Evaluating premises, partial consequences and partial hypotheses *

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Abstract— *To evaluate premises, consequences and hypotheses, on this paper relevance and support ratios are defined for each of them. This allows to distinguish consequences based on the number of premises that support them, and also to reduce the set of premises while maintaining the same consequences. Since the relation between premises and hypotheses is, in some sense, similar to the relation between consequences and premises, analogous ratios are defined for hypotheses and premises.*

Keywords— Conjectures, Consequences, Hypotheses, Relevance, Support.

1 Introduction

The aim of most problems is to make choice between possible solutions, a clear example is a medical diagnosis problem. In this paper we allocate degrees for the elements in the set of consequences, hypotheses or premises in order to choose the consequence, hypothesis or premise with the biggest degree. Papers [2] and [3] had dealt with that idea of graded consequences.

The following section will show Conjectures, Hypotheses, and Consequences (CHC) models introduced in [7], which was suggested, in part by Watanabe in [10], and takes into account the particular case of partial consequence's operator [8] [9]. Partial operators of consequences are that allow to get consequences of each premise, or subset of premises, and obtaining the final set of consequences as the union of all these partial consequences.

To define support for each consequence (section 3) we consider that consequences with bigger support, are those that are supported by more premises or subsets of premises. By the way, different degrees are allocated for consequences. So, for example, following with a medical diagnosis problem in which the premises are diseases and the consequences are symptoms, we can choose between consequences, and select as the stronger, the one with biggest support.

In section 4, we deal with a measure of relevance for premises that is useful for knowing which premises have more importance, in the sense of how many consequences can be deduced from them. Thanks to that measure, the set of premises can be reduced to a smaller set with the same relevance. This reduced set of premises gets rid of superfluous premises and yet allows to work with less premises, while getting the same set of consequences. Till now all premises seemed to have the same importance.

Finally, in section 5, we also consider partial hypotheses, that is, hypotheses of one premise, and not hypotheses of all premises. And analogous measure of support for premises, as well as of relevance for partial hypotheses, are defined. This allows to evaluate subset of partial hypotheses by counting how many premises they give as consequences.

2 Basic concepts

2.1 CHC models

Reasoning can be understood as a process allowing to get conjectures from a set of premises, P. There are three basic types of reasoning: deduction, abduction and induction. A process that allows to get consequences is a deductive reasoning, a process that allows to get hypotheses is an abductive reasoning, and finally, if the process allows to get speculations, it is an speculative reasoning.

In this paper, CHC models are defined on a preordered set $(L, \leq)$, endowed with a negation, $'$. And when it is necessary, the preordered set will be endowed with an infimum, $\cdot$, and a supremum, $+$, operations $(L, \leq, ', \cdot, +)$, a preorder with infimum and supremum operations is a partial ordered set (poset)with these operations. The infimum of L is called first element and it is denoted by 0, the supremum of L is called the last element and it is denoted by 1. The paper only deals with finite algebraic structures, that is, with a finite set L.

CHC models can be based on consequences operators [8, 9],

Definition 2.1 *If L is a set, and $\mathfrak{F} \subset \mathbb{P}(L)$, it is said that $(L, \mathfrak{F}, C)$ is a* structure of consequences*, provided that $C : \mathfrak{F} \to \mathfrak{F}$ verifies,*

1. *$P \subset C(P)$, for all $P \in \mathfrak{F}$ (C is* extensive*)*
2. *If $P \subset Q$, then $C(P) \subset C(Q)$, for all $P, Q \in \mathfrak{F}$ (C is* monotonic*)*
3. *$C(C(P)) = C(P)$, or $C^2 = C$, for all $P \in \mathfrak{F}$ (C is a* clausure*)*

i.e. C is an operator of consequences *(in the sense of Tarski) for $\mathfrak{F}$ in L.*

For each $\{q\} \in \mathfrak{F}$, let us write $C(q) = C(\{q\})$.

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Definition 2.2 *A consequences' operator C is consistent in P, if for all $q \in C(P)$, $q' \notin C(P)$.*
A structure of consequences $(L, \mathfrak{F}, C)$ is consistent if C is consistent for all $P \in \mathfrak{F}$.

