Choquet-integral-based Evaluations by Fuzzy Rules

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Abstract— *Choquet-integral-based evaluation models are proposed. The evaluation parameters – fuzzy measures – are assigned from a fuzzy rule table. There are three variations in this model: TF-, BP-, and AV-type models. The TF-type model is a natural extension of ordinal Choquet integrals. The BP-type model involves an evaluation using a reference point. The AV-type model involves a neutral evaluation method. These methods have a number of advantages: Output values are continuous and piecewise linear; If the fuzzy rules have monotonicity, output values are also monotone. We compare these models with some fuzzy reasoning models.*

Keywords— Choquet Integral, Fuzzy rule, Simplified fuzzy reasoning, Cumulative prospect theory, Choquet integral with respect to bi-capacities

1 Introduction

Fuzzy reasoning models are very useful tools for developing fuzzy control models. A part of a fuzzy control model involves sensing status, evaluation using fuzzy reasoning models, and producing output. By repeatedly applying the feedback process, fuzzy control models help approach the control goal (Approach Principal [1]). However, to be applied in the social sciences, evaluation models must be able to determine one global evaluation value by a single calculation process. It is important that the global evaluation values adhere obsequiously to fuzzy rules. For example, if a fuzzy rule is monotone with respect to the inputs, the output of the evaluation model must satisfy the monotonicity property with respect to the inputs. Almost all the fuzzy reasoning models do not satisfy the monotonicity property [2].

In this paper, we propose Choquet-integral-based evaluation methods by using fuzzy rules. The fuzzy rule table is of the same form as ordinal simplified fuzzy reasoning models, but the calculations use (extended) Choquet integrals instead of min-max calculations or product-sum calculations. The global evaluation values satisfy continuous and piecewise linear outputs. Moreover, if fuzzy rules are monotone respect to the inputs, the output values also satisfy the monotonicity property.

2 Definitions

2.1 Notations

X: Set of evaluation items (n: number of evaluation items)

x_i: Input value of the i^{th} item

y: Global evaluation value

2.2 Fuzzy Space Division Constraint

Each input i is divided into m_i fuzzy sets in which the membership functions are p_i^j for $i = 1, \ldots, n, j = 1, \ldots, m_i$. For each input item, all the membership functions satisfy the following conditions:

1. All fuzzy sets are normal and convex. The vertex of a membership function is unique $\forall i, j$, that is, there is a unique point v_i^j, where $p_i^j(v_i^j) = 1\ \forall i$ and j.
2. The sum of the membership values is 1:
$$\sum_j p_i^j(x_i) = 1, \forall x_i, i. \quad (1)$$
3. There are one or two active membership functions – $p_i^j(x_i) > 0 - \forall x_i, \forall i$.

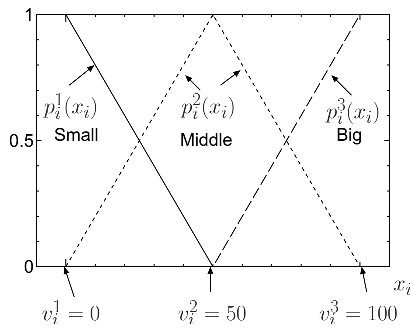

Figure 1: Fuzzy space division ($i = 1, 2$)

An example of the fuzzy space division is shown in figure 1. In this example, there are two inputs, $X = \{1, 2\}$, the membership functions are of the triangle type, and the membership functions are the same for all inputs, $v_i^1 = 0, v_i^2 = 50, v_i^3 = 100, i = 1, 2$.

2.3 Fuzzy Rule Table

Representative points $(k_1, \ldots, k_n)$ are defined as pairs of vertex numbers. For example, the representative point $(2, 3)$ $(n = 2)$ is the point at which input 1 is ”Middle” and input 2 is ”Big.” The fuzzy rule table c is defined as a function from the representative points to the output values:

$$c : \{1, \ldots, m_1\} \times \ldots \times \{1, \ldots, m_n\} \rightarrow \mathbb{R}. \quad (2)$$

Table 1 is an example of the fuzzy rule tables,

- If input 1 is ”Big” and input 2 is ”Big,” the output is 100.

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- If input 1 is "Big" and input 2 is "Middle," the output is 80.
- ...

