

# Fuzzy Logic Applications in Wireless Communications

Maria Erman<sup>1</sup> Abbas Mohammed<sup>2</sup> Elisabeth Rakus-Andersson<sup>3</sup>

<sup>1,2</sup>Department of Signal Processing, Blekinge Institute of Technology  
S-37225 Ronneby, Sweden

<sup>3</sup>Department of Mathematics and Science, Blekinge Institute of Technology  
S-37179 Karlskrona, Sweden

Email: maria.erman@bth.se, abbas.mohammed@bth.se, elisabeth.andersson@bth.se

**Abstract**— A survey of fuzzy logic applications and principles in wireless communications is presented, with the aim of highlighting successful usage of fuzzy logic techniques in applied telecommunications and signal processing. To the best of our knowledge, this is the first such study of its kind. This paper will focus firstly on discerning prevalent fuzzy logic or fuzzy-hybrid approaches in the areas of channel estimation, channel equalization and decoding, and secondly outlining what conditions and situations for which fuzzy logic techniques are most suited for these approaches. Furthermore, after insights gained from isolating fuzzy logic techniques applied to real problems, this paper proposes areas for further research targeted to practice-oriented researchers.

**Keywords**— Channel Equalization, Channel Estimation, Decoding, Fuzzy-hybrid systems, Fuzzy Logic, Wireless Communications

## 1 Introduction

Fuzzy Logic has been successfully applied in various areas pertaining to wireless communication systems. As fuzzy logic is used to model systems and situations, taking into consideration uncertainty and ambiguity, it can be an efficient tool to be utilized in problems for which knowledge of all factors is insufficient or impossible to obtain. Methods furnished with fuzzy logic have been shown to be useful in difficult conditions with respect to non-linear and time-variant systems. Additionally, the often mentioned advantages of using fuzzy logic in practical applications is to reduce complexity as well as to add robustness to the system under study.

Fuzzy logic and, more specifically, fuzzy control traditionally incorporates human expert knowledge into a rule-based framework. It may, however, be further expanded with learning algorithms to derive the fuzzy control parameters from sample data. These parameters may be obtained by combining fuzzy logic with related soft computing disciplines such as, e.g., neural networks, evolutionary computation techniques etc. On the other hand, a method developed by Wang and Mendel [1] derives the fuzzy rule base by using a combination of human experience and numerical data.

Wireless communications is a rapidly evolving industry, constantly challenging researchers for new techniques in order to meet the demands of ever higher performance and efficiency. The most obvious products of wireless communications, e.g., the worldwide adoption of the mobile telephone, wireless local area networks etc., exert a strong influence on many people's lives today.

In a wireless communication system, the *channel* is the medium by which information-bearing signals are transferred

from a transmitter to a receiver. The characteristics of the channel are generally unknown, and, barring any distortion imposed by the channel, i.e., in ideal conditions, the transmitted data will be received without any errors.

In practice, however, it is unavoidable for the channel to be affected by distortion, hence degrading the performance of the receiver, severely limiting the throughput of the system. The wireless channel poses tough challenges for achieving reliable and fast transfers. While interference typically is not a major concern in wired transmission, i.e., in predicting the behavior of the signal in the transmission channel, it poses a great challenge in wireless transmission.

When there is no line-of-sight between the transmitter and the receiver, distortions to the signal in the form of effects such as scattering and reflections, etc. will follow, all of them resulting in a phenomenon called multipath propagation. Due to multipath propagation, the receiver encounters many signal paths from the transmitter, where each of these paths is delayed by an arbitrary amount and attenuated by various factors. As a result there will be a superposition of the different copies of the signal being in different phases, hence causing an amplification or attenuation of the signal power – also referred to as fading. Multipath propagation will cause previously sent data bits to smear into current data bits, referred to as intersymbol interference. The aim of the receiver in the communication system is to overcome the disturbances of the channel, intersymbol interference and noise, and correctly decode the data having been transmitted.

