Around fuzziness. Some philosophical thoughts

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Abstract— *Fuzzy logic has a considerable success in the area of applications. However, this active growth in the implementation in applied electronic devices has not always been accompanied by a discussion about the theoretical grounds underlying those uses. This paper deals with some theoretical aspects that seem convenient to discuss for the purpose of clarifying the status of fuzzy logic, to get into its true nature and to appoint its achievements and limits. In order to carry out this goal, the paper is organized around three questions: 1) Fuzzy logic, is it a logic or a techno-logic?; 2) How fuzzy is fuzzy computing?; 3) Are there fuzzy objects?*

Keywords— Fuzziness, fuzzy computation, fuzzy objects, philosophy of soft computing, vagueness.

1 Introduction

Borderline cases are initially addressed by Ch. S. Peirce in 1902 [1]. In 1923, B. Russell positioned vagueness as an interesting subject in the philosophy arena [2]. Later, Hempel and Copi dealt with vagueness in separate articles published in the journal *Philosophy of Science* [3, 4]. Up till them, vague language was not considered a central topic for the analytic tradition, a school of thought characterized by an emphasis on clarity and argument, achieved by formal logic and analysis of language. M. Black introduced in 1937 [5] the ‘consistency-profiles’ as a way to represent the meaning of a vague predicate, defined by the standard use that a community make of it. Consistency-profiles are represented by curves, showing the degree of consistency in the application of a word to an object or concept. The notion involves three characteristics present in the modern forms to represent vagueness: the user of a language, a situation in which the user is trying to apply a symbol to an object and a way to represent the consistency of this application through a logic formalism.

Among the modern initiatives to deal with the formal representation of vagueness, understood as imprecision, the most successful has been Zadeh’s fuzzy logic [6]. Fuzzy logic enables to symbolize vague propositions and to manage approximate inferences in a suitable way. As a result, it has had a tremendous growth, supported mainly by the success of their applications. However this active growth in the area of applications has not always been accompanied by a reflection on the theoretical grounds involved in the use of the underlying tool. As an applied logic it was not surprising that, in the beginning, Zadeh claimed that *“clearly the problems, the aims and the concerns of fuzzy logic are substantially different from those which animate the traditional logical systems. Thus, axiomatization, decidability, completeness, consistency, proof procedures and other issues which occupy the center of the stage in such systems are, at best, of peripheral importance in fuzzy logic”* [6, p.151]. And perhaps the same could be said of the metatheoretical discussions. In this paper we will deal with some theoretical problems that we believe that they are important for the purpose of clarifying the status of fuzzy logic, to get into its true nature and to draw its achievements and limits. The fundamental issues considered here take the form of three questions that, in turn, will name the next three sections of this work.

2 Fuzzy logic, in its true nature, is it a logic or rather than a *techno*-logic?

The reflection about the logical statute of fuzzy logic is involved with two traditional topics in Philosophy:

i. The classification of sciences.

ii. The fundamentation of sciences.

The classification of sciences is frequently revised in function of new emergent branches of knowledge, forcing to modify standard taxonomies. In the field of deductive sciences, fuzzy logic came as a new discipline that questioned some of the classical principles, prompting a debate about the nature of the logic and provoking a discussion about the status of the deviant logics, among which fuzzy logic was included [7].

Since its birth, fuzzy logic was valued in different ways by engineers (applied scientists) and by mathematicians (theoretical thinkers). Due to the success of the applications, fuzzy logic was welcomed in Engineering Schools. Nevertheless, it also caused rejection among mathematicians because, during a long time, it was a matter under construction, provisional and tentative.

Engineers use fuzzy logic according, at least, three criteria:

- Fuzzy logic allows us to represent complex scenarios that involve uncertainty using few rules. In the same vein, it permits to manage approximate inferences reaching good and quite stable solutions to the pretended conclusions.