Table 1: Fuzzy rule table

Input 2 \ 1	Small (1)	Middle (2)	Big (3)
Small (1)	0 (= $c(1,1)$)	50 (= $c(2,1)$)	80 (= $c(3,1)$)
Middle (2)	30 (= $c(1,2)$)	60 (= $c(2,2)$)	90 (= $c(3,2)$)
Big (3)	70 (= $c(1,3)$)	80 (= $c(2,3)$)	100 (= $c(3,3)$)

2.4 *Choquet Integral and Extended Choquet Integral*

The Choquet integral [3] is a useful tool for use in global evaluation methods. The integral can be used to represent complementary and substitute relations among evaluation items [4]. A fuzzy measure $\mu^\dagger$ is defined as

$$\mu^\dagger : 2^X \to \mathbb{R} \tag{3}$$

$$\mu^\dagger(\emptyset) = 0. \tag{4}$$

In ordinal fuzzy measure definitions, this definition is known as a non-monotonicity fuzzy measure, but in this paper, the monotonicity of the fuzzy measure is not assumed.

The Choquet integral is defined as

$$y = (\mathrm{C}) \int h d\mu \equiv \int_0^\infty \mu(\{x \mid h(x) > r\}) dr. \tag{5}$$

The extended fuzzy measure and extended Choquet integral [5, 6] are proposed for handling the cases in which $\mu(\emptyset) \neq 0$. An extended fuzzy measure μ is defined as

$$\mu : 2^X \to \mathbb{R}. \tag{6}$$

Since it is not assumed by definition that $\mu(\emptyset) = 0$, the integration interval is limited to $[0, 1]$. The extended Choquet integral is defined as

$$y = (\mathrm{EC}) \int h d\mu \equiv \int_0^1 \mu(\{x \mid h(x) > r\}) dr. \tag{7}$$

The extended Choquet integral can be calculated by using an ordinal Choquet integral:

$$(\mathrm{EC}) \int h d\mu = (\mathrm{C}) \int h d\mu^\dagger + \mu(\emptyset) \tag{8}$$

where $\mu^\dagger(A) = \mu(A) - \mu(\emptyset), \forall A \in 2^X$.

3 Segment Division Calculation Method

3.1 *Segmentation and Segment Selection*

Segmentation is performed at vertex points v_i^j for all inputs. Figure 2 shows an example of the segmentation. Segment $S_{(k_1,\ldots,k_n)}(k_i < m_i, \forall i)$ is the n-th rectangle whose vertices are the representative points $(k_1 + l_1, \ldots, k_2 + l_n), \forall l_i \in \{0, 1\}, i = 1, \ldots, n$. In the example, there are four segments, $S_{(1,1)}$, $S_{(2,1)}$, $S_{(1,2)}$ and $S_{(2,2)}$.

First, we select the segment that includes the input values $((x_1, \ldots, x_n))$. If $x_1 = 40$ and $x_2 = 20$, $S_{(1,1)}$ is selected.

In this model, each segment has a different extended fuzzy measure $\mu^{(k_1,\ldots,k_n)}$ and integrand $h^{(k_1,\ldots,k_n)}$, but the Choquet integral is caclculated only for the selected segment.

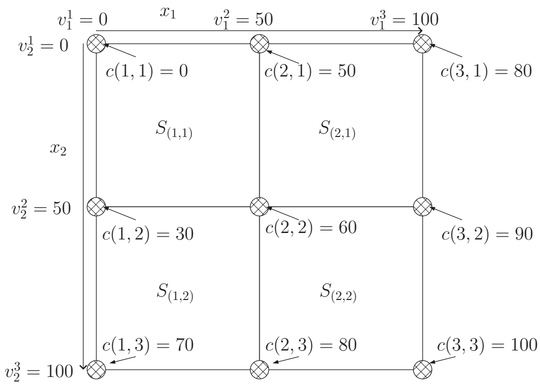

Figure 2: Segmentation

3.2 *Segment Calculation*

Figure 3 shows segment $S_{(1,1)}$. The output is calculated by interpolating the output values of four representative points; those values are $c(1,1)$, $c(2,1)$, $c(1,2)$, and $c(2,2)$.