Let P be the set of premises, $P \neq \emptyset$, and $C(P)$ a set of consequences for P. Conjectures of P can be defined from a consistent consequences operator C, as those elements whose negation is not in $C(P)$, $Conj_C(P) = \{q \in L; q' \notin C(P)\}$. Hypothesis can be defined by $Hyp_C(P) = \{h \in Conj_C(P) - C(P); \{h\} \in \mathfrak{F}, P \subset C(h)\}$.

Finally, speculations are those conjectures that are neither consequences, nor hypotheses. Hence, $Sp_C(P) = Conj_C(P) - [C(P) \cup Hyp_C(P)] = \{q \in L, q \notin C(P), q' \notin C(P), q \notin Hyp_C(P)\}$.

2.2

Definition 2.3 *A consequences' operation C is a partial consequences operator if $C(P) = \bigcup_{R \subset P, R \in \mathfrak{F}} C(R)$.*

Definition 2.4 *A decomposable consequences operator is a consequences' operator such that $C(P) = \bigcup_{p \in P} C(R)$.*

This paper considers *partial hypothesis*, elements that are hypotheses of a subset of the set of premises P. This idea comes from that of *partial consequences*.

Definition 2.5 *For each set P of premises the partial hypotheses set is,*

$$Hyp^*_C(P) = \{h \in \{L - 0\} - P; \{h\} \in \mathfrak{F}, \exists R \subset P,$$
$$R \subset C(h)\}.$$

Obviously, hypotheses are partial hypotheses, since $P \subset P$ and $P \subset C(h)$, provided h is a hypothesis.

Remark 2.6 *Although hypotheses are anti-monotonic ($P_1 \subset P_2$, implies $Hyp(P_2) \subset Hyp(P_1)$), partial hypotheses are (as it is easy to prove) monotonic ($P_1 \subset P_2$, implies $Hyp^*(P_1) \subset Hyp^*(P_2)$). That is why they can not be considered classical hypotheses.*

2.3

The paper deals with a general concept of measure [6], defined in a preordered set $(L, \leq)$. A measure is a mapping $m : L \to [0, 1]$, such that:

- There exists a minimal $x_0 \in L$, such that $m(x_0) = 0$
- There exists a maximal $x_1 \in L$, such that $m(x_1) = 1$
- If $x \leq y$, then $m(x) \leq m(y)$.

3 Consequences support

This section introduces a ratio in order to distinguish which consequences are the more supported by a given set of premises. And proof in which cases is a measure

Let's recall that in this paper L is assumed to be a finite set.

Definition 3.1 *The support of $q \in L$ is the ratio of subsets of premises that allow getting q as a consequence, to all possible subsets of premises.*

$$Supp_{C,P}(q) = \frac{|\{R \in \mathbb{P}(P); q \in C(R)\}|}{2^{|P|} - 1} = \qquad (1)$$
$$\frac{|\{R \in \mathbb{P}(P); q \in C(R)\}|}{|\mathbb{P}(P) - \emptyset|}.$$

Since $P \neq \emptyset$, it is $|P| > 0$ and the quotient in the definition is possible.

The bigger support a consequence has, the more subsets of premises allow deducing it.
Notice that if $q \notin C(P), Supp_{C,P}(q) = 0$, since if there were $R \in \mathbb{P}(P)$ such that $q \in C(R)$, because of the monotonicity of the consequence operator, $C(R) \subset C(P)$ would imply $q \in C(P)$.

This ratio verifies the following properties,

- If $P \subset Q$, it is $Supp_{C,P}(q) \leq Supp_{C,Q}(q)$, for all $q \in L$.
- If $P = \{p\}, \forall q \in C(P)$, it is $Supp_{C,P}(q) = 1$.
- For all $q \in C(P)$, $Supp_{C,P}(q) > 0$.
- $Supp_{C,P}(q) = 1$ means that q is a consequence for all $R \in \mathbb{P}(P)$. Particularly, q is consequence of all $p \in P$.

The support defined by (1), is not a measure in general. For example, if $C(P) = P, \ \forall P \in \mathbb{P}(L)$, let P be a set with more than one element. If $q \in L$, it is either $q \in P$, or $Supp(q) = 0$. If $q \in P$, there exists $p \in P$ such that $p \neq q$, and $q \notin C(p)$ and $Supp(q) \neq 1$. Therefore, there is no $q \in L$ such that $Supp_{C,P}(q) = 1$.

