Image reduction with interval-valued fuzzy sets and OWA operators

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Abstract— *In this paper we propose a generalization of Atanassov's operators and we prove that these generalized operators and OWA operators of dimension* 2 *provide the same numerical results. We apply Atanassov's operators to image compression and use different families of OWA operators in order to calculate the coefficient* α *of the Atanassov's operator.*

Keywords— Image compressing, OWA operators, Atanassov's operator, Interval-valued fuzzy set.

1 Introduction

Image compression consists of reducing the resolution of an image in order to display it, print it, or even speed up some other computations (see [10, 11, 18]). The target of image reduction algorithms is to reduce the size of the image with the smallest possible quality loss.

Atanassov's operator was given in 1983 ([1, 2]) allowing to associate a fuzzy set to each Atanassov's intuitionistic fuzzy set or interval-valued fuzzy set ([9, 12, 14]). Later, in 1988, Yager presented the Ordered Weighted Averaging (OWA) operators ([16])). Our paper is based on the idea that, under certain conditions, the numerical results of both operators are the same. For this reason, we define an operator generalizing Atanassov's operators, being an OWA operator of dimension 2([3, 4, 5]).

We apply this operator in image compression. In this way, we divide the image in blocks. We associate each block with an interval, getting an interval-valued fuzzy set associated with the image. We use the Atanassov's operator to obtain a single point from each interval. The result is a new (fuzzy) set representing the reduced image.

In order to calculate the α parameter of the Atanassov's operator, we focus on several families of OWA operators of dimension 2. We calculate the α coefficient according to the type of OWA operator, with the aim of getting different reductions from a single image.

This paper is organized in the following way. First, we start recalling some concepts related to interval-valued fuzzy sets and Atanassov's operator. In Section 3, we see the relation between OWA operators of dimension 2 and generalized Atannasov's operators. In Section 4 we present different methods to calculate α, based on different families of OWA operators. In Section 5 we propose an algorithm to compress images and in Section 6 we see different experimental results. We finish with some conclusions and references .

2 Preliminary definitions

We denote by L([0,1]) the set of all closed subintervals of the unit interval [0,1], that is,

$$L([0,1]) = \{\mathbf{x} = [\underline{x}, \overline{x}] | (\underline{x}, \overline{x}) \in [0,1]^2 \text{ and } \underline{x} \leq \overline{x}\}.$$

L([0,1]) is a partially ordered set with respect to the order relationship $\leq_L$ defined in the following way: given **x**,**y** $\in$ L([0,1])

$$\mathbf{x} \leq_L \mathbf{y} \text{ if and only if } \underline{x} \leq \underline{y} \text{ and } \overline{x} \leq \overline{y}.$$

With this order relationship, (L([0,1]),$\leq_L$) is a complete lattice ([7, 8, 3, 4]), where the smallest element is $0_L = [0, 0]$ and the largest is $1_L = [1, 1]$.

Definition 1. *An interval-valued fuzzy set (IVFS) A on the universe* $U \neq \emptyset$ *is a mapping* $A : U \to L([0, 1])$.

M_A(u)=[$\underline{A}$(u),$\overline{A}$(u)] $\in$ L([0,1]) is the membership degree of u $\in$ U, with $\underline{A}$(u), $\overline{A}$(u) $\in$ [0,1] denoting the lower bound and the upper bound respectively of the membership associated to u.

In 1983, Atanassov proposed an operator to associate a fuzzy set with each interval-valued fuzzy set (see [1, 2]). This operator associates each interval with a point.

Definition 2. *Let* $\alpha \in [0, 1]$. *The Atanassov's operator* K_α *is a mapping* $K_\alpha : L([0, 1]) \to [0, 1]$ *defined by*

1. $K_0(x) = \underline{x}$ *for all* $x \in L([0, 1])$,
2. $K_1(x) = \overline{x}$ *for all* $x \in L([0, 1])$,
3. $K_\alpha(x) = K_\alpha([K_0(x), K_1(x)]) = K_0(x) + \alpha(K_1(x) - K_0(x)) = \underline{x} + \alpha(\overline{x} - \underline{x})$ *for all* $x \in L([0, 1])$.