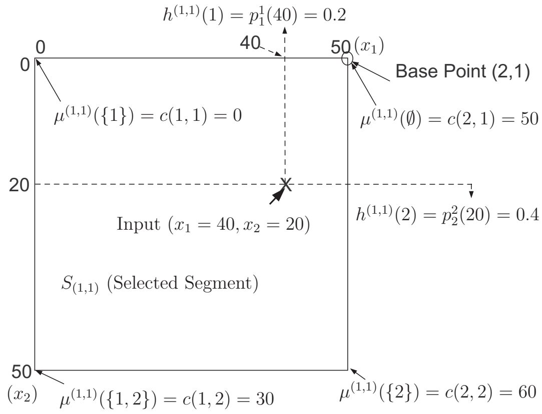

Figure 3: Calculation process (Base Point: $(2,1)$)

Base point selection The base point is the origin of the fuzzy measure and integrand for the segment. First, the base point is selected from among the representative points in the segment. Alternative base points of segment $S_{(k_1,\ldots,k_n)}$ are $(k_1 + l_1, \ldots, k_n + l_n), \forall l_i \in \{0, 1\}, i = 1, \ldots, n$.

Alternative base points of segment $S_{(1,1)}$ in figure 3 are $(1,1)$,$(2,1)$,$(1,2)$, and $(2,2)$.

Extended fuzzy measure $\mu^{(k_1,\ldots,k_n)}$ assignment When the segment $S_{(k_1,\ldots,k_n)}$ and the base point $(q_1, \ldots, q_n)$ are selected, extended fuzzy measures $\mu^{(k_1,\ldots,k_n)}$ are assigned as follows:

$$\mu^{(k_1,\ldots,k_n)}(A) = c(l_1, \ldots, l_n), \forall A \in 2^X \tag{9}$$

$$\text{where } l_i = \begin{cases} q_i & \text{if } i \notin A \\ k_i & \text{if } i \in A \text{ and } k_i \neq q_i \\ k_i + 1 & \text{if } i \in A \text{ and } k_i = q_i. \end{cases}$$

If $(2,1)$ is selected as the base point in figure 3,

$$\mu^{(1,1)}(\emptyset) = c(2,1),\ \mu^{(1,1)}(\{1\}) = c(1,1)$$
$$\mu^{(1,1)}(\{2\}) = c(2,2),\ \mu^{(1,1)}(\{1,2\}) = c(1,2)$$

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Integrand $h^{(k_1,\ldots,k_n)}$ assignment When the segment $S_{(k_1,\ldots,k_n)}$ and the base point $(q_1,\ldots,q_n)$ are selected, integrands $h^{(k_1,\ldots,k_n)}$ are assigned as follows:

$$h^{(k_1,\ldots,k_n)}(i) = p_i^j(x_i), i = 1,\ldots,n \qquad (10)$$

$$\text{where } j = \begin{cases} k_i & \text{if } k_i \neq q_i \\ k_i + 1 & \text{otherwise.} \end{cases}$$

If $(2,1)$ is selected as the base point of segment $S_{(1,1)}$,

$$h^{(1,1)}(1) = p_1^1(x_1), \;\; h^{(1,1)}(2) = p_2^2(x_2) \qquad (11)$$

Extended Choquet integral The output value y is calculated from the extended Choquet integral by using the fuzzy measure $\mu^{(k_1,\ldots,k_n)}$ and integrand $h^{(k_1,\ldots,k_n)}$ of the selected segment.

$$y = (\text{EC}) \int h^{(k_1,\ldots,k_n)} d\mu^{(k_1,\ldots,k_n)} \qquad (12)$$

The output values change with the methods for selection the base points. We propose the following type of Choquet-integral-based evaluation methods: TF-, BP- and BP-type.

3.3 TF-type

In TF-type models, the smallest (upper left) representative points are selected as base points for all segments, that is, the base point of a segment $S_{(k_1,\ldots,k_n)}$ is $(k_1,\ldots,k_n)$.