Remark 3.2 *$Supp_{C,P}$ is monotonic with respect to the preorder given by C, $q_1 \leq_C q_2$ iff $q_2 \in C(q_1)$ [1],*

Proof. Since if $q_1 \leq_C q_2$, for each R such that $q_1 \in C(R)$, it is $C(q_1) \in C(C(R)) = C(R)$, and, as $q_2 \in C(q_1)$, it is also $q_2 \in C(R)$. So, $Supp_{C,P}(q_1) \leq Supp_{C,P}(q_2)$.
□

Since, given P, the relation defined between the pairs of elements in L with the same value of $Supp_{C,P}$, is an equivalence, the classes

$$[q] = \{v \in L; \ Supp_{C,P}(v) = Supp_{C,P}(q)\}$$

give a partition on L in a number of parts that is, at most, $2^{|P|}$.

3.1 The case of the operator $C_\bullet$

Let $(L, \leq)$ be now a preordered set in which is defined an infimum operation denoted by '$\cdot$'.

The partial consequences operator $C_\bullet$ gives consequences that are consequences for a subset of the set of premises P, it is $C_\bullet(P) = \{q \in L; \exists\{p_1, p_2, ..., p_n\} \subset P : p_1 \cdot p_2 \cdot \ldots \cdot p_n \leq q\}$. It is a partial consequences' operator and it obviously verifies $C_\bullet(P) = \bigcup_{R \subset P, R \in \mathfrak{F}} C_\bullet(R)$.

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Notice, that as L is finite, P is also finite and $C_\bullet(P) = \{q \in L; \inf P \leq q\}$, which equal to the infimum operator of consequences $C_\wedge$.

$$Supp_{\,C_\bullet,P}(q) = \frac{|\{R \in \mathbb{P}(P) \in P; \inf(R) \leq q\}|}{2^{|P|} - 1} \tag{2}$$

Let's see what specific properties are verified by $Supp_{\,C_\bullet,P}$,

- If L has last element, it implies $1 \in L$, then $1 \in C_\bullet(P)$ and $Supp_{\,C_\bullet,P}(1) = 1$.
- If $q_1 \leq q_2$, then $Supp_{\,C_\bullet,P}(q_1) \leq Supp_{\,C_\bullet,P}(q_2)$. That is the function $Supp_{\,C_\bullet,P}$ is monotonic.
- $Supp_{\,C_\bullet,P}(\sup\{q_1, q_2\}) \geq \max\{Supp_{\,C_\bullet,P}(q_1), Supp_{\,C_\bullet,P}(q_2)\}$, provided $\sup\{q_1, q_2\}$ exists.
- $Supp_{\,C_\bullet,P}(\inf\{q_1, q_2\}) \leq \min\{Supp_{\,C_\bullet,P}(q_1), Supp_{\,C_\bullet,P}(q_2)\}$

Corollary 3.3 *Let $(L, \leq, ', \cdot, +)$ be a partial ordered set with infimum and supremum operations and first and last elements. If $P \neq \{0\}$, the function $Supp_{\,C_\bullet,P} : L \rightarrow [0, 1]$ is a measure.*

Proof. It is monotonic, and it verifies the boundary conditions, since 0 is not a consequence $Supp_{\,C_\bullet,P}(0) = 0$ and $Supp_{\,C_\bullet,P}(1) = 1$. □

3.2 The case of the operator $C_\leq$

$C_\leq$ is the partial consequences operator that gives as consequences those elements that follow from at least one premise in P, formally, it is $C_\leq(P) = \{q \in L; \exists p \in P : p \leq q\}$, see [8]. Hence, it can be considered as a decomposable consequences' operator, since allows getting consequences that are not deduced from all premises. It is straightforward that $C_\leq(P) = \bigcup_{R \subset P, R \in \mathfrak{F}} C(R) = \cup_{p \in P} C_\leq(p)$.

In this case, a different definition of the support's ratio seems to be more convenient, since it deals only with consequences of each $p \in P$, nor with consequences of each subset of $\mathbb{P}(P)$.

Definition 3.4 *The support of $q \in C_\leq(P)$ is the ratio of premises that allow getting q as consequence to all premises.*

$$\widehat{Supp}_{\,C_\leq,P}(q) = \frac{|\{p \in P; p \leq q\}|}{|P|} \tag{3}$$

Since $P \neq \emptyset$, it is $|P| > 0$ and the quotient in the definition is possible.