## 2 Contemporary Uses of Fuzzy Logic in Wireless Communications

In this section we will focus on fuzzy logic applications in channel estimation, channel equalization and decoding. The purpose of channel estimation is to accurately describe the channel and track its variations, and with the aid of channel equalization and decoding recover the original transmitted data. In the case of time-varying channels, adaptive techniques have to be employed and it is in this area that fuzzy techniques and/or neural networks find their main uses.

### 2.1 Channel Estimation

In [2] channel estimation is performed by tracking the channel coefficients, applying a fuzzy tracking method in a multipath fading Code Division Multiple Access (CDMA) [3] channel. CDMA is a spread-spectrum technology that makes it possible for transmitters to share the same frequency range. The

fuzzy tracking method is based on Kosko's fuzzy associative memory models [4]. The fuzzy associative memory models combines fuzzy logic and a single-layer feed-forward neural network that saves the fuzzy logic rule-base in matrix form. The tracking used in [2] is iterative with the estimated symbol being used in the prediction of the coefficient. There are two inputs to the fuzzy tracker: the difference between the measured and predicted coefficients, and the change of difference between current and previous differences between the measured and predicted coefficients. The output of the fuzzy tracker yields a correction term for the next coefficient.

The motivation in [2] for using fuzzy channel estimation is due to fuzzy estimation not needing exact process models. Comparisons are made between the fuzzy tracker and a non-fuzzy tracker, a.k.a. the alpha tracker, where it is shown that the fuzzy tracker performs better under noisier multipath conditions.

Channel estimation using a fuzzy approach has also been performed with a multi-carrier modulation technique called Orthogonal Frequency Division Multiplexing (OFDM) [5]. In OFDM, multiple orthogonal subcarriers are used for the same channel. The data stream is divided into lower bit-rate data streams, each modulating a separate subcarrier.

There are two different methods for estimating the channel parameters at each subcarrier: blind channel estimation techniques and pilot assisted channel estimation, with the pilot being a reference signal used by the transmitter and the receiver. The blind channel estimation techniques do not use pilot samples and are thus more spectrally efficient, but at the cost of higher computational complexity and slower convergence rate. Pilot assisted channel estimation has typically been based either on the Minimum Mean-Square-Error (MMSE), the LS (Least Square) or the LMS (Least Mean-Square) algorithms [6], with the MMSE algorithm being more robust and performing better in time-varying channels.

In [7] a Takagi-Sugeno-Kang (TSK) [8] model is used for the pilot assisted channel estimation in an OFDM system. The TSK method is similar to that of the traditional Mamdani method – the difference being that the output of the TSK model is a linear function of the input variables instead of fuzzy sets. Pilot symbols are used in order to train the TSK fuzzy model as well updating the fuzzy model in order to track the channel. A Gaussian membership function is chosen, as it has been shown that a fuzzy system with bell-shaped Gaussians can approximate any continuous functions on compact sets to any degree of accuracy [9]. The TSK learning algorithm consists of defining the center and width of the rules dependent on the number of rules chosen, after which an adjustable parameter of the TSK model is trained from the first snapshot of the pilot subcarriers. Finally the channel transfer function is estimated and the adjustable parameter updated to track the channel. Simulation results show that the proposed TSK channel estimation model performs closely to the ideal MMSE, but with lower computational complexity.

The TSK fuzzy modeling technique used in [7] has been updated for a multiple-input multiple-output (MIMO) OFDM system in [10] with two transmit and two receive antennas. MIMO [11] is a way to increase system throughput without the need for higher transmit power or bandwidth, and has hence become a highly popular research topic. MIMO

achieves increased throughput by using multiple antennas at the transmitter and the receiver. It is shown in [10] with computer simulations that the Word Error Rate (WER) is close to the MMSE method with lower computational complexity.

In [12] an MMSE linear receiver was proposed in which a fuzzy inference system was inserted into the LMS algorithm. The motivation to use fuzzy logic was for convergence and stability reasons. The LMS algorithm was modified with a fuzzy logic controlled adaptive step size and partial update. This modified algorithm was then used in simulations of noise cancellation in a space-time joint direct-sequence (DS) CDMA system in a dynamic fading multipath channel. It was shown that the performance of the authors' modified LMS algorithm was superior to that of the LMS algorithm.