- Although fuzzy logic is a (precise) logic about imprecision, it manages outputs that provides enough information to make good decisions. As Popper says: *“Although clarity is valuable in itself, exactness or precision is not: there can be no point in trying to be more precise than our problem demands”* [8, p.28]. So, in presence of imprecision, alternatives to the classical bivalent logic should be suggested.

- Systems with incorporated fuzzy rules are quite robust. Fuzzy logic admits a tolerance fringe between positive and negative outputs, although it is difficult to supply *a priori* guaranties about the stability of a fuzzy controller.

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These points agree with the Zadeh's *dictum*, known as 'Principle of Incompatibility': *"Stated informally, the essence of this principle is that as the complexity of a system increases, our ability to make a precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics"* [9, p. 28].

In summary, from the applied sciences point of view, as engineering, fuzzy logic is a useful discipline. Its success are the achievements of its applications and any further evidence is not requested about the logical properties of the formal tool. For engineers, fuzzy logic is a techno-logic, i.e., a logic to be implemented in electronic machines.

But beyond applications, fuzzy logic is a kind of logic and, therefore, a branch of mathematics. Mathematics is a theoretical science, not an applied one and its currency are precision, rigor and proof. So, frequently, fuzzy logic was not welcomed in mathematical circles while it was used as an *ad hoc* tool for solving quite particular problems in a large extent and specially in early applications. This partial and temporary knowledge was not well received in the field of mathematics, accustomed to dealing with eternal truths. Attending to its provisional character, mathematicians think that fuzzy logic is not a logic, but a kind of formalism for managing specific problems that involves imprecision; fuzzy logic, they say, does not manage any rationality, even not that it say to grasp: the imprecise or vague reasoning.

Attending to classical logic, the Verificability Principle allows us to substitute a proposition by its truth value. But in the field of fuzzy logic, this unanimous correspondence is problematic. A vague predicate can be represented, in a reasonable way, by similar linguistic labels or fuzzy numbers. As a matter of degree, fuzzy meaning is grasped in an imprecise way; hence, some margins of tolerance should be admitted in order to represent it. Accordingly, in fuzzy logic it is not possible to forget the world and focus only in the proofs. We need to reconcile both of them. Any attempt to give a fuzzy logic axiomatic approach hits with the absence of the stability of truth, with the lack of a perfect truth. Therefore, for mathematicians, fuzzy logic is not a true logic.

From the point of view of the fundamentation of sciences, logic is a main part of the scientific method. The hypothetico-deductive system is considered, frequently, as the scientific method *par excellence*. From Aristotle, Science is strongly related to deductive reasoning. In [10], it is defined a deductive science as a set S of sentences that satisfies four conditions:

1. *Reality postulate*: Sentences of S are about a particular subject matter, about real entities.
2. *Truth postulate*: Each sentence of S is true.
3. *Deductive postulate*: If something can be deduced from a subset of S, then it is also a member of S –closure property under logical consequence.
4. *Evidence postulate*:
   (a) *Definability*: The sentences of S are built from a finite set of terms, which have a meaning immediately obvious.
   (b) *Axiomatizability*: The truth of every sentence of S can be inferred logically from a certain finite subset S_1 of S whose truth is also obvious.

Postulates 1 and 2 require comments. It is known that both mathematics and physics are scientific theories. But while mathematics is an ideal science, physics is an experimental one. So, the objects of mathematics are not real objects, but ideal entities. Therefore, their statements can be absolutely true; timeless true. This forces to not cast doubts about the postulate 3. The simplification of the reality and the idealism related to the mathematical thinking lead to not questioning the postulate 4. But empirical statements are about real entities, and material objects change and vary over time. So, postulate 2 –the *truth postulate*– is not immediately obvious. Sometimes empirical sentences are true, sometimes are partially true (true in a degree, true with a probability,...) but they are always temporarily true; that is, true in absence of falsation. Postulate 2 is known as the Principle of Bivalence and it was traditionally rejected in other applications which are different from the logic of mathematical entities.