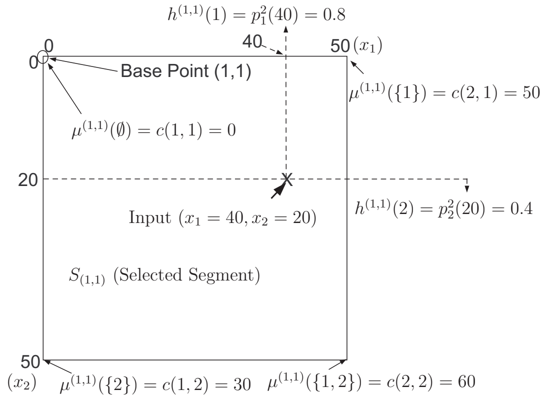

Figure 4: Extended fuzzy measure assignment in TF-type models

When $x_1 = 40$ and $x_2 = 20$, from figure 2, segment $S_{(1,1)}$ is selected. In the TF-type model, $(1,1)$ is selected as the base point. Figure 4 shows an example of the calculation process. Since the origin is $(1,1)$, the membership functions "Middle" ($p_1^2(x_1)$ and $p_2^2(x_2)$) are used. Therefore, the integrand $h^{(1,1)}(1) = p_1^2(x_1) = 0.8$ and $h^{(1,1)}(2) = p_2^2(x_2) = 0.4$. Since the origin is $(1,1)$, $\mu^{(1,1)}(\emptyset) = c(1,1) = 0$, $\mu^{(1,1)}(\{1\}) = c(2,1) = 50$, $\mu^{(1,1)}(\{2\}) = c(1,2) = 30$, and $\mu^{(1,1)}(\{1,2\}) = c(2,2) = 60$ (figure 4). The output is calculated as

$$y = (\text{EC}) \int h^{(1,1)} d\mu^{(1,1)}. \qquad (13)$$

Since $\mu^{11}(\emptyset) = 0$, this equation is calculated by using the ordinal Choquet integral.

$$\begin{aligned} y =& [h^{(1,1)}(1) - h^{(1,1)}(2)]\mu^{(1,1)}(\{1\}) \\ &+ h^{(1,1)}(2)\mu^{(1,1)}(\{1,2\}) = 44. \end{aligned} \qquad (14)$$

Table 2 lists integrands and extended fuzzy measures for all segments. Figure 5 shows a graph corresponding to Table1.

Table 2: Integrand and extended fuzzy measure for TF ($n = 2$)

		Integrand		μ			
Segment	Base Point	$h(1)$	$h(2)$	$\emptyset$	$\{1\}$	$\{2\}$	$\{1,2\}$
$S_{(1,1)}$	$(1,1)$	$p_1^2(x_1)$	$p_2^2(x_2)$	$c(1,1)$	$c(2,1)$	$c(1,2)$	$c(2,2)$
$S_{(2,1)}$	$(2,1)$	$p_1^3(x_1)$	$p_2^2(x_2)$	$c(2,1)$	$c(3,1)$	$c(1,2)$	$c(3,2)$
$S_{(1,2)}$	$(1,2)$	$p_1^2(x_1)$	$p_2^3(x_2)$	$c(1,2)$	$c(2,2)$	$c(1,3)$	$c(2,3)$
$S_{(2,2)}$	$(2,2)$	$p_1^3(x_1)$	$p_2^3(x_2)$	$c(2,2)$	$c(3,2)$	$c(2,3)$	$c(3,3)$

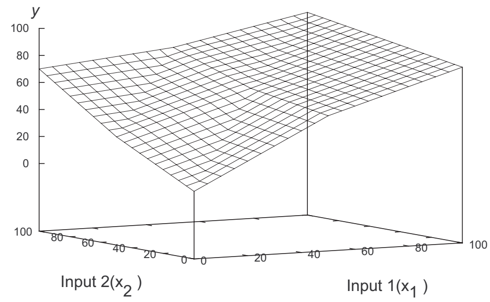

Figure 5: 3D graph corresponding to Table1 (TF-Type)

3.4 Base Point Change

The output values y are dependent on the selection of the base point. If the base point $(2,1)$ is selected for segment $S_{(1,1)}$, the extended fuzzy measure $\mu^{(1,1)}$ is

$$\mu^{(1,1)}(\emptyset) = c(2,1) = 50, \;\; \mu^{(1,1)}(\{1\}) = c(1,1) = 0,$$
$$\mu^{(1,1)}(\{2\}) = c(2,2) = 60, \;\text{ and } \mu^{(1,1)}(\{1,2\}) = c(1,2) = 30.$$

If the input values are $x_1 = 40, x_2 = 20$, $h^{(1,1)}(1) = p_1^1(x_1) = 0.2$ and $h^{(1,1)}(2) = p_2^2(x_2) = 0.4$. The output value is

$$y = (\text{EC}) \int h^{(1,1)} d\mu^{(1,1)} = 48,$$

which is not the equal to the output value 44 given by the TF-type models (14).

3.5 BP-type

In cumulative prospect theory [7], the evaluation methods differ for values above and below the reference point. The model has been developed for analyzing such situations. BP-type calculations can represent the generalized Choquet integral with respect to bi-capacity [8, 9], and the generalized Choquet integral is an extension of cumulative prospect theory.