To generalize this operator, the following definition was proposed in [4]:

Definition 3. *Let* $\alpha \in$ *[0,1]. An operator* D_α *is a mapping* $D_\alpha : L([0, 1]) \to [0, 1]$ *such that it satisfies the following conditions:*

1. *If* $\underline{x} = \overline{x}$, *then* $D_\alpha(\boldsymbol{x}) = \underline{x}$,
2. $D_0(\boldsymbol{x}) = \underline{x}$, $D_1(\boldsymbol{x}) = \overline{x}$ *for all* $\boldsymbol{x} \in L([0, 1])$,
3. *If* $\boldsymbol{x} \leq_L \boldsymbol{y}$ *with* $\boldsymbol{x}, \boldsymbol{y} \in L([0, 1])$, *then* $D_\alpha(\boldsymbol{x}) \leq D_\alpha(\boldsymbol{y})$,
4. $D_\alpha([0, 1]) = \alpha$ *for any* $\alpha \in [0, 1]$.

Example 1.

$$D_\alpha([\underline{x}, \overline{x}]) = \begin{cases} \overline{x} & if\, \overline{x} \leq \alpha \\ \underline{x} & if\, \underline{x} \geq \alpha \\ \alpha & otherwise \end{cases}$$

Proposition 1. *For any* $\alpha \in [0, 1]$, K_α *is a* D_α *operator.*

Next, we propose a theorem to construct D_α operators using one-variable real functions.

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Theorem 1. *Let $\alpha \in [0,1]$ and let $f : [0,1] \to [0,1]$ be a continuous and strictly increasing function. Then the operator*

$$D_\alpha : L([0,1]) \to [0,1] \text{ given by}$$
$$D_\alpha(\boldsymbol{x}) = f^{-1}\Big(pf(\underline{x}) + (1-p)f(\overline{x})\Big)$$

with $p = \frac{f(1)-f(\alpha)}{f(1)-f(0)}$, is a D_α operator in the sense of Definition 3.

Proof:

1. If $\underline{x} = \overline{x}$, then $D_\alpha(\mathbf{x}) = f^{-1}(pf(\underline{x}) + (1-p)f(\underline{x})) = = f^{-1}(f(\underline{x})) = \underline{x}$.
2. If $\alpha = 0$, then $p = 1$. In these conditions $D_0([\underline{x}, \overline{x}]) = = f^{-1}(f(\underline{x})) = \underline{x}$.
   If $\alpha = 1$, then $p = 0$ and therefore $D_1([\underline{x}, \overline{x}]) = \overline{x}$.
3. If $[\underline{x}, \overline{x}] \leq_L [\underline{y}, \overline{y}]$, as f is continuous and strict, we have that $D_\alpha([\underline{x}, \overline{x}]) = f^{-1}(pf(\underline{x}) + (1-p)f(\overline{x})) \leq f^{-1}(pf(\underline{y}) + (1-p)f(\overline{y})) = D_\alpha([\underline{y}, \overline{y}])$.
4. If $\mathbf{x} = [0,1]$, then $D_\alpha([0,1]) = f^{-1}(p(f(0) + (1-p)f(1)) = f^{-1}(\frac{f(0)(f(1)-f(\alpha)) - f(1)(f(1)-f(\alpha)) + f(1)(f(1)-f(0))}{f(1)-f(0)}) = f^{-1}(\frac{f(\alpha)(f(1)-f(0))}{f(1)-f(0)}) = \alpha$.

Remark: In this paper, we take $f(x) = x$. Under this condition, by Theorem 1, we have that

$$D_\alpha([\underline{x}, \overline{x}]) = \underline{x} + \alpha(\overline{x} - \underline{x}) = K_\alpha([\underline{x}, \overline{x}])$$

3 K_α and OWA operators

In [16], Yager introduced the Ordered Weighted Aggregation Operator (OWA operator) in the following way:

Definition 4. *A mapping $F : [0,1]^n \to [0,1]$ is called an OWA operator of dimension n if there exists a weighting vector W,$W = (w_1, w_2, \ldots, w_n) \in [0,1]^n$ with $\sum_i w_i = 1$ and such that*

$$F(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j$$

with b_j the j-th largest of the a_i.

In the original definition, Yager considered the OWA operators as a mapping from the whole euclidean space $\mathbb{R}^n$ to $\mathbb{R}$. However, for us it is better to reduce the domain only to $[0,1]^2$.

There exist three important special cases of OWA operators that coincide with well known aggregation functions, as we can see in [17].