In twentieth century, with the birth of artificial intelligence, a new way to see deductive sciences came. The logic is not only applied to specific languages, such as the mathematical language –crisp in nature–, or to regimented natural language –specified artificially–, but also to statements that verbalize normal situations related to ordinary life. So, the *Reality Postulate* was extended to any real entity, including these with ill-defined borders and the *Principle of Bivalence* was replaced by the *Principle of Valence* [11], which states that every proposition has a truth value, not fixing how many or which. The relaxation of principles 1 and 2 casts doubt on the fourth: in real life it is difficult to think about meanings or truths immediately obvious. More attention is demanded by the third principle. Fuzzy logic is a deductive logic that grasps imprecise consequences. As a logic, it aims to conclude deductively, but as a theory of imprecise consequences, calls for margins of tolerance or imprecise intervals of plausible conclusions. Whether fuzzy logic verifies, or not, the deductive postulate depends on how the meanings of vague sentences are represented, as well as the approach to combine them into complex propositions. For example, Trillas et al. showed that the unique way to obtain consequences in the Tarski's sense from the Zadeh's compositional rule of inference is using the bold t-norm if the rule is represented by the Reichenbach's conditional definition [12].

From the beginning until now, fuzzy logic has evolved dramatically. Today it is still a formal tool with successful applications, but also a logic that seeks its theoretical consolidation. In this goal highlights –among others– the contributions made by the Spanish school of fuzzy logic, particularly those made by Esteva & Godo and Trillas. Esteva & Godo deal with fuzzy logic as a symbolic logic developed in the spirit of classical logic (syntax, semantics, axiomatization, truth-preserving deduction, completeness, etc. both in propositional and predicate logic) [13]. Hajek and Novak et al. contributed too to credit fuzzy logic in this vein [14, 15]. Hence, their work was directed to confer credit to fuzzy logic as a kind of logic. Trillas has been from a long time devoted to evolve a fuzzy metatheory of fuzzy logic; so, he has contributed to develop fuzzy logic as a fuzzy theory [12]. All of them, helped to

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credit fuzzy logic as an area of interest in the contemporary scientific panorama.

Despite these developments, fuzzy logic needs to assume new challenges. In the field of applications is a truism the success of the fuzzy controllers in many household devices. But to avoid falling in auto-complacency, it would be desirable to develop applications where vagueness or imprecision are the main component of a complex system; e.g., implementing fuzzy logic into devices in order to summarize perceptions [16]. In the theoretical perspective, fuzzy logic needs to debate about the ways to represent the meaning of a vague proposition, how to justify one choice or another among the various alternatives. Its credibility will be critically linked to this task. Another job will be to mix fuzzy logic with others logics while imprecision in not the only characteristic of human language or human decision-making. Context, time, intentionality, beliefs and its corresponding logics are jointly relevant to model human reasoning and, therefore, to build electronic devices that simulate human behavior.

The new fuzzy logic proposed by Zadeh under bright titles as 'Computing with Words and Perceptions', 'Precisiated Natural Languages', 'Protoforms' and so on [17], should make progress in this regard. Until now we have ashy headlines but low content.

3 How fuzzy is fuzzy computing?

'Turing machine' (TM) is perhaps the main reference in computer science. So, it is said that any function is computable if it is Turing computable. If a problem is computable, it admits an algorithmic definition; i.e., its solution can be reached by a finite sequence of instructions that ends in a finite time.

A TM requires that:

- Each cell of the tape contains only a symbol, although there can be an infinite number of cells.
- States are finite and discrete: the machine is in one state or another, but never in two states at the same time or in no state.

According to the discrete nature of the alphabet and the states, the computation performed by a TM is linear and precise. But although the representation and the computation of a problem are bivalent in nature, the output may be trivalent: in effect, a TM computation can conclude in three different states:

- Final state or success: The problem is decidable and it has a solution.
- Non-final state or failure: The problem is decidable but it does not have solution.
- Loop state or auto-referential state: The machine computes indefinitely and never stop. The problem is indecidable and it does not have any solution. Hence, a TM may have not solution because it stops on a not final state or because it does not stop.