In the BP-type model, each input has three membership functions: negative, reference, and positive; $m_i = 3, \forall i$. In prospect theory, it is important to evaluate the distance from the reference point, and therefore, the base point for the all segments is the reference point (figure 6 and table 4), namely $(2,\ldots,2)$.

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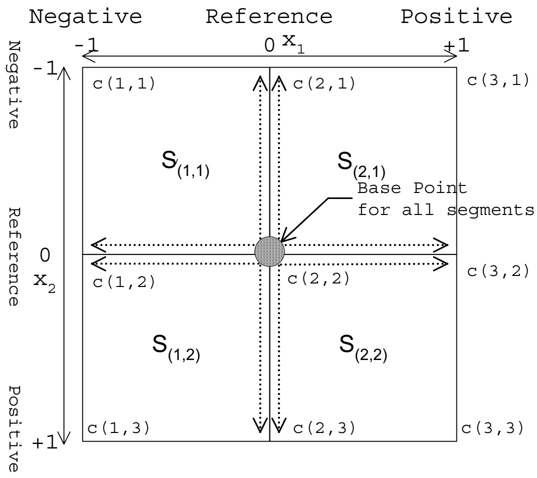

Figure 6: BP-type

BP type model Table 3 is an example of a BP-type evaluation rule table. In thoose BP-type models, membership functions are of the triangle type, such as figure 1; the base points for all segments is the reference point $(2,\ldots,2)$, and the output of the reference point is 0, that is, $c(2,\ldots,2) = 0$ and $x_i \in [-1,1]$. The integrands $h^{(k_1,\ldots,k_n)}$ are

$$h^{(k_1,\ldots,k_n)}(i) = \mid x_i \mid, \forall (k_1,\ldots,k_n), i = 1,\ldots,n. \qquad (15)$$

Table 4 lists the integrands and extended fuzzy measures for all segments when $n = 2$. If $x_1 = 0.2$ and $x_2 = -0.6$, $S_{(2,1)}$ is selected and $h^{(2,1)}(1) = 0.2$ and $h^{(2,1)}(2) = 0.6$;

$$y = (\mathrm{EC}) \int h^{(2,1)} d\mu^{(2,1)} = -0.44.$$

Figure 7 shows a graph corresponding to table 3.

Table 3: BP-type fuzzy rule table

Input 2 \ 1	Negative	Reference	Positive
Negative	-1.0	-0.8	-0.6
Reference	-0.8	0	$+0.6$
Positive	-0.6	$+0.8$	$+1.0$

Table 4: Integrands and extended fuzzy measures for BP

		Integrands		μ			
Segments	Base Points	$h(1)$	$h(2)$	$\emptyset$	$\{1\}$	$\{2\}$	$\{1,2\}$
$S_{(1,1)}$	$(2,2)$	$-x_1$	$-x_2$	$c(2,2)$	$c(1,2)$	$c(2,1)$	$c(1,1)$
$S_{(2,1)}$	$(2,2)$	x_1	$-x_2$	$c(2,2)$	$c(3,2)$	$c(2,1)$	$c(3,1)$
$S_{(1,2)}$	$(2,2)$	$-x_1$	x_2	$c(2,2)$	$c(1,2)$	$c(2,3)$	$c(1,3)$
$S_{(2,2)}$	$(2,2)$	x_1	x_2	$c(2,2)$	$c(3,2)$	$c(2,3)$	$c(3,3)$

Bi-capacity Model Let $Q(X) = \{(A,B) \in 2^X \times 2^X \mid A \cap B = \emptyset\}$. Bi-capacity [8] is defined as a function $v : Q(X) \to [-1,1]$ that satisfies the following conditions:

- If $A \subset A'$, $v(A,B) \le v(A',B)$ and if $B \subset B'$, $v(A,B) \ge v(A,B')$,
- $v(\emptyset,\emptyset) = 0$ and

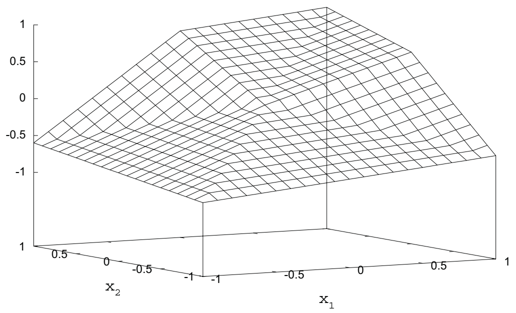

Figure 7: Outputs corresponding to Table 3 (BP-type))

- $v(X,\emptyset) = 1, v(\emptyset,X) = -1$.