1. F^*: The weighting vector, denoted as W^*, is defined as $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$.
   $F^*(a_1, \ldots, a_n) = max\{a_1, \ldots, a_n\}$
2. F_*: The weighting vector, denoted as W_*, is defined as $w_n = 1$ and $w_j = 0$ for all $j \neq n$.
   $F_*(a_1, \ldots, a_n) = min\{a_1, \ldots, a_n\}$.
3. F_A: The weighting vector, denoted as W_A, is defined as $w_j = 1/n$ for all $j \in 1, \ldots, n$.
   $F_A(a_1, \ldots, a_n) = \frac{1}{n}\sum_{i=1}^{n} a_i$.

If we focus on two-dimensional OWA operators using as weighting vector $W = (\alpha, 1-\alpha)$, we can think of applying it to the bounds of an interval. In this case, the numerical result of applying the OWA operator on the bounds of an interval and the operator D_α acting on that interval is the same. While D_α operator acts over elements in L([0,1]), the domain of OWA operators is [0,1]x[0,1]. For this reason, OWA operators acting on the unit square require an ordering operation to ensure the elements to be the extremes of an interval defined in L([0,1]). That's the reason why we need the following theorems to study the relationship between these operators.

We define a new operator $\mathbb{D}_\alpha$ by composing the D_α operator with the map

$$\begin{array}{rcl} i : [0,1]^2 & \to & L([0,1]) \\ (x,y) & \to & [min(x,y), max(x,y)] \end{array}$$

Theorem 2. *1. Let $\alpha \in [0,1]$ and $\mathbb{D} = D_\alpha \circ i$ where D_α is the operator given in Definition 3. Then, if F is the OWA operator (of dimension 2) defined by the weighting vector $W = (\alpha, 1-\alpha)$, we have that*
$\mathbb{D}_\alpha(x,y) = F(x,y)$ for all $x, y \in [0,1]$.

2. Let F be an OWA operator (of dimension 2) with weighting vector $W = (w_1, w_2)$. Then for any $(x,y) \in [0,1]^2$ we have that
$F(x,y) = \mathbb{D}_\alpha(x,y)$, with $\alpha = w_1$.

4 Calculation methods for α coefficient

In this section, we study how to calculate the α coefficient basing on different families of OWA operators. First, we study two measures defined in [17] and associated with any OWA operator: the orness and the dispersion.

Definition 5. *Let F be an OWA operator and W its weighting vector. The orness measure is defined as*

$$orness(F) = \frac{1}{(n-1)}\sum_{i=1}^{n}(n-i)w_i.$$

With this definition it is easy to see that $orness(F^*) = 1$, $orness(F_*) = 0$ and $orness(F_A) = 0.5$.

Proposition 2. $orness(\mathbb{D}_\alpha) = \alpha$

Proof:

$$orness(\mathbb{D}_\alpha(x)) = \frac{1}{2-1}\sum_{i=1}^{2}(2-i)w_i = w_1 = \alpha$$

Definition 6. *Let F be an OWA operator and W its weighting vector. The dispersion measure is defined as*

$$Disp(F) = -\sum_{i=1}^{n} w_i ln w_i.$$

Proposition 3. $Disp(\mathbb{D}_\alpha) = \alpha ln(\frac{1-\alpha}{\alpha}) + ln(1-\alpha)$

Proof:

$$Disp(\mathbb{D}_\alpha) = -\textstyle\sum_{i=1}^{2} w_i ln w_i =$$

$$= \alpha(ln(1-\alpha) - ln(\alpha)) - ln(1-\alpha) =$$

$$= \alpha ln(\tfrac{1-\alpha}{\alpha}) + ln(1-\alpha)$$

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Next, it is shown the relation between the coefficient α of the operator $\mathbb{D}_\alpha$ and some families of OWA operators.

Definition 7. *F is called a ME-OWA operator if F is an OWA operator such that given a desired value of orness β, it maximizes the dispersion (entropy). In particular we solve the following problem:*

$$\begin{array}{ll} \textit{Max} & -\sum_{i=1}^n w_i ln w_i \\ \textit{subject to} & \beta = 1/(n-1)\sum_{i=1}^n (n-i)w_i \\ \textit{where} & \sum_{i=1}^n w_i = 1,\ w_i \in [0,1]. \end{array}$$

Proposition 4. *The operator $\mathbb{D}_\alpha$ is a ME-OWA for all $\alpha \in [0,1]$, with $orness(\mathbb{D}_\alpha) = \alpha$.*

Definition 8. *F is called a generalized S-OWA operator if F is an OWA operator where*

$$\begin{array}{lcll} w_1 & = & a + \frac{1}{n}(1-(a+b)), & \\ w_i & = & \frac{1}{n}(1-(a+b)), & \textit{with } i = 2,\ldots,n-1, \\ w_n & = & b + \frac{1}{n}(1-(a+b)). & \end{array}$$

with $a, b \in [0,1]$ and $a + b \leq 1$.

