Colour Image Segmentation using A-IFSs

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Abstract— *The problem of segmentation is an important research field and often a critical and decisive preprocessing step for many image processing applications. During the past years, the use of colour images instead of gray images is growing fast in many research areas. Moreover, the use of segmentation over colour images is advised when additional colour information conveyed is necessary to further discriminate the image regions. Recently, fuzzy logic theory, and Atanassov's intuitionistic fuzzy sets (A-IFSs), have been successfully applied to image segmentation and, in this paper we propose a general methodology for RGB colour image segmentation based on A-IFSs. The proposed methodology is based on a multilevel colour thresholding framework that uses Atanassov's intuitionistic index values for representing the uncertainty present in the assigning of the pixels to the different regions. This framework is to be applied to each RGB component separately to finally aggregate the results of the three components. The number of thresholds is auto established by the methodology based on the image entropy. Experimental results are presented.*

Keywords— Atanassov's Intuitionistic Fuzzy Sets, Color Image Segmentation, Fuzzy Logic, Multi-Thresholding.

1 Introduction

Color image segmentation deals with the subdivision of an image into disjoined regions according to the pixels colour with the goal of finding the image objects. As a result of the segmentation, the original set of pixels in the image is partitioned into a finite set of separated regions (objects).

Within the framework of fuzzy theory [1, 2, 3, 4], segmentation algorithms that use the concept of fuzzy entropy [1, 3, 5] are widely used in image processing applications. In previous works [6] we have proposed methodologies for image segmentation that use A-IFSs [7, 8, 9]. In those methodologies, Atanassov's intuitionistic fuzzy index values are used for representing the uncertainty/impresision on determining whether a pixel of the image belongs to an object of that image.

The general framework presented in [6] is the basis for colour segmentation framework proposed in this paper where it is extended to a multi-level thresholding framework since, often, images include several regions and, thus, it is necessary to compute more than one threshold in order to correctly segment the image.

In this methodology we use RGB images in such way that each RGB component is primarily segmented separately. After the three components are segmented we then aggregate the three segmented components into the final segmented image. For each component, the number of computed thresholds is automatically determined by the methodology on the basis of the images intuitionistic fuzzy entropy. This entropy is obtained trough the Atanassov's intuitionistic fuzzy index values.

The remainder of this paper is organized as follows: In Section 2 we describe the proposed methodology. Section 3 presents the experimental results obtained and, finally, in Section 4 some conclusions and some lines of future work are presented.

2 Proposed methodology

In this section we present the proposed methodology for colour image segmentation that besides its low computational cost is also able to auto determine the number of required thresholds based on the image intuitionistic fuzzy entropy.

Since this methodology deals with each RGB component separately, we will only describe the algorithm for one of the components. The exact same algorithm is to be applied to the other two components.

This methodology successively applies the bi-level thresholding algorithm proposed in [6], for the computation of one threshold, an unspecified number of times to an image, resulting in a set of thresholds for each RGB component. After the number of required thresholds needed for each RGB component are computed, the total number of thresholds is calculated and, on the basis of that number, the three component segmentations are combined in order to create a resulting gray segmented image. The combination of the components is made by adding all three intensities and afterwards normalizing them back to the original scale. Different ways of combining the three RGB thresholds can be explored.

Thus, in the following sub-sections, we will first describe the bi-level thresholding algorithm used and, in the last subsection the multi-level thresholding algorithm.

2.1 Bi-level Thresholding Algorithm

In this section we describe the general framework for image thresholding using A-IFSs and restricted dissimilarity functions proposed in [6]. The goal is to obtain the better threshold in order to separate the object from the background.

We will denote by (x, y) the coordinates of each pixel on the image Q, being $q(x, y)$ the gray level of the pixel (x, y) so that $0 \leq q(x, y) \leq L - 1$ for each $(x, y) \in Q$ where L is the image grayscale.

In this methodology Atanassov Intuitionistic Fuzzy Sets are used in the following way: In order to choose/construct the

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membership function of each pixel of the image to the associated fuzzy set, three numerical values are assigned to each one of them.