The proposed algorithm in [12] was further used in [13] for channel estimation and tracking in OFDM systems for a time-variant channel. Simulation results indicated that the modified LMS algorithm had lower steady-state Mean-Square Error (MSE) and faster convergence speed compared to the ordinary LMS algorithm.

In [14], an adaptive neuro-fuzzy inference system (ANFIS) was evaluated for channel estimation in OFDM systems. The ANFIS uses a hybrid learning algorithm based on the LS and the gradient descent methods in order to train the parameters of the membership functions of a TSK fuzzy inference system. Clustering is used in order to group data and from this generate the TSK fuzzy rule-base. From the results of the computer simulations performed in [14] it can be seen that the ANFIS performs very closely to that of the MMSE algorithm albeit with less computational complexity.

## 2.2 Channel Equalization

Channel equalization is the process of removing the degradation caused by the channel with the aim of reconstructing the transmitted data. The wireless channel is time-variant, and in this kind of channel, non-linear distortion is usually encountered. Due to linear equalization not performing very well in these channels, efficient equalization should be both adaptive and non-linear.

The first results that seem to appear in the literature with regards to fuzzy logic applications in channel equalization are [15, 16]. Here fuzzy adaptive filters based on both the LMS and the Recursive Least Squares (RLS) algorithms are constructed and applied to channel equalization. Initially the fuzzy sets are defined over the filter input space, after which linguistic information from human experts and numerical data are combined and incorporated into the filter. The algorithms are then used to update the free parameters. The objective of using fuzzy adaptive filters is to improve the adaptation speed of the algorithms with the extra help of linguistic inference rules. The results indicate that the bit error rates of the fuzzy equalizer is close to that of the optimal equalizer.

Whereas Wang and Mendel in [15, 16] assumed a fixed delay, in [17] the delay and the membership functions are derived from training data. Two membership functions and two rules are defined that correspond to the input of the channel, which is binary, i.e. it takes on two different values. Together they form an output grounded on the conclusion of a fuzzy rule, from which an independent decision results. Test symbols are transmitted and the correlation between the de-

sired outputs and the received signals are determined and form weights assigned to fuzzy outputs. A weighted sum of all the rules is used in the fuzzy inference, after which the defuzzified value is fed through a thresholding device for the final decision of the equalizer. The results from the simulations performed in [17] show that the fuzzy logic equalizer outperforms the LMS in non-linear channels as well as a neural network equalizer using the backpropagation algorithm. The fuzzy logic equalizer also needs fewer training samples compared to the LMS in linear channels for the same error performance.

The Wang-Mendel RLS Fuzzy Adaptive Filter [16] is extended in [18] to a complex fuzzy filter that can handle complex channel models and signals. In [19] human expert knowledge and heuristic reasoning are replaced altogether by a Multilayer Perceptron (MLP) preprocessor unit. The MLP unit consists of a 3-layer network, the training of which provides information to the fuzzy logic system. The LMS algorithm, less computationally complex than the RLS algorithm, is then used to update the free parameter of the system.

In [20] a Bayesian equalization architecture has been developed by using a fuzzy adaptive filter construction as in [15]. The adaptive equalization is visualized as a classification problem in which an observation vector is mapped to signal constellations. In contrast to a channel equalizer such as the maximum likelihood sequence estimation (MLSE) there is no need to include a channel estimator, thus making the equalization process less computationally complex. The derived fuzzy filter function in [20] using fuzzy basis functions [21], product inference, a center of gravity (COG) defuzzifier, and Gaussian membership functions, is able to properly represent the Bayesian decision solution. The performance of the fuzzy equalizer is close to the Bayesian, with the advantage of reduced computational complexity.