Proposition 5. *Let $a, b \in [0,1]$ such that $a+b \leq 1$. If we take*

$$\alpha = \frac{1+a-b}{2}$$

then $\mathbb{D}_\alpha$ is a generalized S-OWA operator.

Definition 9. *F is called a BADD-OWA operator if F is an OWA operator where*

$$w_i = \frac{b_i^\beta}{\sum_{j=1}^n b_j^\beta}$$

with $\beta \geq 0$ and being b_i as in Definition 4.

Proposition 6. *Let $\beta \geq 0$. If we take*

$$\alpha = \frac{(\mathbb{D}_1(x,y))^\beta}{(\mathbb{D}_0(x,y))^\beta + (\mathbb{D}_1(x,y))^\beta}$$

with $x, y \in [0,1]$, then $\mathbb{D}_\alpha$ is a BADD-OWA operator.

Definition 10. *Let $\beta \geq 0$. F is called a modified BADD-OWA operator if F is an OWA operator where*

1. $w_i = \frac{(1/b_i)^\beta}{\sum_{j=1}^n (1/b_j)^\beta}$ *or*
2. $w_i = \frac{(1-b_i)^\beta}{\sum_{j=1}^n (1-b_j)^\beta}$ *or*
3. $w_i = \frac{1}{n-1}(1 - \frac{b_i^\beta}{\sum_{j=1}^n b_j^\beta})$ *or*
4. $w_i = \frac{(b_{n-i+1})^\beta}{\sum_{j=1}^n b_j^\beta}$

being b_i as in Definition 4.

Proposition 7. *Let $\beta \geq 0$. The following items are satisfied:*

1. *If we take*

$$\alpha = \frac{(1/\mathbb{D}_1(x,y))^\beta}{(1/\mathbb{D}_1(x,y))^\beta + (1/\mathbb{D}_0(x,y))^\beta} =$$

$$= 1 - \frac{(\mathbb{D}_1(x,y))^\beta}{(\mathbb{D}_1(x,y))^\beta + (\mathbb{D}_0(x,y))^\beta} = \frac{(\mathbb{D}_0(x,y))^\beta}{(\mathbb{D}_1(x,y))^\beta + (\mathbb{D}_0(x,y))^\beta}$$

then $\mathbb{D}_\alpha$ is a modified BADD-OWA operator in the sense of items 1, 3 y 4 of Definition 10.

2. *If we take*

$$\alpha = \frac{(1-\mathbb{D}_1(x,y))^\beta}{(1-\mathbb{D}_1(x,y))^\beta + (1-\mathbb{D}_0(x,y))^\beta}$$

then $\mathbb{D}_\alpha$ is a modified BADD-OWA operator in the sense of item 2 of Definition 10.

Proof:

1. Obviously, taking into account items 1,3 and 4 of Definition 10 and that $\mathbb{D}_\alpha$ is an OWA operator of dimension 2.
2. Direct □.

5 Image reduction

We consider an image Q as a $N \times M$ matrix. Each coordinate of the pixels in the image Q is denoted as (i,j). The intensity or gray level of the pixel located in (i,j) is represented as q_{ij}, with $0 \leq q_{ij} \leq L-1$ for each $(i,j) \in Q$.

In our approach to image reduction we use interval-valued fuzzy sets and D_α operator. It has been shown (see [13, 6, 15]) that interval-valued fuzzy sets used in images allow the development of algorithms in several topics as edge detecion, contrast or thresholding with very good results. We propose the following algorithm:

1. Divide the image Q in blocks of size $n \times n$. If M or N are not multiple of n, we delete the minimum number of rows/columns in the boundary of the image until the new size of the image satisfies the property.
2. Associate each block with an interval in the following way: the lower bound of the interval is given by the minimum of the intensities in the block and the upper bound by the maximum.
3. Choose the α parameter in the operator D_α.
4. Associate each interval with the number obtained after applying the operator D_α.