If $q \notin C_\leq$, then $\widehat{Supp}_{\,C_\leq,P}(q) = 0$.

So, the bigger Support an element has, the more premises allow to reach it.

$\widehat{Supp}_{\,C_\leq,P}$ verifies,

- If $P = \{p\}, \forall q \in C_\leq(p)$, it is $\widehat{Supp}_{\,C_\leq,P}(q) = 1$.
- If L has last element, 1, then $1 \in C_\leq(P)$ and $\widehat{Supp}_{\,C_\leq,P}(1) = 1$.
- For no $q \in C_\leq(P)$ is $\widehat{Supp}_P(q) = 0$. That is, for all $q \in C_\leq(P)$, $\widehat{Supp}_{\,C_\leq,P}(q) > 0$.
- $\widehat{Supp}_{\,C_\leq,P}(q) = 1$ means that q is a consequence for all $p \in P$.
- If $q_1 \leq q_2$, then $\widehat{Supp}_{\,C_\leq,P}(q_1) \leq \widehat{Supp}_{\,C_\leq,P}(q_2)$. That is the function $\widehat{Supp}_{\,C_\leq,P}$ is monotonic with respect to $\leq$.

Remark 3.5 *In order to know what happens if we calculate the support for the infimum or supremum, of two consequences, provided it exists and it is a consequence, weak boundaries are found,*

- $\widehat{Supp}_{\,C_\leq,P}(\sup\{q_1, q_2\}) \geq \max\{\widehat{Supp}_{\,C_\leq,P}(q_1), \widehat{Supp}_{\,C_\leq,P}(q_2)\}$
- $\widehat{Supp}_{\,C_\leq,P}(\inf\{q_1, q_2\}) \leq \min\{\widehat{Supp}_{\,C_\leq,P}(q_1), \widehat{Supp}_{\,C_\leq,P}(q_2)\}$

Obviously, if the operator is consistent, that is, if $q \in C_\leq(P)$, then $q' \notin C_\leq(P)$, it follows $\widehat{Supp}_{\,C_\leq,P}(q') = 0$.

Theorem 3.6 *Let $(L, \leq, ', \cdot, +)$ be a partial ordered set with first and last elements and $P \neq \{0\}$. Function $\widehat{Supp}_{\,C_\leq,P} : L \rightarrow [0, 1]$ is a measure.*

Proof. It is monotonic as it is stated above. Since 0 is not a consequence $\widehat{Supp}_{\,C_\leq,P}(0) = 0$. Finally, it is obvious that $1 \in C_\leq(P)$ and $\widehat{Supp}_{\,C_\leq,P}(1) = 1$. □

From $\widehat{Supp}_{\,C_\leq,P}$ we can calculate $Supp_{\,C_\leq,P}$. If $\widehat{Supp}_{\,C_\leq,P}(q) = k$ and $|P| = n$, it is $|\{R \in \mathbb{P}(P); q \in C(R)\}| = k{\cdot}n$. Hence,

$$Supp_{\,C_\leq,P} = \frac{2^n - \sum_{i \in \{0,\ldots,n-k\cdot n\}} \frac{(n-k\cdot n)!}{i!(n-k\cdot n-i)!}}{2^n - 1}. \tag{4}$$

The numerator in (4) is the number of subsets of premises that contain at least one of the premises supporting q.

Corollary 3.7 *Let $(L, \leq)$ be a preordered set with first and last elements and $P \neq \{0\}$. Function $Supp_{\,C_\leq,P} : L \rightarrow [0, 1]$ is a measure.*

Proof. It is monotonic as it is proven at the beginning of this section, and it verifies boundary conditions, since 0 is not a consequence $Supp_{\,C_\leq,P}(0) = \frac{2^n - 2^n}{2^n - 1} = 0$ and $Supp_{\,C_\leq,P}(1) = \frac{2^n - 1}{2^n - 1} = 1$. □

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Example 3.8 *Figure 1 represents a preordered set of medical symptoms and deseases for patients. Let's calculate the support for the consequences for a patient with* $P = \{antibody, bacterium\}$. *To such an end, let us notice that,* $C_{\leq}(P) = \{antibody, bacterium, fever, eruption, 1\}$. *Then,*