There are so many TM as problems can be putted in an algorithmic form. But a nice achievement of the theory of computation is that any TM admits to be simulated by a special one: the binary TM. A binary TM works as follows: any symbol or state is represented by binary digit or a set of binary digits. So, an instruction as $10 \longrightarrow 10001l$ denote that the machine is in the internal state *1*, reads *0* in the tape, change to the state *1000*, writes *1* and it moves the tape one step to the left. A binary TM can simulate any other specific TM; so, it is an universal computing machine.

At this point, we wish to emphasize the capability of the binary TM to simulate the behavior of any other TM. But the TM not only support be simulated. It is also an excellent simulator; in particular, mimic goods solutions to problems which was not designed to. For example, several TM working simultaneously can imitate a parallel computation, although the way in which TM process is sequential and not parallel.

Fuzzy TM goes back to the late 1960s, when Zadeh spoke about the notion of fuzzy algorithm [18]. Santos gave the formal description of a fuzzy algorithm by fuzzy variants of Turing machines, Markov algorithms and finite automata [19]. He established the fundamentals of the fuzzy language too. Fuzzy TM are linked to classical TM through the support and crisp part of a fuzzy set. Fuzzy algorithms were drawn to perform imprecise computations.

What is meant by imprecise computations? At least, two answers are possible:

- In a soft sense, imprecise computations refer to problems whose working process can be expressed in an algorithm form through imprecise instructions; that is, rules that are or are not completely satisfied, but only fulfilled at some extent. From this perspective, a typical imprecise computation is a cooking recipe: add a pinch of salt, cook quite slowly, serve very cold,...
- In a hard sense, imprecise computations should refer to a genuinely form to compute vague instructions. If so, there would be a new model of computing, but up till today, the TM is the only machine for computing. Afterwards, we come back to this topic.

Imprecise computations in a soft sense can be simulated by a TM. A fuzzy instruction, as "add a pinch of salt', can be represented by a generalized membership function. Fuzzy logic provides a fuzzy arithmetic for managing fuzzy commands. So, a TM with a fuzzy arithmetic module can operate fuzzy computations and output a final state of partial success; that is, a solution qualified by a degree. The appearance of this fuzzy output may be continuous and not discrete. Discrete outputs passing serially, one after the other, in a small temporary sequence, may emerge as a continuous appearance –cartoons are a god metaphor: snapshot sequences of static pictures seem to have movement by themselves.

In a soft sense, a fuzzy computation can be performed by a classical TM, at least, in two ways:

- Weighting the symbols of the tape alphabet.
- Qualifying the states with degrees of preference.

So, from a theoretical point of view, fuzzy computation is classical computation augmented with a module that operates with degrees. "Fuzzy" refers to data high level representation not to a form of computing. Therefore, there is not a new or genuine way of computation [20]. However, from an applied

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perspective, the important thing is not to valuate the nature of the process, but its outcome. Today there is an enormous amount of computational processes that simulate approximate solutions to interesting problems. Still having the TM as a substrate, all of them have a very realist appearance, as it is showed by the software that control the smoothness of a train brake or the camera that stabilizes the frame for taking a good picture show. From this point of view, fuzzy computation is a fact regardless if the true nature of the computational process is fuzzy or not.

As we have said before, if it is possible to get good simulations of fuzzy instructions using classical methods, would still be desirable a true fuzzy computer? The answer of this question leads to discuss the differences and analogies between the real and the artificial, between the authentic and the simulated, directing, ultimately, the old philosophical question of what is 'being'. But this inquiry is out the limits of this paper.

