior and Cognition*. 4<sup>th</sup> chapter, MIT Press, 1995.
- [6] S. Khanmohammadi, R. A. Ribeiro, J. Jassbi. Multi criteria decision making using dynamics of criteria. Proceedings of the 11<sup>th</sup> Mediterranean Conference on Control and Automation (MED03), June 2003, med.ee.nd.edu/MED11/pdf/papers/t3-013.pdf
- [7] T. C. Pais, R.A. Ribeiro, Y. Devouassoux, S. Reynaud Dynamic ranking algorithm for landing site selection. In: Proceedings of the Int. Conference on Information Processing and Management of Uncertainty (IPMU'08), Malaga, June 2008.
- [8] Y. Devouassoux, S. Reynaud, G. Jonniaux, R. A. Ribeiro, T. C. Pais. Hazard avoidance developments for planetary exploration. In: Proceedings of the 7<sup>th</sup> International ESA Conference on Guidance, Navigation & Control Systems, Tralee, Ireland, 2-5 June 2008
- [9] Richardson, G. P. & Pugh III, A. L. *Introduction to System Dynamics Modeling with DYNAMO*. Portland, Oregon: Productivity Press, 1981.
- [10] R. Marques-Pereira, Rita A. Ribeiro. Aggregation with generalized mixture operators using weighting functions. *Fuzzy Sets and Systems*, 137 (2003) 43-58.
- [11] Rita A. Ribeiro, R. Marques-Pereira. Generalized mixture operators using weighting functions: a comparative study with WA and OWA. *European Journal of Operational Research*, 145 (2), March 1 (2003) 329-342.