- $\widehat{Supp}_{C_{\leq},P}(antibody) = \frac{1}{2}$, *and* $Supp_{C_{\leq},P}(antibody) = \frac{2^2-(1+1)}{2^2-1} = \frac{2}{3}$.
- $\widehat{Supp}_{C_{\leq},P}(bacterium) = \frac{1}{2}$, *and* $Supp_{C_{\leq},P}(bacterium) = \frac{2}{3}$.
- $\widehat{Supp}_{C_{\leq},P}(fever) = 1$, *and* $Supp_{C_{\leq},P}(fever) = \frac{3}{3} = 1$.
- $\widehat{Supp}_{C_{\leq},P}(eruption) = \frac{1}{2}$, *and* $Supp_{C_{\leq},P}(eruption) = \frac{2}{3}$.
- $\widehat{Supp}_{C_{\leq},P}(1) = Supp_{C_{\leq},P}(1) = 1$.

Hence, the consequence with greatest support is 'fever'.

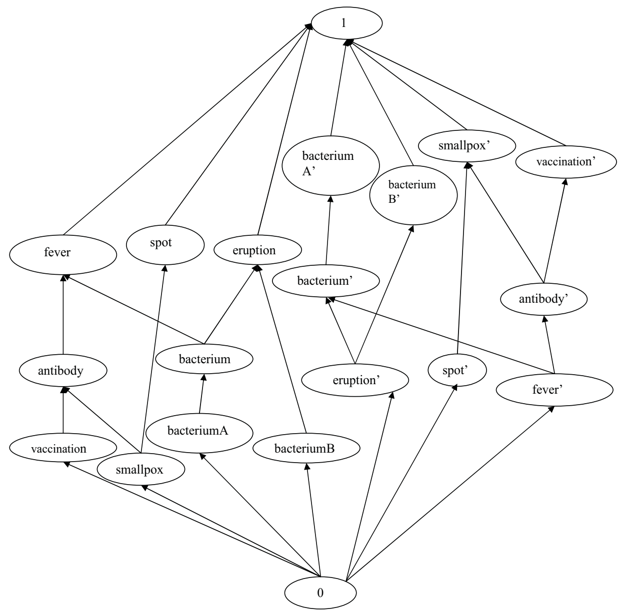

Figure 1: Preorder

4 Relevance for premises

This section introduces a measure to calculate the proportion of consequences that are gotten from a subset of premises, and what is more, it is shown how to reduce the set of premises using this ratio in order to give the same set of consequences.

Definition 4.1 *The relevance of a subset of premises* $R \in \mathbb{P}(P) - \{\emptyset\}$ *is the ratio of consequences deduced from a* R, *to all consequences.*

$$Rel_{C,P}(R) = \frac{|\{q \in L; q \in C(R)\}|}{|C(P)|} = \frac{|C(R)|}{|C(P)|}, \quad \textit{if } R \in \mathbb{P}(P) - \{\emptyset\}, \tag{5}$$

and, $Rel_{C,P}(\emptyset) = 0$.

Since $|P| > 0$ *and* $P \subset C(P)$, *it is* $|C(P)| > 0$, *the quotient does exist.*

If a subset of premises allows to deduce all consequences, the set of premises can be reduced to it, since both have the same set of consequences.

There are many properties that $Rel_{C,P}$ verifies,

- If there exists $R \in \mathbb{P}(P)$, such that $Rel_{C,P}(R) = 1$, it means that all consequences for P are consequences of R. So, $C(P) = C(R)$.
- If $R_1 \subset R_2$, then $Rel_{C,P}(R_1) \leq Rel_{C,P}(R_2)$. That is function $Rel_{C,P}$ is monotonic.
- It is $Rel_{C,P}(P) = 1$, and $Rel_{C,P}(\emptyset) = 0$.

Theorem 4.2 *Function* $Rel_{C,P} : \mathbb{P}(P) \to [0, 1]$ *is a measure.*

Proof. Straightforward, by the last properties. □

Remark 4.3 *In this case, the concept of fuzzy measure is defined in the preordered set* $(\mathbb{P}(L), \subset)$, *since relevance is defined for all subsets of premises and not only for single premises. Remember that the support is defined for each element.*

The ratio of relevance applying for each premise allows to define a partition into the set of premises, in classes whose elements have the same relevance, $[q] = \{p \in L;\ Supp_{C,P}(p) = Supp_{C,P}(q)\}$. Analogously, it can be built a partition in the set $\mathbb{P}(P)$, defining each class as $[S] = \{R \in \mathbb{P}(P);\ Supp_{C,P}(R) = Supp_{C,P}(S)\}$. The maximum number of classes that can exist is $|C(P)| + 1$.