The Choquet integral with respect to bi-capacity[9] is defined as

$$C_v(x) = \sum_{i=1}^{n} \mid x_{\sigma(i)} \mid [v(A_{\sigma(i)} \cap X^+, A_{\sigma(i)} \cap X^-) \\ -v(A_{\sigma(i+1)} \cap X^+, A_{\sigma(i+1)} \cap X^-)] \qquad (16)$$

where $X^+ \equiv \{i \in X \mid x_i \ge 0\}$, $X^- = X \setminus X^+$ and σ is a permutation on X satisfying $\mid x_{\sigma(1)} \mid \le \ldots \le \mid x_{\sigma(n)} \mid$ and $A_{\sigma(i)} \equiv \{\sigma(i),\ldots,\sigma(n)\}$.

BP type and bi-capacity model If a fuzzy rule table c is assigned from a bi-capacity v in the following manner:

$$c(k_1,\ldots,k_n) = v(A,B), \qquad (17)$$
$$\text{where } A = \{i \mid k_i = 3\}, B = \{i \mid k_i = 1\},$$

the output y is equal to $C_v(x)$. Table 5 shows the correspondence when $n = 2$.From (17), the extended fuzzy measures are as follows:

$$\mu^{(k_1,\ldots,k_n)}(A) = v(E,F), \forall (k_1,\ldots,k_n), \forall A \in 2^X \qquad (18)$$
$$\text{where } E = \{i \mid k_i = 2\} \cap A \text{ and } F = \{i \mid k_i = 1\} \cap A$$

For the input $x_1,\ldots,x_n$, the selected segment is $(k_1,\ldots,k_n)$, where

$$k_i = \begin{cases} 1 & \text{if } x_k < 0 \\ 2 & \text{otherwise} \end{cases} \qquad (19)$$

Since $h^{(k_1,\ldots,k_n)}(i) = \mid x_i \mid$, $\mu^{(k_1,\ldots,k_n)}(\emptyset) = 0$, and from (18) and (19), $\mu^{(k_1,\ldots,k_n)}(A_{\sigma(i)}) = v(A_{\sigma(i)} \cap X^+, A_{\sigma(i)} \cap X^-)$,

$$y = (\mathrm{C}) \int h^{(k_1,\ldots,k_n)} d\mu^{(k_1,\ldots,k_n)} \\ = \sum_{i=1}^{n} \mid x_{\sigma(i)} \mid [\mu^{(k_1,\ldots,k_n)}(A_{\sigma(i)}) - \mu^{(k_1,\ldots,k_n)}(A_{\sigma(i+1)})] \\ = C_v(x).$$

3.6 AV-type

TF-type and BP-type calculations depend on the selection of the base points. Since the output value in the AV-type model is the average value for all base point selections, this output

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Table 5: Fuzzy rule table by a bi-capacity ($n = 2$)

Input 2 \ 1	Negative	Reference	Positive
Negative	$v(\emptyset, \{1,2\})$	$v(\emptyset, \{2\})$	$v(\{1\}, \{2\})$
Reference	$v(\emptyset, \{1\})$	$v(\emptyset, \emptyset)$	$v(\{1\}, \emptyset)$
Positive	$v(\{2\}, \{1\}))$	$v(\{2\}, \emptyset)$	$v(\{1,2\}, \emptyset)$

does not depend on the base point selection. Therefore, the AV-type method is a neutral evaluation method in terms of the selection of base points.

If $n = 2$, there are four options for selections the base point for each segment. For $S_{(1,1)}$, $(1,1)$, $(2,1)$, $(1,2)$, and $(2,2)$ are the options for selecting the base points. Table 6 lists the integrands and extended fuzzy measures for each base point selection.

Table 6: Integrand and extended fuzzy measure for AV ($S_{(1,1)}$)

	Integrand		μ			
Base Point	$h(1)$	$h(2)$	$\emptyset$	$\{1\}$	$\{2\}$	$\{1,2\}$
$(1,1)$	$p_1^2(x_1)$	$p_2^2(x_2)$	$c(1,1)$	$c(2,1)$	$c(1,2)$	$c(2,2)$
$(2,1)$	$p_1^1(x_1)$	$p_2^2(x_2)$	$c(2,1)$	$c(1,1)$	$c(2,2)$	$c(1,2)$
$(1,2)$	$p_1^2(x_1)$	$p_2^1(x_2)$	$c(1,2)$	$c(2,2)$	$c(1,1)$	$c(2,1)$
$(2,2)$	$p_1^1(x_1)$	$p_2^1(x_2)$	$c(2,2)$	$c(1,2)$	$c(2,1)$	$c(1,1)$

The calculation process when $x_1 = 30$, $x_2 = 40$, and the rule in table 1 is followed is given in Table 7. The output value of AV-type model, 44, is the average value for four selected base points. Figure 8 shows the graph of the AV-type outputs.