- A value for representing the expert knowledge of the membership of the pixel to the background. A membership function, constructed using dissimilarity functions, is used to obtain this value (see [10]).
- Dissimilarity functions are also used by the expert to construct a membership function to retrieve a value for representing the expert knowledge of the membership of the pixel to the object.
- The expert knowledge/ignorance, in determining the above mentioned membership functions, is represented by a third value obtained trough Atanassov's intuitionistic index.

In this sense, L fuzzy sets Q_{Bt} associated with the background and L fuzzy sets Q_{Ot} associated with the object are constructed. Each one of these fuzzy sets is associated with a gray level t of the grayscale L used. The membership functions of these sets are defined by means of restricted dissimilarity functions and, two of the possible expressions one might use are:

$$\mu_{Q_{Bt}}(q) = F\left(d\left(\frac{q}{L-1}, \frac{m_{\mathrm{B}}(t)}{L-1}\right)\right)$$

$$\mu_{Q_{Ot}}(q) = F\left(d\left(\frac{q}{L-1}, \frac{m_{\mathrm{O}}(t)}{L-1}\right)\right)$$

where

$$m_{\mathrm{B}}(t) = \frac{\sum_{q=0}^{t} qh(q)}{\sum_{q=0}^{t} h(q)} \quad (1)$$

$$m_{\mathrm{O}}(t) = \frac{\sum_{q=t+1}^{L-1} qh(q)}{\sum_{q=t+1}^{L-1} h(q)} \quad (2)$$

being $h(q)$ the number of pixels of the image with the gray level q, $F(x) = 1-0.5x$ and, the restricted dissimilarity function $d(x,y) = |x-y|$ (see [6, 11, 12]).

Due to these constructions, the membership functions constructed are always greater than or equal to 0.5 and, a pixel unequivocally belongs to one of the regions (object or background) if and only if its intensity q is equal to the mean of intensities of the that region ($m_O(t)$ or $m_B(t)$). When the difference between the pixel's intensity q and the mean of intensities of the region is maximal, then the value of its membership function to that region is minimal. Thus, the lesser the distance between a pixel's intensity q and the mean of intensities of the region, the greater the value of its membership to that region.

In this methodology, it is considered that the membership functions ($\mu_{Q_{Bt}}$ and $\mu_{Q_{Ot}}$) indicate the expert's degree of knowledge of the pixel belonging to the regions, meaning that, the Atanassov's intuitionistic fuzzy index (π) represents the ignorance/intuition present in the construction of these fuzzy sets.

Thus, the value represented by Atanassov's intuitionistic index indicates the knowledge/ignorance of the expert when assigning a pixel either to the background or the object. The Atanassov's intuitionistic index value associated with a pixel has the value of zero, when the expert is absolutely sure of that pixel belonging (either to the background or the object). The Atanassov's intuitionistic index value increases with respect to the ignorance/intuition of the expert as to whether the pixel belongs to the background or the object. If the expert doesn't know if a pixel belongs to the background or the object, its membership to both must be represented with the value 0.5 and, it is said that the expert used the greatest ignorance/intuition allowed in the construction of the membership functions, of the set associated with that pixel, to the background and the object resulting in a Atanassov's Intuitionistic Fuzzy Index maximum value. For this reason, A-IFSs (Atanassov's Intuitionistic Fuzzy Set [8, 13]) are used.

However, the ignorance/intuition should have the least possible influence on the choice of the membership degree. In this methodology the ignorance/intuition will have a maximum influence of 25 percent.

Under these conditions, within the set of all the possible expressions(see [6]), the following expression is used to calculate π:

$$\pi(q) = \wedge(1 - \mu_{Q_{Bt}}(q), 1 - \mu_{Q_{Ot}}(q)).$$

This π is now used to construct an A-IFS with each one of the fuzzy sets Q_{Bt} and Q_{Ot}.

$$\tilde{Q}_{Bt} = \{(q, \mu_{\tilde{Q}_{Bt}}(q), \nu_{\tilde{Q}_{Bt}}(q)) | q = 0, 1, \cdots, L-1\},$$

given by

$$\mu_{\tilde{Q}_{Bt}}(q) = \mu_{Q_{Bt}}(q)$$
$$\nu_{\tilde{Q}_{Bt}}(q) = 1 - \mu_{\tilde{Q}_{Bt}}(q) - \pi(q)$$

and

$$\tilde{Q}_{Ot} = \{(q, \mu_{\tilde{Q}_{Ot}}(q), \nu_{\tilde{Q}_{Ot}}(q)) | q = 0, 1, \cdots, L-1\},$$

given by

$$\mu_{\tilde{Q}_{Ot}}(q) = \mu_{Q_{Ot}}(q)$$
$$\nu_{\tilde{Q}_{Ot}}(q) = 1 - \mu_{\tilde{Q}_{Ot}}(q) - \pi(q)$$

At this stage, the Atanassov's intuitionistic fuzzy index is used to calculate the entropy IE of each one of the L Atanassov's intuitionistic fuzzy sets associated with the image.