4.1 Using the operator $C_{\leq}$

For the operator $C_{\leq}$ it is useful to calculate the relevance for each $p \in P$, since it is sufficient to get consequences for each premise and then join them. So, in this case is enough to deals with

$$Rel_{C_{\leq},P} : P \to [0, 1]$$

Definition 4.4 *The relevance for a premise* $p \in P$ *is the proportion of consequences deduced from* p.

$$Rel_{C_{\leq},P}(p) = \frac{|\{q \in L; p \leq q\}|}{|C_{\leq}(P)|} = \frac{|C(p)|}{|C(P)|}. \tag{6}$$

If a premise allows to deduce all consequences, the set of premises can be reduced to that premise, since both give the same set of consequences.

There are many properties that $Rel_{C_{\leq},P}(p)$ verifies,

- For all $p \in P$, $Rel_{C_{\leq},P}(p) > 0$, since $p \in \{q \in L; p \leq q\}$ implies $|\{q \in L; p \leq q\}| > 0$.
- If there exists $p \in P$ such that $Rel_{C_{\leq},P}(p) = 1$, it means that all consequence for P is consequence of p. So, $C_{\leq}(P) = C_{\leq}(p)$.

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- If $p_1 \leq p_2$, then $Rel_{C_{\leq},P}(p_2) \leq Rel_{C_{\leq},P}(p_1)$. That is the function $Rel_{C_{\leq},P}$ is anti-monotonic in this sense. Then, the function $1 - Rel_{C_{\leq}}$ is monotonic.
- Let L be endowed with an infimum operation. If $\inf P \in P$, as $Rel_{C_{\leq},P}(\inf P) = 1$, because $\forall q \in C_{\leq}(P)$ there exists $p \in P$ such that $p \leq q$, and $\inf P \leq p \leq q$. Then, $C_{\leq}(P) = C_{\leq}(\inf P)$.
  A common consequence's operator is $C_{\wedge}$, defined by $C_{\wedge}(P) = \{q \in L; \inf P \leq q\}$, that can be defined as $C_{\wedge}(P) = C_{\leq}(\inf P)$, so in that case $C_{\leq}(P) = C_{\wedge}(P)$.

Example 4.5 *Using the same preset in figure 1. Let's calculate the relevance for premises. Here, we have an example that allows us to quantify the relevance of deseases of a patient.*

If the patient has $P = \{antibody, smallpox\}$*, then,* $C_{\leq}(P) = \{antibody, smallpox, spot, fever, 1\}$*. Hence,*

- $Rel_{C_{\leq},P}(antibody) = \frac{|\{antibody, fever, 1\}|}{|C_{\leq}(P)|} = \frac{3}{5}$.
- $Rel_{C_{\leq},P}(smallpox) = \frac{|\{antibody, smallpox, spot, fever, 1\}|}{|C_{\leq}(P)|} = \frac{5}{5} = 1$
- *Obviously,* $Rel_{C_{\leq},P}(P) = 1$.

This example shows the case that a premise allows to deduce all consequences of P*, since* $C_{\leq}(P) = C_{\leq}(smallpox)$.

Theorem 4.6 *If* $(L, \leq, ', \cdot, +)$ *is a partial ordered set and* $P \subset L$ *such that* $\inf P \neq 0$.
There exists $p \in P$ *such that* $Rel_{C_{\leq},P}(p) = 1$*, if and only if* $p = \inf P$.