Table 7: Calculation process for $x_1 = 30$ and $x_2 = 40$

	Integrand		μ				output
Base Point	$h(1)$	$h(2)$	$\emptyset$	$\{1\}$	$\{2\}$	$\{1,2\}$	y
$(1,1)$	0.6	0.8	0	50	30	60	42
$(2,1)$	0.4	0.8	50	0	60	30	46
$(1,2)$	0.6	0.2	30	60	0	50	46
$(2,2)$	0.4	0.2	60	30	50	0	42
		Average (y)					44

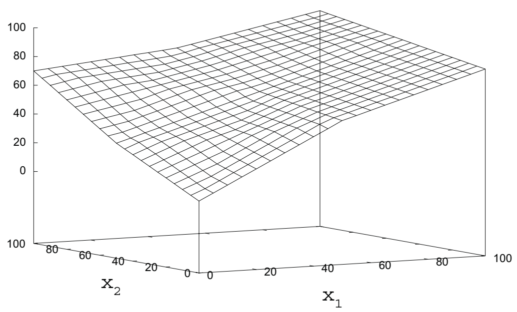

Figure 8: AV-type corresponding to Table 1

4 Properties

Continuity Choquet integral models satisfy the continuity property. Therefore, the proposed models satisfy the continuity property in each segment. For calculating the output values of a point on a bordering hyperplane, there are two or more options for selecting the calculation segment. However, in TF-, BP- and AV-type models, the output values remains the same regardless of the selected segment, that is, the output value on a bordering hyperplane does not depend on the selection of segments (The proof is omitted.) Therefore, Choquet integral based evaluations satisfy the continuity property.

Monotonicity It is known that almost all fuzzy reasoning models do not satisfy the monotonicity property when the fuzzy rule table is monotone. Choquet-integral-based evaluations satisfy the monotonicity property when the fuzzy rule table is monotone. A monotone fuzzy rule table is defined as $c(k_1, \ldots, k_n) \geq c(k'_1, \ldots, k'_n), \forall k_i \geq k'_i$. Examples of monotone rule tables are Table 1 and 3. When the fuzzy rule table is monotone, it is easy to confirm that the values of the extended Choquet integral are monotone and continuous for each segment. From the continuity of the proposed model, it follows that the Choquet-integral-based evaluation models have monotonicity if the fuzzy rule table is monotone.

Piecewise linear If the all membership functions are linear in that case in figure 1, the outputs are piecewise linear. This property is derived from the piecewise linearity of the Choquet integral.

5 Comparisons with Fuzzy Reasoning models

Fuzzy-rule-based global evaluation methods have been developed by using fuzzy reasoning models [10] such as ”Min-Max Gravity Method,” ”Product-Sum Gravity Method,” and ”Simplified Fuzzy Reasoning.” The outputs of the fuzzy rules for the min-max gravity method and product-sum gravity method are fuzzy sets. Therefore, to obtain the output (non fuzzy) value, a defuzzfication (gravity) calculation is performed. However, these calculations are complex. To analyze properties, the simplified fuzzy reasoning is utilized.

Simplified fuzzy reasoning is based on the product-sum method [11]. In the simplified method, it is possible to use min-max calculations instead of product-sum calculations. To demonstrate the properties of min-max and product-sum calculations, we utilize the two types of simplified fuzzy reasoning models.

5.1 Product-sum-type Simplified Fuzzy Reasoning

Simplified fuzzy reasoning is the most popular type of fuzzy reasoning. If the membership function shown in figure 1 and monotone fuzzy rules are assumed, product-sum-type simplified fuzzy reasoning satisfies the monotonicity property.