Computing in the traditional sense involves inputs with numbers and symbols that are crisp. However, human beings use words to solve daily problems that attend to fuzzy rules; so to speak, humans use words to make computations. In the last years, some new idea has being emerging in this field. Fuzzy logic is emphasized to play a key role in granular computing, the methodological base for computing with words and perceptions. So, while in classical theory a sensor records a temperature by a number (30 C^o), in fuzzy theory a device should to convert the numerical input in the name of a perception: its hot. According to Ying [21], the start point of models for computing with words is to deal with fuzzy finite-state and fuzzy pushdown automata through extending their inputs to be strings of fuzzy subsets of input an alphabet. However, a study about the correlation between the strings or words and the symbols as inputs has not taken place.

In the context of translating human perceptions to computational perceptions and in the frame of a practical application, Triviño et al. [16] developed a methodology to generate a linguistic summary from data provided by sensors. A chain of transducers perform this task. A 'transducer' is a kind of automata capable of translating perceptions between two levels of granularity. In the project of a Linguistic Fuzzy Transducer the following actions are involved:

- Representing the granular word of the input by its associated vector of linguistic state variables.
- Representing the granular word of the output by its associated vector of linguistic state variables.
- Defining a set of rules that describes the evolution of the system in order to reach the outputs as a function of the values of the inputs.
- Generating a generic representation of the system by the methodology of Zadeh's protoform.

Besides fuzzy computation, there are other initiatives to deal with imprecision. The most remarkable is quantum computing. In quantum physics, vagueness or imprecision take the form of indetermination. But quantum indetermination and linguistic vagueness or imprecision are quite different concepts in nature. So, today quantum computing does not seem an adequate model for imprecise computing. Despite the different nature of vagueness and indetermination, the *continuum* and not deterministic nature of quantum computing encouraged to find alternative models in the general area of imprecision. Although today TM is the computational tool *par excellence*, other emerging tools, impelled by the problem of imprecision, are coming. If they can be an alternative to the classical way of computation is still an enigma, as they are in a starting point.

4 Are there fuzzy objects?

As previously noted, in 1923 Bertrand Russell published a seminal paper about vagueness. In that article, he provided a starting point to introduce the vagueness as a topic of discussion in the field of the analytic philosophy, that attempt to clarify, by analysis, the meaning of statements and concepts. After Russell, other philosophers like Hempel, Black, Dummet or Fine wrote about this subject [22].

Russell discussed the role of vagueness in the language and wrote: *"Vagueness and precision alike are characteristics which can only belong to a representation, of which language is an example. The have to do with the relation between a representation and that which it represents. Apart from representation, whether cognitive or mechanical, there can be no such thing as vagueness or precision; things are what they are, and there is an end of it"*[2, p. 2]. So, vagueness is confined to the realm of the language and excluded as a property of the objects.

Saying that there are only vague sentences and not vague objects seems to be a truism. Objects are what they are. Only the way to access them by the language is imperfect or imprecise in nature. In the vein of positivism, some philosophers thought the opposite: so, the first Wittgenstein said in the *Tractatus* that sentences, if they are to mean anything, must mirror reality in the same way that a picture does. But even in natural sciences, words picture objects in different ways: in that task, some words are more vague that others.

So, we can say unequivocally that the Everest is higher than Mont Blanc. But is it is intrinsically imprecise where the Mont Blanc starts or which area encompasses. A cloud is too a typical case of object with fuzzy boundaries. Quantum mechanics brought a new field of examples of vague objects. The 'spin' is a fundamental property of atomic nuclei, hadrons, and elementary particles. 'Having a spin direction x' is a property clearly defined and it is possible to know exactly when a particle is or not in a given spin direction. Making an experiment we can identify a number of electrons that have this property and the result obtained will be the same if the experiment is repeated. Nevertheless, we cannot notice if the particles that had the property in the first experiment are the same as the second. The particles fall or not under the extension of the predicate 'Having a spin direction x', but it is impossible to fix the identity of the resulting set.

We can refer to vague objects from three points of view:

- Objects are well defined but there is not enough information or appropriate knowledge for grasping them precisely.
- Objects are well defined but it is not possible to know their true being.
- Objects are ill defined because we cannot observe them

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directly and the instruments to watch them distort irremediably their nature.