Proof. If $Rel_{C_{\leq}}(p) = 1$, it is $p \leq q$ for all $q \in C_{\leq}(P)$. And since $C_{\leq}(P) \subset C_{\wedge}(P)$, it is $\inf P \leq q \; \forall q \in C_{\leq}(P)$. The infimum is the greatest lower bound of a set, then $p \leq \inf P$. It is also $\inf P \leq p$. Thus, $p = \inf P$, because L is a lattice, so verifies antisymmetric property and has an infimum for each subset.
On the other hand if $p = \inf P$, implies $\inf P \in P$, and as it is said, $Rel_{C_{\leq},P}(\inf P) = 1$. $\square$

In the theorem and in the above example, it is shown that the set of premises can be reduced to an only premise with relevance one, but if there is no one premise with relevance one, it could be found a subset of premises that allows to obtain the same consequences as the initial set of premises. When models deal with not a very big number of premises, a simple program can be used in order to find a minimal set of premises by calculating all combination of premises.

This algorithm is exponential in the number of premises. So, others algorithms can be designed in order to deal with a big number of premises.

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The algorithm is as follows.

First of all, we look for premises with greatest relevance, we put one of these premises (p_1) into the set of reduced premises, then we calculate a relative relevance

$$Rel_{C_{\leq},P-\{p_1\}}(p) = \frac{|\{q \in C_{\leq}(P) - C_{\leq}(p_1); p \leq q\}|}{|C_{\leq}(P) - C_{\leq}(p_1)|},$$

and we introduce a premise with the greatest relative relevance (p_2), and then we calculate other relative relevance,

$$Rel_{C_{\leq},P-\{p_1,p_2\}}(p) =$$

$$\frac{|\{q \in (C_{\leq}(P) - C_{\leq}(p_1)) - C_{\leq}(p_2); p \leq q\}|}{|(C_{\leq}(P) - C_{\leq}(p_1)) - C_{\leq}(p_2)|},$$

and this process is repeated till the lowest r that verifies $C_{\leq}(P) = \bigcup_{i \in \{1,\dots,r\}} C_{\leq}(p_i)$. Then the reduced set searched in this way will be $\{p_1, ..., p_r\}$.

4.2 Using the operator $C_{\bullet}$

In this case we can particularize the definition of relevance for each subset of premises.

The relevance for a subsets of premises $R \subset \mathbb{P}(P) - \{\emptyset\}$ is the ratio of consequences deduced from R, to consequences deduced from P.

$$Rel_{C_{\bullet},P}(R) = \frac{|\{q \in L; q \in C_{\bullet}(R)\}|}{|C_{\bullet}(P)|} =$$

$$\frac{|\{q \in L; \exists \tilde{R} \subset \mathbb{P}(R), \inf \tilde{R} \leq q\}|}{|C_{\bullet}(P)|}$$

Example 4.7 *Let* P *be* $\{c, d, a'\}$ *defined in the preorder in figure 2. So,* $C_{\bullet}(P) = \{c, d, f, g, b, e, d', a', 1\}$*. Relevance for all subset of premises are,*

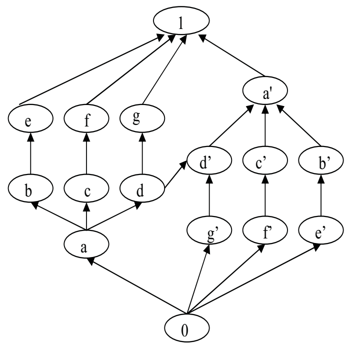

Figure 2: Preorder

- $Rel_{C_{\bullet},P}(\{c\}) = \frac{3}{7}$, $Rel_{C_{\bullet},P}(\{d\}) = \frac{5}{7}$, $Rel_{C_{\bullet},P}(\{a'\}) = \frac{2}{7}$
- $Rel_{C_{\bullet},P}(\{c,d\}) = 1$, $Rel_{C_{\bullet},P}(\{c,a'\}) = 1$, $Rel_{C_{\bullet},P}(\{d,a'\}) = \frac{5}{7}$

This example gives two reduced sets of premises, $\{c, d\}$ *and* $\{c, a'\}$*. Obviously* $C_{\bullet}(P) = C_{\bullet}(\{c, d\}) = C_{\bullet}(\{c, a'\})$.

5 Validating premises and partial hypotheses

Each premise is supported, or explained by hypotheses, so in this section, a support for each premise is defined. Then, we define the relevance of each partial hypothesis. This is analogously to the above section, because premises are consequences of hypotheses.

Let $Hyp^*_C(P) \neq \emptyset$.