Example: Let Q be a matrix of dimension 6×6 and let $n = 3$

$$\begin{pmatrix} q_{1,1} & q_{1,2} & q_{1,3} & q_{1,4} & q_{1,5} & q_{1,6} \\ q_{2,1} & q_{2,2} & q_{2,3} & q_{2,4} & q_{2,5} & q_{2,6} \\ q_{3,1} & q_{3,2} & q_{3,3} & q_{3,4} & q_{3,5} & q_{3,6} \\ q_{4,1} & q_{4,2} & q_{4,3} & q_{4,4} & q_{4,5} & q_{4,6} \\ q_{5,1} & q_{5,2} & q_{5,3} & q_{5,4} & q_{5,5} & q_{5,6} \\ q_{6,1} & q_{6,2} & q_{6,3} & q_{6,4} & q_{6,5} & q_{6,6} \end{pmatrix}$$

Then, the interval-valued fuzzy set associated with Q is formed by 4 elements:

$$\begin{pmatrix} \left[\bigwedge_{\substack{i=1,2,3\\j=1,2,3}} q_{i,j} \quad \bigvee_{\substack{i=1,2,3\\j=1,2,3}} q_{i,j}\right] & \left[\bigwedge_{\substack{i=1,2,3\\j=4,5,6}} q_{i,j} \quad \bigvee_{\substack{i=1,2,3\\j=4,5,6}} q_{i,j}\right] \\ \left[\bigwedge_{\substack{i=4,5,6\\j=1,2,3}} q_{i,j} \quad \bigvee_{\substack{i=4,5,6\\j=1,2,3}} q_{i,j}\right] & \left[\bigwedge_{\substack{i=4,5,6\\j=4,5,6}} q_{i,j} \quad \bigvee_{\substack{i=4,5,6\\j=4,5,6}} q_{i,j}\right] \end{pmatrix}$$

Remark: Notice that the symbols $\wedge$ and $\vee$ stand for minimum and maximum respectively.

Once the interval-valued fuzzy set associated with the image has been obtained, we build a fuzzy set. Applying the operator D_α to each interval we get the reduced image of size 2×2.

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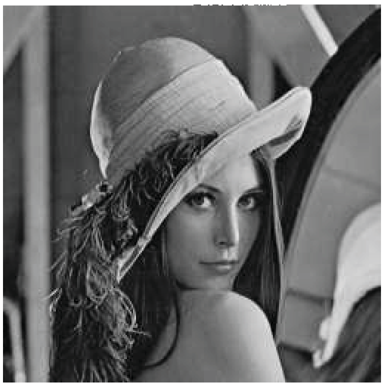

Figure 1: Original Image Lena

Figure 2: Original image Cameraman

6 Experimental results

In this section we use the algorithm proposed in Section 5. Using the relation between the operator D_α and OWA operators of dimension 2, we propose several methods of calculating the α coefficient based on families of OWA operators.

We get two different situations according to the OWA operator used. In the first one, the value of α is constant for the whole image. This situation happens with ME-OWA operators and generalized S-OWA operators, due to their definition. In the second one, the value of α depends on the bounds of each interval, so its value varies for each block. This happens when we take BADD-OWA operators and modified BADD-OWA operators.

The tests of this section have been made with the image Lena (Figure 1) and Cameraman (Figure 2). The size of the block is $n = 3$ (submatrices 3×3). In this way, the size of the reduced image is 9 times smaller than the original.

6.1 *Calculation with constant α*

In this section we use ME-OWA operators and generalized S-OWA to calculate a constant value of α for the whole image. Once the value has been calculated, we apply the D_α operator to each interval in the image.

6.1.1 *Reduction with ME-OWA operators*

As we have seen in Section 4, the construction of ME-OWA operators is direct. For this reason, we analyze three specific cases: $\alpha = 0$, $\alpha = 0.5$ and $\alpha = 1$.

With $\alpha = 0$, we associate the lower bound of the interval to each block. With $\alpha = 0.5$, we take the mean point of the interval. Finally, with $\alpha = 1$, we associate the upper bound of the interval.

Obviously, the higher the value of α, the higher the membership degree and therefore the intensity of the image. The image is darker with $\alpha = 0$ than with $\alpha = 0.5$, which is also darker than with $\alpha = 1$.

$\alpha = 0$ $\qquad$ $\alpha = 0.5$ $\qquad$ $\alpha = 1$

Figure 3: Reduction using ME-OWA operators

6.1.2 *Reduction with generalized S-OWA operator*

To construct a generalized S-OWA operator, it is necessary, as we have seen in Definition 8, the election of two parameters $a, b \in [0, 1]$ such that $a + b \leq 1$. If we focus on the contribution of the parameters in the S-OWA operator, we can study several possibilities:

1. If $a = b$, then we have that $\alpha = 0.5$, getting the average of the bounds of the interval.
2. If $a = 1$ and $b = 0$, we get $\alpha = 1$.
3. If $a = 0$ and $b = 1$, we get $\alpha = 0$.
4. If $a + b < 1$ and $a \neq b$, when $a > b$, the upper bound of the interval is predominant over the lower bound, while when $a < b$, the importance is given to the lower.