The first two points are related to the representation of reality through language. The third refers to the essential nature of the objects. We approach them in an inverted order.

The mental experiment of Schrödinger's cat provides a good example of essential vagueness: suppose a cat into a box. Suppose you cannot see in it and the cat cannot see out. Into the box there are a rock, a Geiger counter and a vial of poison. Now, the rock is slightly radiactive, with an exactly 50% chance of emiting a subatomic particle during an hour. If the rock emits a particle, the Geiger counter activates a device that breaks the vial of poison, killing the cat. Quantum mechanics suggests that after an hour the cat is simultaneously alive and dead, so it is an indeterminate or vague object.

From a philosophical point of view, an essentialist perspective is approached by G. Evans, who address the impossibility of vague objects in the frame of the Kripke's theory of rigid designators. A term is a rigid designator if it always designates the same object in any world where the object exists. Proper nouns are rigid designators. Like this, we can imagine a world where the name 'cat' do not refer to a domestic, hairy, medium size, enemy of the mice,. . . , animal, but we cannot imagine that the name 'Lotfi Aliasker Zadeh' do not refer to Lotfi Aliasker Zadeh. If a word denotes always to the same object in every possible world, we can say that there are a strong correspondence between the word and the object: when we are talking about something, we can say that the words replace objects and the words have an unambiguous and precise meaning. Evans uses this preconception for showing the impossibility of vague objects.

In a short paper published in *Mind* in 1978, he introduced the question about the existence of vague objects [23]. The underlying question posed under the title: *Can there be vague objects?* This question adress the possibility of a vague identity relationship when both sides of the equal sign are occupied by terms that are rigid designators. Evans answers 'no' arguing the following proof (see table 1): Denote ∇ 'indeterminate',

Table 1: Evans' Theorem.

(1)	$\nabla(a = b)$	$a = b$ is indeterminate	(assumption)
(2)	$\lambda x[\nabla(a = x)]b$	b is indeterminely equal to a	(from 1)
(3)	$\neg\nabla(a = a)$	$a = a$ is determinate	axiom
(4)	$\neg\lambda x[\nabla(x = b)]a$	a is not indeterminely equal to b	(from 3)
(5)	$a \neq b$	a is not equal to b	(from 2 and 4)

Evans' argument shows that there are not vague entities if the terms are rigid designators: briefly, if it is indeterminate that *a* is equal to *b* and it is determinate that *a* and *a* are equals, whatever *a* be, *a* and *b* differ at least in one property; so, are different. But the Evans' argument entails a mistake: as D. Lewis point out in [24], Evans' proof start attaching vagueness to the identity relation involving rigid designators. But the own proof demonstrates that the premise is false; therefore, the starting point of the argument was inadequate. Evans' argument shows the inconvenience to judge the imprecision in a small subset of the language –that of rigid designators–, and the mistake to extend this conclusion to the overall language.

In general, language is imprecise in nature and its formal approach, a matter of degree. Furthermore, some words are essentially vague. In the sequel, we analyse the second way in which an object can be vague, according to the bottom clasification. There are objects of which we know all that it is required for knowing their meaning, for attributing them a valuation, but, in some cases, we cannot do that in a unique way. A 'Bald man' or a 'heap of sand' are some examples.

Sorites or *Falakros* is a well-known paradox since Greek philosophers that comes from the use of predicates as 'bald' or 'heap' in the argumentation. To analyse a *Sorites* argument with a classical look leads to paradoxical and unacceptable conclusions. Note that baldness can be mainly referred in terms of hairs and their distribution, that is, in terms of numerable instances. So, a man with 0 hairs is absolutely bald and a man with 10^{10} hairs, not. But counting does not ever clarify the status of a vague predicate. Adding to every man that is bald one hair does not modify its attribute and preserve bald. But the key point is how many hairs are neccesary to change a man from bald to not bald. If you choose a number, I can select another very close to the previous and to claim the truth with the same legitimacy. There is not a bald object that determines, in a unanimous way, a unique use of the word 'bald', but only different uses depending on the context. Different uses mean diverse representations, leading to variations –between some margins– in the conclusions reached by argumentations. Trillas and Urtubey proposed a reasonably cut point that divide when a person is going to be more bald that not bald [25]. But following a kind of metatheoretical game and applying a type-*sorites* reasoning to it, the solution comes irremeably affected by high-order vagueness: the objection that emerge is why not suggest a very similar, but distinct, cut point, to the one supplied. So, it seems quite difficult to solve problems of vagueness with precise measures.