In [22] a further development of the fuzzy adaptive filters is presented: the type-2 fuzzy adaptive filter, which based on an unnormalized type-2 TSK fuzzy logic system using a training sequence. This is used to implement the Bayesian equalizer with a decision feedback structure, reducing the complexity of the equalizer compared to that of the transversal equalizer (TE). In contrast to a traditional transversal equalizer or filter, the decision feedback equalizer uses previous detector decisions to cancel intersymbol interference. The type-2 fuzzy sets [23] is an extension of ordinary fuzzy sets in that the membership grades are fuzzy as well. It is shown that an unnormalized output type-1 TSK fuzzy logic system is able to implement a Bayesian equalizer for a time-invariant channel, albeit being model free and not based on a Gaussian probability model. This is further developed into a more generalized form with the type-2 fuzzy adaptive filter to accommodate a time-varying channel.

To a lesser extent, work has also been conducted on blind methods for channel equalization. Blind methods are distinguished by only using information contained in the received signal, thus making both channel estimation and training data unnecessary, with the advantage of higher spectral efficiency. However, this also means that they are strongly dependent on the obtained statistical data.

In [24] the fuzzy-C-means (FCM) algorithm is used to perform joint equalization and demodulation of a signal modulated with the Quadrature Amplitude Modulation (QAM)

scheme. The receiver mapping the signal onto a set of symbols can be reduced to a classification problem, rendering a clustering analysis useful. The aim of clustering analysis is to classify objects into groups or classes (clusters) with the objects in the same group having similarities. The FCM algorithm is an unsupervised algorithm, i.e., no external information outside the data itself is needed on which the algorithm operates. The membership functions are used as a measure of what degree the data is connected to the clusters, which in this particular application depends on the amplitude and the phase distances between the received symbols, i.e. the signal constellation points. The results, for a test environment with a static channel, indicate that the algorithm converges very quickly, is robust and has lower computational complexity than that of conventional MLSE receivers. One of the problems with the FCM algorithm however is that it forces points seriously degraded by noise to belong to one or more clusters with some degree, instead of giving it low or no membership in any cluster.

The FCM algorithm in [24] is improved on in [25] by introducing what is referred to as a fuzzy possibilistic C-means (FPCM) assisted blind channel equalization scheme for time-varying channels. The FPCM algorithm solves the problem with the FCM algorithm above by making it less sensitive toward the highly noisy symbol samples. The scheme also allows the receiver to take into account cluster center information in previous data, thereby improving the accuracy of the cluster centers with more data samples. However, it is also suggested that a forgetting factor might be taken into consideration to reduce the significance of cluster centers in previous data in a time-varying channel. Due to a rather large coherence time in high speed wireless transmission systems, it is stated in [25], that the accumulation of cluster centers is feasible even under time-variant conditions. The FPCM outperforms the FCM algorithm due to the former's capability of rejecting the interference of data seriously degraded by noise. It is also shown that the performance of both algorithms depend on the amount of data involved, as can be expected.

In [26] and [27] a blind equalization algorithm based on a fuzzy neural network is outlined. Equalization is performed with a combination of channel estimation and a fuzzy neural network classifier. The algorithm first blindly estimates the channel by using the fourth cumulants of the received sequences [28]. Afterward, an approximate deconvolution is carried out. The output from the deconvolution is then fed into a fuzzy neural network classifier. Simulations undertaken in time-invariant channels with 64-QAM, indicate that the convergence speed as well as BER are improved compared to that of a feedforward neural network blind equalization algorithm.

Similarly, in [29] an improvement of a feedforward neural network blind equalization algorithm is proposed by using a fuzzy neural network consisting of an input layer, a fuzzification layer, a rule layer, a normalization layer and a defuzzification layer with the aim of improving the convergence rate. Simulation results for 16-QAM shows that the fuzzy neural network has faster convergence speed and lower BER compared to that of the feedforward neural network blind algorithm.



### 2.3 Decoding and Equalization

In contrast to conventional communication systems where encoding/decoding and channel equalization are performed separately, turbo equalization schemes combine the two mechanisms. This combination is carried out by iterating the equalizer and the channel decoder on the same set of received data. Since turbo equalizers have been implemented with the Bayesian algorithm, and it was shown that a TSK fuzzy logic system is able to implement a Bayesian model in [22], a turbo-fuzzy equalization approach should be feasible. Such an approach was introduced in [30], where a turbo equalizer using fuzzy filters is proposed. The fuzzy turbo equalizer is introduced with the motivation that fuzzy filters could deal with uncertainty characterized by impulse noise, and also has lower computational complexity compared to the Bayesian equalizer. To adapt the parameters of the fuzzy equalizer, the back-propagation algorithm is used.