Cases 1, 2 and 3 have been already analyzed in Section 6.1.1, so we focus in case 4. For this, we start taking $a = 0.5, b = 0.25$. As $a > b$, the importance is given to the upper bound of the interval, getting a value of $\alpha = 0.625$. If we increase the value of a, then α tends to 1. For example, if we take $a = 0.9, b = 0$, then $\alpha = 0.95$. With this value of α we get a reduced image very similar to the images studied in the last column of Figure 3. On the other side, if we take $a = 0.25, b = 0.5$, we get a value of $\alpha = 0.375$. If we increase the value of b, $\alpha \to 0$. We can see this if we take $a = 0, b = 0.9$ ($\alpha = 0.05$). Notice that as α increase, the image becomes lighter.

6.2 *Calculation with variable α*

In this section, the value of α is calculated by means of the bounds of the interval. This means that the value of α is variable. For this reason we use BADD-OWA and modified BADD-OWA operators.

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a=0,b=0.9 $\alpha = 0.05$ | a=0.25,b=0.5 $\alpha = 0.375$ | a=0.5,b=0.25 $\alpha = 0.625$ | a=0.9,b=0 $\alpha = 0.95$

Figure 4: Reduction using generalized S-OWA operators

6.2.1 Reduction with BADD-OWA operators

As we see in Definition 9, to construct a BADD-OWA operator it is necessary to select a value of $\beta \geq 0$. We know that if we take $\beta = 0$, then we get $\alpha = 0.5$, already studied in Section 6.1.1. In other case, if we take $\beta = 1$, the value of α is calculated as follows:

$$\alpha = \frac{\overline{x}}{\overline{x} + \underline{x}}$$

being $[\underline{x}, \overline{x}] \in L([0,1])$ the interval representing each block. We also know that $\alpha \geq 0.5$, and if we increase the value of β, $\alpha \to 1$. That is, the result tends to the upper bound of the interval. For this reason, with a high value of β, we get reduced images similar as the images analyzed in third column of Figure 3.

$\beta = 1$ | $\beta = 10$ | $\beta = 20$ | $\beta = 100$

Figure 5: Reduction using BADD-OWA operators

6.3 Reduction with modified BADD-OWA operators

If we use modified BADD-OWA operators, bearing in mind items 1 and 2 of Proposition 7, we can take two different expressions for α. As we have analyzed in BADD-OWA operators, the value of α depends on the interval and on the β parameter. As in Section 6.2.1, if $\beta = 0$, then $\alpha = 0.5$.

Under conditions of item 1 of Proposition 7 and taking $\beta = 1$, the value of α is as follows:

$$\alpha = \frac{\underline{x}}{\underline{x} + \overline{x}}$$

being $[\underline{x}, \overline{x}] \in L([0,1])$ the interval representing each block. In this case, $\alpha \leq 0.5$ and as β increases, $\alpha \to 0$ when $\beta \to \infty$ (notice that the image becomes darker as β increases and α decreases).

$\beta = 1$ | $\beta = 10$ | $\beta = 20$ | $\beta = 100$

Figure 6: Reduction using modified BADD-OWA operators (item 1)

If we base on item 2 of Proposition 7, for $\beta = 1$, α is as follows:

$$\alpha = \frac{1 - \overline{x}}{(1 - \overline{x}) + (1 - \underline{x})}$$

being $[\underline{x}, \overline{x}] \in L([0,1])$ the interval representing each block. In this case, the value of α also decreases when β increases.

$\beta = 1$ | $\beta = 10$ | $\beta = 20$ | $\beta = 100$

Figure 7: Reduction using modified BADD-OWA operators (item 2)

7 Conclusions and future research

In this paper, we have proved that we can obtain OWA operators of dimension 2 from Atanassov's operators. This fact has allowed us to use several families of OWA operators to get D_α operators for image reduction.

Some future lines of research can be:

1. Compare the different reductions obtained according to the value of α.

2. Compare our results with other reduction algorithms.

3. Consider different methods of reconstruction of the original image from the reduced one. Analyze which of the reduced images leads to the best reconstructed image.

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