The following expression is used to calculate the entropy IE, so that $0 \le IE(\tilde{Q}_{Bt}) \le 0.25$.

$$IE(\tilde{Q}_{Bt}) = \frac{1}{N \times M} \sum_{q=0}^{L-1} h(q) \cdot \pi(q) \quad (3)$$

where $N \times M$ are the image dimensions in pixels.

This way, the entropy on A-IFSs is interpreted as a measure of the degree of a A-IFS that a set has with respect to the fuzzyness of the said set (see [14]). Under these conditions, the entropy will be null when the set is a FSs and will be maximum when the set is totally intuitionistic.

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Finally, the gray level associated with the Atanassov's intuitionistic fuzzy set $\tilde{Q}_{Bt}$ of lowest entropy IE is selected as the best threshold.

2.2 *Multi-level Thresholding Algorithm*

In this approach, the goal is to obtain a set of thresholds in order to separate all the image objects. This method is based on a divide and conquer strategy and its main idea is to successively apply the algorithm proposed in [6] and presented in the last section, for the computation of one threshold, an unspecified number of times to an image. In Fig. 1 we illustrate the methodology computational process progress.

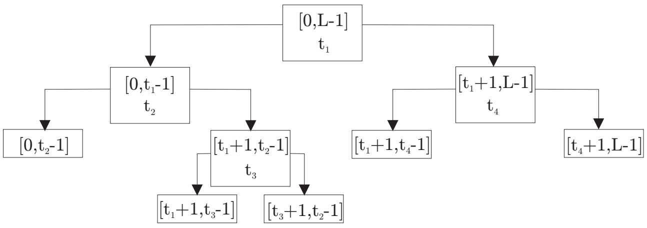


Figure 1: Computational process progress

First, the algorithm is applied to the original image resulting in the determination of the threshold value t_i. This threshold value is then used to create two sub-images: the sub-image with intensity values lower than t_i and the sub-image with intensity values greater than t_i. Then, the algorithm is to be applied to the sub-image which has the greater amplitude between gray-levels entropy values (difference between the intensity value of maximum entropy and the one of minimum entropy).

Hence, The proposed algorithm consecutively divides the resulting sub-images by means of the threshold value t_i obtained through the application of the algorithm for the computation of one threshold to each one of them.

The algorithms stopping criteria is based on the sub-images gray-levels entropy values in such way that the process stops when the difference between the entropy of the computed threshold value t_i and the maximum entropy of the sub-image that generated it, is inferior to the difference between the maximum and minimum entropies of the entire image divided by a constant K.

After experimental evaluation, the value of the constant K was set to 2.

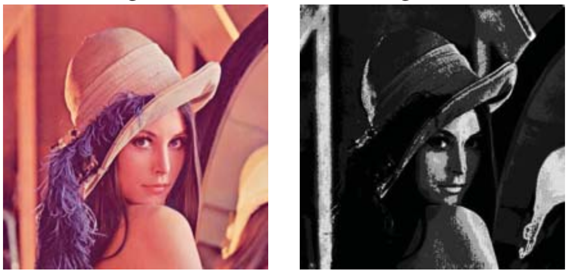
The number of computed thresholds is closely related to the value of this constant K. This value is directly proportional to the number of computed thresholds since, the maximum difference of global entropy allowed for each sub-image to produce a new threshold is inversely proportional to the value of K. Thus, the value of the constant K can be chosen according to the purpose of the application where the algorithm is to be applied.

3 Experimental results

In order to illustrate the methodology we used the "Lena" image in Fig. 2 and Fig. 3.