A critique against the turbo-fuzzy equalizer in [30] can be found in [31] where it is stated that the turbo-fuzzy equalizer in [30] is unable to use the *a priori* information provided by the decoder, hence not having an iterative extrinsic information exchange between the fuzzy system and the decoder. This is improved on in [31] and [32] where fuzzy turbo equalization schemes with low complexity are proposed.

In [32] a turbo equalization scheme, based on the radial basis functions (RBF), is proposed by using an extended FCM algorithm. An emphasis is made on the low computational complexity this scheme provides compared to a turbo equalization scheme based on the Jacobian RBF in the context of binary phase-shift keying (BPSK) modulation in a Rayleigh-fading channel. Simulation results show that the scheme proposed performs closely to that of the Jacobian RBF based turbo equalization scheme but with a significant reduction in computational complexity.

In [31] the Jacobian RBF turbo equalization scheme is modified by using the same Bayesian equalization architecture based on a fuzzy adaptive filter structure as introduced in [20]. The simulations are performed for BPSK and QAM in a Rayleigh fading channel, and indicate that the proposed fuzzy adaptive filter TEQ scheme considerably reduces the computational complexity with only a slight degradation in performance compared to the Jacobian RBF turbo equalization scheme, providing a trade-off for highly low complexity-oriented circuit implementation.

## 3 Conclusions

In this paper we have traced the research being conducted over the last two decades, leading up to current research, in which the usage of fuzzy logic in wireless communications has yielded successful results. To the best of our knowledge, this is the first such study of its kind. The three areas in wireless communications focused on in this paper have been: channel estimation, channel equalization and decoding.

In channel estimation, the fuzzy based methods to have been applied have ranged from fuzzy tracking based on Kosko's fuzzy associative memory models and the TSK model, as well as fuzzy logic used in combination with adaptive algorithms such as the LMS and RMS algorithms or a neuro-fuzzy inference system. The neural network and adap-

tive algorithms are commonly used in order to train the parameters of membership functions in a fuzzy inference system.

In channel equalization the research in fuzzy adaptive filters, both type-1 and more recently type-2 TSK fuzzy logic systems, from Wang and Mendel have been highly influential. These fuzzy adaptive filters are able to use input from both human experts and/or training data. Using as a foundation the adaptive fuzzy filters, a Bayesian architecture has been developed which incorporates fuzzy basis functions and Gaussian membership functions, being able to properly represent the Bayesian decision solution. Another category in channel equalization is the blind methods which uses variants of the fuzzy-C-means algorithm or a fuzzy neural network. Turbo equalization has either been based on the Bayesian equalization architecture or the clustering approach with the fuzzy-C-means algorithm.

Conclusions that can be drawn from the research collated and presented in this paper with regards to the main benefits of using fuzzy logic based methods are:

- Fuzzy logic based methods particularly perform well under non-linear and time-variant conditions, where adaptive techniques have to be employed.
- When dealing with complex models that are not completely known and varying with time, fuzzy logic based methods can be used for faster convergence and reduced complexity with a slight degradation in performance compared to that of standard methods.
- When human expert knowledge is available, a fuzzy approach is highly suitable to incorporate this knowledge to complement available numerical data.

Building on the study we have presented in this paper, there are a few research areas that we consider merit further attention. These are: fuzzy adaptive equalization techniques for time-varying MIMO-channels, and fuzzy power control in MIMO-OFDM systems. Furthermore, another interesting area in which research is being conducted, is in cognitive radio [33]. Cognitive radio is an intelligent wireless communication system that adapts to its environment with the purpose of improving the spectrum efficiency. Both signal processing and machine learning techniques are of interest in cognitive radio, with, e.g., game theory being a commonly used method to model the transmit-power control problem. Research has also been conducted in fuzzy based game theory with the aim of application in cognitive radio in [34].