Finally, there are objects well defined but there is not enough information or appropriate knowledge for grasping them precisely; that is, objects that are imprecisely verbalized. Linguistic imprecision should be analysed with fuzzy armamentarium. Fuzzy logic provides tools for managing problems that involve imprecise statements or approximate reasoning. In the field of fuzzy logic, there is not a debate about what is a vague object but how to represent words with vague meaning. From some philosophical traditions (phenomenology, e.g.), an object is how we describe it using words in context. Objects are represented by words, and sentences are represented by logic. If objects have a crisp shape, perhaps precise sentences can designate them and classical membership functions should be employed to formalize them. If the objects have loose frontiers, the way to speak about them are vague in nature and generalized membership functions should be used.

For fuzzy logic there are vague objects as there are fuzzy representations of them. There is no other consideration about its true nature. Fuzzy logic is not a physical theory, but a formal one. So, it does not pretend to reveal the essential nature of the objects, but knowing how people verbalize and represent them in a manageable way in order to solve problems. For fuzzy logic, fuzzy objects are mental or physical objects acceded by sentences uttered in a culture or context that point out the consensus required for an adequate formalization.

Fuzzy logic repairs on linguistic vagueness, not vague objects, anything they that be. Therefore, words or predicates

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that are instead of the objects are the objective of fuzzy logic. So, its function is to be faithful to how that ordinary people describe reality trough language, generating models to catch the meaning of the predicates according their use in context, generalising solutions whenever possible, making hypothesis on its explanatory character and contrasting the reached conclusions with those awaited. This is a dialectical process, typical of experimental sciences. As a formal tool, fuzzy logic must pursue eternal truths –even imprecise truths–, but like an experimental one, facts must show that truth is obtained only in a degree.

In addition to pursuing affordable challenges, as providing models for a large number of vague sentences in real scenarios, fuzzy logic must address others questions that are still elusive. One of them is how to justify and to manage a suitable specification of the context, basic for representing, in an adequate way, the meaning of vague predicates. There are some formal theories of context, but seems too complex to be manageable or too simple to account the complexity of the meaning. Other challenge is how to summarize a complex perception, where attention is frequently directed to an issue, with a unique membership function. Those are applied inquiries that, however, also require a theoretical debate.

5 Final Remarks

Philosophical discussion about fuzzy logic and its applications is convenient. The popularity of fuzzy logic is based, in a large extent, on the success of its applications. Applications show the working face of a theory, but its consolidation and credit come through the development of a meta-theory, that shed light over its foundations and limits. More work should be done in this direction.

Although vagueness is treated from fuzzy logic mostly by the side of applications, theoretical studies are useful if shed intriguing doubts or suggest illuminating challenges to applied developments. Theory without applications is lame, but applications without theory are blind.

So, to justify the election of a membership function consistent with the true context of a used word or to argue about alternatives for combining atomic sentences into a composite one are tasks that will help to approach more credible solutions to problems every time more generals; so, to transfer credit to fuzzy logic as logic.

The discussion about alternatives to classical computation and the characteristic of a genuine fuzzy TM serve to distinguish the real and the imitated behavior. Also it is interesting to emphasize the simulation power of the classical TM to get fuzzy outputs managing approximate algorithms with crisp implementations.

Finally, to consider how we represent objects permits us to focus on tools as fuzzy logic that manage the meaning of the words that verbalize not only objects, but objects in context.

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