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Definition 5.1 *The support of $p \in P$ is the ratio of subsets of hypotheses that allow getting p as consequence, to all subsets of partial hypotheses.*

$$Supp_{C,Hyp^*_C(P)}(p) = \frac{|\{H \subset Hyp^*_C(P); p \in C(H)\}|}{2^{|Hyp^*_C(P)|} - 1} \quad (7)$$

Since $|Hyp^*_C(P)| > 0$ the quotient in the definition is possible.

The bigger Support a premise has, the more hypotheses allow to deduce it.

The $Supp_{C,Hyp^*_C}$ verifies,

- If $P = \{p\}$, it is $Supp_{C,Hyp^*_C(P)}(p) = 1$.
- If $1 \in P$, then $Supp_{C,Hyp^*_C(P)}(1) = 1$.
- $Supp_{C,Hyp^*_C(P)}(p) = 1$ means that p is explained by all $h \in Hyp^*_C(P)$, in particular for all $h \in Hyp(P)$.
- If $p_1 \leq p_2$, then $Supp_{C,Hyp^*_C(P)}(p_1) \leq Supp_{C,Hyp^*_C(P)}(p_2)$. That is, function Sup is monotonic.
- $Supp_{C,Hyp^*_C(P)}(\sup\{p_1, p_2\}) \geq \max\{Supp_{C,Hyp^*_C(P)}(p_1), Supp_{C,Hyp^*_C(P)}(p_2)\}$.
- $Supp_{C,Hyp^*_C(P)}(\inf\{p_1, p_2\}) \leq \min\{Supp_{C,Hyp^*_C(P)}(p_1), Supp_{C,Hyp^*_C(P)}(p_2)\}$.

$Supp_{C,Hyp^*_C(P)}$, allows to compare premises in relation to hypotheses and to allocate different degrees to each premise.

Definition 5.2 *The relevance for a subset of partial hypotheses $H \subset Hyp^*_C(P)$ is the proportion of premises deduced from H.*

$$Rel_{\ C,Hyp^*_C(P)}(H) = \frac{|\{p \in P; p \in C(H)\}|}{|P|}, \quad (8)$$

if $H \in \mathbb{P}(Hyp^*_C(P)) - \{\emptyset\}$,

and, $Rel_{\ C,Hyp^*_C(P)}(\emptyset) = 0$.

Since $|P| > 0$, the quotient is possible.

There are many properties that $Rel_{\ C,Hyp^*_C(P)}$ verifies,

- If $P = \{p\}$, it is $Rel_{\ C,Hyp^*_C(P)}(H) = 1$, for all $H \subset Hyp^*_C(P)$.
- For all $H \subset Hyp^*_C(P)$, $Rel_{\ C,Hyp^*_C(P)}(H) > 0$.
- If $H_1 \subset H_2$, then $Rel_{\ C,Hyp^*_C(P)}(H_1) \leq Rel_{\ C,Hyp^*_C(P)}(H_2)$. That is, function $Rel_{\ C,Hyp^*_C(P)}$ is monotonic.
- If there exists $h \in Hyp^*_C(P)$ such that $Rel_{\ C,Hyp^*_C(P)}(\{h\}) = 1$, it means that h is a partial hypothesis that can explain all premises, so it is hypothesis.

Theorem 5.3 *If $Hyp_C(P) \neq \emptyset$, then $Rel_{\ C,Hyp^*_C(P)}$ is a measure.*

Proof. It is monotonic as it is stated above, $Rel_{\ C,Hyp^*_C(P)}(\emptyset) = 0$, and since there exists $h \in Hyp_C(P)$ $Rel_{\ C,Hyp^*_C(P)}(\{h\}) = 1$.
□

The measure, $Rel_{\ C,Hyp^*_C(P)}$, allows to compare partial hypotheses, and to distinguish which partial hypotheses are hypotheses in the classical sense, that are those one with relevance one.

6 Conclusions

In this paper, it is built a measure in the set of consequences, premises and partial hypotheses. That can be useful in decision problems in order to choose the consequence, premise or hypotheses with the biggest measure.

It should be also pointed out that by using a relevance measure, we can get rid of premises that do not add information, and still get the same set of consequences.

It is also introduced the concept of set of partial hypotheses, that contains the classical hypotheses one. The measure built allocate value one to that partial hypotheses that are hypotheses in the classical sense.

As future work it can be proposed to apply these measures to practical problems, for examples medical diagnosis problems much more bigger than the one that appears in this paper.

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