The purpose of this paper has been to give a background to common problems in wireless communication systems. Moreover, a survey of relevant research has been presented in wireless communication systems in which fuzzy methods have been used successfully. The aim has been to give a condensed and clear overview of conducted research as well as highlighting the common features of the problems in which fuzzy logic has been used in order to discern future areas to be investigated further by the ambitious researcher.

## References

- [1] L-X Wang and Jerry M. Mendel. Generating fuzzy rules by learning from examples. *IEEE Transactions on Systems, Man, and Cybernetics*, 22(6):1414–1427, Nov-Dec 1992.

- [2] Antti Niemi, Jyrki Joutsensalo, and Tapani Ristaniemi. Fuzzy channel estimation in multipath fading CDMA channel. *The 11th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, 2:1131–1135, 2000.
- [3] Andrew J Viterbi. *CDMA: Principles of Spread Spectrum Communication*. Prentice Hall PTR., 1995.
- [4] Bart Kosko. *Neural Networks and Fuzzy Systems. A Dynamic Systems Approach to Machine Intelligence*. Prentice Hall, 1992.
- [5] A. R. S. Bahai, B. R. Saltzberg, and M. Ergen. *Multi Carrier Digital Communications: Theory and Applications of OFDM*. Springer, 2004.
- [6] M. Michele and M. Umberto. A comparison of pilot-aided channel estimation methods for OFDM systems. *IEEE Transactions on Signal Processing*, 49(12):3065–3073, December 2001.
- [7] Jian Zhang, Zhi ming He, Xue gang Wang, and Yuan yuan Huang. A TSK fuzzy approach to channel estimation for OFDM systems. *Journal of Electronic Science and Technology in China*, 4(2), June 2006.
- [8] T. Takagi and M. Sugeno. Fuzzy identification of systems and its applications to modelling and control. *IEEE Transactions on Systems, Man, and Cybernetics*, 15(1):116–132, Jan-Feb 1985.
- [9] L-X Wang. Fuzzy systems are universal approximators. *IEEE International Conference on Fuzzy Systems*, pages 1163–1170, March 1992.
- [10] Jian Zhang, Zhi ming He, Xue gang Wang, and Yuan yuan Huang. TSK fuzzy approach to channel estimation for MIMO-OFDM systems. *IEEE Signal Processing Letters*, 14(6), June 2007.
- [11] D. Gesbert, M. Shafi, Shiu Da-shan, P.J. Smith, and A. Naguib. From theory to practice: an overview of MIMO space-time coded wireless systems. *IEEE Journal on Selected Areas in Communications*, 21(3):281–302, April 2003.
- [12] Hsuan-Yu Lin, Chia-Chang Hu, Yu-Fan Chen, and Jyh-Horng Wen. An adaptive robust LMS employing fuzzy step size and partial update. *IEEE Signal Processing Letters*, 12(8):545–548, August 2005.
- [13] Jyh-Horng Wen, Cheih-Yao Chang, Gwo-Ruey Lee, and Ching-Yao Huang. Ofdm channel prediction using fuzzy update lms algorithm in time-variant mobile channel. *IEEE 64th Vehicular Technology Conference*, pages 1–5, September 2006.
- [14] M. Nuri Seyman and Necmi Taspinar. Channel estimation based on adaptive neuro-fuzzy inference system in OFDM. *IE-ICE Trans. Commun.*, E91-B(7):2426–2430, July 2008.
- [15] L-X Wang and Jerry M. Mendel. Fuzzy adaptive filters, with application to nonlinear channel equalization. *IEEE Transactions on Fuzzy Systems*, 1(3):161–170, August 1993.
- [16] L-X Wang and Jerry M. Mendel. An RLS fuzzy adaptive filter with applications to nonlinear channel equalization. *Second IEEE International Conference on Fuzzy Systems*, 2:895–900, Mar-Apr 1993.
- [17] P. Sarwal and M. D. Srinath. A fuzzy logic system for channel equalization. *IEEE Transactions on Fuzzy Systems*, 3(2):246–249, May 1995.