Product-sum-type simplified fuzzy reasoning can also be perfoermed using the segment division method. If $S_{(1,1)}$ in figure 2 is selected, the output given by the method is

$$\begin{aligned} y = & \quad [1 - p_1^2(x_1)][1 - p_2^2(x_2)]c(1,1) \\ + & \quad p_1^2(x_1) \times [1 - p_2^2(x_2)]c(2,1) \\ + & \quad [1 - p_1^2(x_1)]p_2^2(x_2)c(1,2) + p_1^2(x_1)p_2^2(x_2)c(2,2) \end{aligned} \quad (20)$$

This equation involves an expectation calculation, that is, $p_1^2(x_1)$ and $p_2^2(x_2)$ are random variables, and the expected value of four events ($c(1,1), c(2,1), c(1,2)$, and $c(2,2)$) is calculated by this eqation.

In the segment, if the fuzzy rules are monotone, the outputs satisfy the monotonicity and continuities. Therefore, product-sum methods with monotone fuzzy rules satisfy the monotonicity property.

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Since product-sum-type calculations involve random variables calculation, output values vary unpredictably. Table 8 lists the AND-type fuzzy rule "If Input 1 and 2 are big, the output is 1." Figure 9 shows the output values for the rule table. Product-sum-type calculations are nonlinear because of product calculations. The input variation from $x_1 = x_2 = 0$ to $x_1 = x_2 = 0.01$ increases the output value by only 0.001, but the variation from $x_1 = x_2 = 0.99$ to $x_1 = x_2 = 1$ increases the output value by 0.0199. These results are inconsistent with the rule.

Table 8: AND-type rule table

Input 2 \ Input 1	Small	Big
Small	0.0	0.0
Big	0.0	1.0

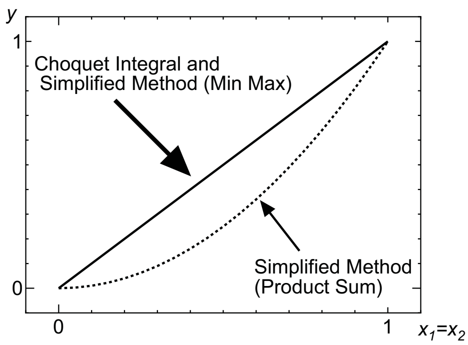

Figure 9: Comparison with product-sum-type method

Since $x_1 = x_2 = 0.5$ only satisfies half of the rule condition, it is expected that the output value is 0.5, but while the product-sum-type is 0.25. Moreover, if the number of inputs for the condition is increased, the output given by the product-sum method decreases. For example, if the number of inputs is 3 or 4, the product-sum output is $0.5^3 = 0.125$ or $0.5^4 = 0.0625$, respectively.

5.2 Min-max type Simplified Fuzzy Reasoning

In ordinal simplified fuzzy reasoning, product-sum calculation is performed, but it is possible to use min and max operators instead of product and sum operators.

Table 9: Monotone fuzzy rule table

Input 2 \ Input 1	Small	Big
Small	0.6 ($c(1,1)$)	0.7 ($c(2,1)$)
Big	0.8 ($c(1,2)$)	1.0 ($c(2,2)$)

Table 9 is represented by the fuzzy switching function

$$y = (c(1,1) \wedge x_1^1 \wedge x_2^1) \vee (c(2,1) \wedge x_1^2 \wedge x_2^1)$$
$$\vee (c(1,2) \wedge x_1^1 \wedge x_2^2) \vee (c(2,2) \wedge x_1^2 \wedge x_2^2),$$

where $x_1^1 + x_1^2 = 1$ and $x_2^1 + x_2^2 = 1$. The equation does not satisfy the monotonicity property because fuzzy switching functions do not satisfy the complementary laws. For example, if $x_1^2 = 0.5$ and $x_2^2 = 1.0$, $y = 0.5$. However, if $x_1^2 = 0.4$ and $x_2^2 = 1.0$, $y = 0.6$.

Therefore, if fuzzy-value calculations are necessary, it is preferable to use Choquet-integral-based calculations.

6 Conclusions

This model is very useful for applications in social science, such as performance evaluation and educational evaluations. Since social science applications require strict validity of the output values, the global evaluation functions should not exhibit non-monotonicity and unexpected nonlinearity, such as that seen in figure 9. The Choquet integral models can be used to ensure the validity of the outputs. However, the attitude of evaluators might change according to the situation or the input values, and this might change the weights and the relationships among the evaluation parameters. The Choquet integral models keep those validities. Fuzzy rule tables can be used to represent these attitudes. Therefore, the proposed model inherits the merits of both the Choquet integral model and the fuzzy rule table.

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