In Fig. 2 the original "Lena" image and the segmentation result are presented. In Fig. 3, in the first column, we present

Original Segmented

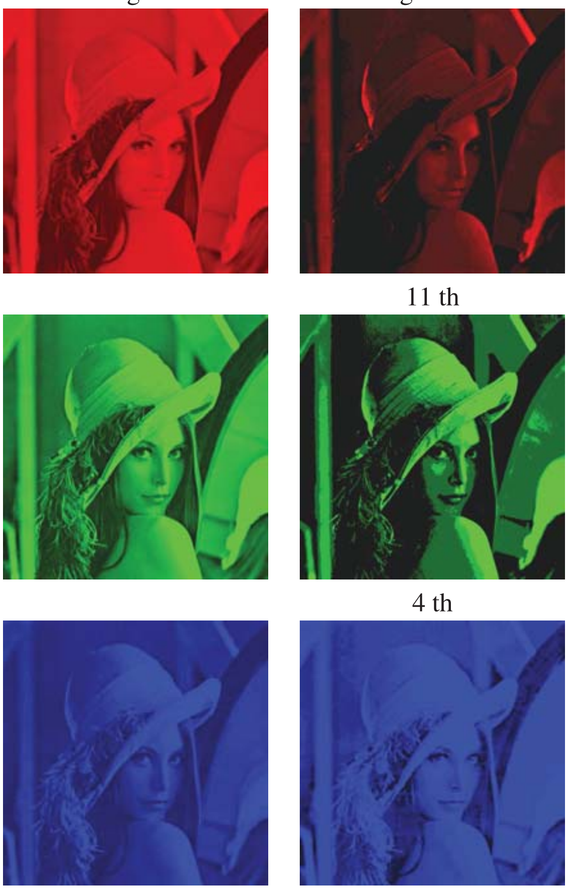


23 th

Figure 2: Original and segmented "Lena" image.

the image decomposed in its RGB components and, in the second column, each RGB component segmentation result is presented. For all the "Lena" segmented images the number of computed thresholds is also presented.

Original Segmented



11 th

4 th

8 th

Figure 3: Original and segmented RGB components of "Lena" image.

Regarding the number of computed thresholds in each RGB component we can conclude that, as one can see looking at he original image, the red component is the one that has more information regarding the segmentation of the image and, therefore the red component was the one with more computed thresholds. We think that this is an indicator of the goodness of the methodology since it was able to accurately auto determine the required number of thresholds in each component

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separately.

Looking at Fig. 3, we can also see that the RGB components are highly related. We consider this to be the major drawback of segmenting images in the RGB colour space.

To test the performance of the proposed approach, more than a hundred images where segmented. However, in order to illustrate the obtained results, 5 images from the original set of images (taken form the Berkeley Segmentation Dataset: `http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/`) were selected and used as test images (see Fig. 4).

Figure 4: Original and segmented test images.

In Fig 4 we present the results from the application of the proposed methodology to complex images where, the value of the constant K was also set at 2 and, where we can see that the main features of the images are preserved.

4 Conclusions and Future Work

Since, an image usually contains more than one region, in this paper we presented a new extension of an existing method, that uses A-IFSs for multi-level colour image segmentation. The algorithm first segments each RGB component separately and finally combines them in order to obtain the final segmented image. The use of A-IFSs allowed us to use entropy in the same sense as fuzzy entropy is used in the fuzzy algorithms and, consequently, allowed us to endow the methodology with the capability of dealing with uncertainty.

The proposed methodology successively applies a bi-level thresholding algorithm an undetermined number of times to an image RGB component subdividing it into several disjoined regions. Moreover, the algorithm is able to auto determine the number of thresholds required to segment each RGB image component.

Considering the experimental results, we can say that, in general terms, the proposed algorithm provides suitable results and therefore can be considered when colour image segmentation applications are needed.

The proposed methodology is parameterizable through the setting of a constant K. The value of this constant is directly proportional to the optimal number of thresholds computed. Thus, the value of the constant K should always depend of the application needs. Future work is intended to study the incorporation of heuristics in the algorithm through the setting of the constant K value. Different combinations of the resulting thresholds of each one of the RGB components can be an object of future work in order to incorporate the perceptual differences among the three components.

Due to the high correlation between the three RGB components, further work is also intended to apply an extension of this algorithm to images in the HSV colour space. Being the HSV colour space a more intuitive color space, it's our belief that a more coherent segmentation will be achieved.

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