- [18] K.Y. Lee. Complex RLS fuzzy adaptive filter and its application to channel equalization. *Electronics Letters*, 30(19):1572–1573, September 1994.
- [19] Chee Seng Tang and Chin Leonard. Using multilayer perceptron fuzzy adaptive filter in non-linear channel equalization. *IEEE Global Telecommunications Conference, Globecom '95*, 2:884–887, November 1995.
- [20] Sarat Kumar Patra and Bernard Mulgrew. Efficient architecture for bayesian equalization using fuzzy filters. *IEEE Transactions on Circuits and Systems*, 45(7):812–820, July 1998.
- [21] L-X Wang and Jerry M. Mendel. Fuzzy basis functions, universal approximation, and orthogonal least-squares learning. *IEEE Transactions on Neural Networks*, 3(5):807–814, September 1992.
- [22] Qilian Liang and Jerry M. Mendel. Equalization of nonlinear time-varying channels using type-2 fuzzy adaptive filters. *IEEE Transactions on Fuzzy Systems*, 8(5):551–563, October 2000.
- [23] L. A. Zadeh. The concept of a linguistic variable and its application to approximate reasoning. *Inform. Sci.*, 8:199–249, 1975.
- [24] Eugenio Costamagna, Lorenzo Favalli, and Pietro Savazzi. Blind recovery of M-QAM signals for fading channels using the fuzzy-C-means algorithm. *IEEE 49th Vehicular Technology Conference*, 1:516–520, May 1999.
- [25] Jenn-Kaie Lain and Chia-Tai Huang. FPCM-Assisted blind channel equalization of M-QAM signals for time-varying channels. *IEEE International Conference on Networking, Sensing and Control*, pages 787–791, April 2007.
- [26] Shen Fang, Sun Yunshan, Zhang Liyi, Li Yanqin, and Li He. Blind equalization algorithm based on fuzzy neural network in QAM system. *2nd IEEE Conference on Industrial Electronics and Applications*, pages 484–487, May 2007.
- [27] Sun Yunshan, Zhang Liyi, Li Yanqin, Li He, and Yan Junwei. A QAM blind equalization algorithm based on fuzzy neural network. *IEEE International Conference on Control and Automation*, pages 1420–1423, May-June 2007.
- [28] D. Gansawat and T. Stathaki. Blind identification of volterra models using minimax optimization with higher order cumulants. *Proceedings. 7th International Conference on Signal Processing*, 1:164–167, Aug-Sep 2004.
- [29] Yongzong Hou and Qinghua Bu. A new blind equalization algorithm based on the fuzzy neural network controller. *11th IEEE International Conference on Communication Technology Proceedings*, pages 466–469, Nov 2008.
- [30] Murilo Bellezoni Loiola, Moiss Ribeiro, and Joao Marcos T. Romano. A turbo equalizer using fuzzy filters. *14th IEEE Signal Processing Society Workshop on Machine Learning for Signal Processing*, pages 695–704, Sep-Oct 2004.
- [31] Jenn-Kaie Lain, Kun-Huang Kuo, and Yuan-Kai Zhang Liao. Low-complexity fuzzy filter-assisted turbo equalization for dispersive rayleigh-fading channels. *European Transactions on Telecommunications*, 19(2):137–148, 2008.
- [32] Kun-Huang Kuo, Jenn-Kaie Lain, and Chia-Tia Huang. Reduced-complexity radial basis function-assisted turbo equalization for dispersive rayleigh-fading channels. *IEEE International Conference on Systems, Man, and Cybernetics*, 5:3691–3696, October 2006.
- [33] Simon Haykin. Cognitive radio: brain-empowered wireless communications. *IEEE Journal on Selected Areas in Communications*, 23(2):201–220, February 2005.
- [34] Elisabeth Rakus-Andersson, Maria Salomonsson, and Hang Zettervall. Ranking of weighted strategies in the two-player games with fuzzy entries of the payoff matrix. *Eighth International Conference on Hybrid Intelligent Systems*, pages 186–191, September 2008.