Multi-Goal Aggregation of Reduced Preference Relations Based on Fuzzy Interactions between Decision Goals

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Abstract *– Many aggregation operators like fuzzy integrals and similar require a preference relation as input. If the number of decision alternatives increases it becomes almost impossible to provide the exponentially increasing number of preference statements which are needed because a preference relation is defined upon the power set of the set of decision alternatives. If, as in many real world applications, the number of decision goals increases, it is also almost impossible to supervise the consistency of preference statements with respect to the interactions between the decision goals. In this paper it is discussed in which way a decision making approach based on interactions between goals is applied in order to use preference information that is not defined upon the power set of the set decision alternatives but simply on the decision set itself. Instead of defining the preferences upon the power set of the decision alternatives with respect to all the goals, for every decision goal the preference of the decision alternatives for this goal is defined upon the set of decision alternatives by a linear preference ranking of the decision alternatives. Consequences for both decision modeling based on classical preference relations and based on weighted sums are discussed.*

Keywords– Aggregation complexity, decision making, interactions between goals, reduced preference relation, weighted sum.

1 Introduction

Many decision making approaches are limited with respect to the management of complexity [4], [5], [8], [9], [10]. The consequence of this limitation is that these models are rather not applicable for real world problems. Many aggregation approaches require a preference relation as input. For more complex decision problems such preference relations are difficult to obtain because a preference relation is defined upon the power set of the set of the decision alternatives. If the number of decision alternatives increases it becomes almost impossible to provide the exponentially increasing number of preference statements needed. If additionally, as in many real world applications, the number of decision goals increases, it is almost impossible to supervise the consistency of preference statements with respect to the interactions between the decision goals. In this paper it is discussed in which way a decision making approach based on interactions between goals is adapted in order to aggregate in complex decision situations requiring only reduced linear preference information. The required input preference information is not defined upon the power set of the decision alternatives. Instead of this, for every single decision goal a linear preference ranking of the decision alternatives with respect to that goal is required. Of course, the single goal preference rankings may rank the decision alternatives in different way for different goals as the goals usually are partly conflicting. The presented aggregation approach ascertains for each pair of goals their conflicts (and correlations) by computing the so called interactions between the goals from the initial input single goal rankings. The only additional information needed when the interactions between the goals have been calculated is a linear importance information of each goal which is expressed in terms of goal priorities. The goal priorities are numbers of [0,1] and are comparable with a fuzzy measure that ranks not the decision alternatives but the decision goals itself with respect to their importance (priority).

It turned out that the decision making based on interactions between goals is less limited because the complexity of both the input information required and the aggregation process is not higher than polynomial. Since the model has successfully been applied to many real world problems [3] it clearly helped to manage many complex aggregation processes in which the number was varying between 50 and 100. The number of decision alternatives was in some applications even more than several thousands.

In the subsequent sections first for better readability of the paper we repeat a brief description of the decision making approach based on interactions between decision goals. Then we show how the approach is applied in order to work with single goal preference rankings and a statement with respect to classical preference based decision modeling is made. Subsequently it is discussed under which conditions decision models based on weighted sums help and how they are related to decision situations with interacting decision goals. Finally, the consequences of the results are discussed.

2 Decision Making based on Interactions between Goals

In the following it is shown how an explicit modeling of interaction between decision goals that are defined as fuzzy sets of decision alternatives helps to manage complexity of the decision making and aggregation. This modeling of the decision making and aggregation process significantly differs from the related approaches and the way they manage complex decision situations. First the notion of positive and negative impact sets is introduced. Then different types of interaction between goals are defined. After this it is shown how interactions between goals are used in order to aggregate pairs of goals to the so called local decision sets. Then it is described how the local decision sets are used for the aggregation of a final decision set. The complexity of the different steps is discussed.

2.1 Positive and Negative Impact Sets

Before we define interactions between goals as fuzzy relations, we introduce the notion of the positive impact set and the negative impact set of a goal. A more detailed discussion can be found in [1],[2] and [3].

Def. 1a) Let A be a non-empty and finite set of decision alternatives, G a non-empty and finite set of goals, $A \cap G$

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$=\emptyset, a \in A, g \in G, \delta \in (0,1]$. For each goal g we define the two fuzzy sets S_g and D_g each from A into [0, 1] by:

1. Positive impact function of the goal g: $S_g(a) := \delta$, if a affects g positively with degree δ, $S_g(a) := 0$ else.
2. Negative impact function of the goal g: $D_g(a) := \delta$, if a affects g negatively with degree δ, $D_g(a) := 0$ else.

Def. 1b) Let S_g and D_g be defined as in Def. 1a). S_g is called the positive impact set of g and D_g the negative impact set of g.

The set S_g contains alternatives with a positive impact on the goal g and δ is the degree of the positive impact. The set D_g contains alternatives with a negative impact on the goal g and δ is the degree of the negative impact.

2.2 Interactions between Goals

Let $\mathsf{P}(A)$ be the set of all fuzzy subsets of A. Let X, Y ∈ $\mathsf{P}(A)$, x and y the membership functions of X and Y respectively. Assume now that we have a binary fuzzy inclusion $\mathrm{I}: \mathsf{P}(A) \times \mathsf{P}(A) \rightarrow [0,1]$ and a fuzzy non-inclusion $\mathrm{N}: \mathsf{P}(A) \times \mathsf{P}(A) \rightarrow [0,1]$, such that $\mathrm{N}(X,Y) := 1 - \mathrm{I}(X,Y)$. In such a case the degree of inclusions and non-inclusions between the impact sets of two goals indicate the degree of the existence of interaction between these two goals. The higher the degree of inclusion between the positive impact sets of two goals, the more cooperative the interaction between them. The higher the degree of inclusion between the positive impact set of one goal and the negative impact set of the second, the more competitive the interaction. The non-inclusions are evaluated in a similar way. The higher the degree of non-inclusion between the positive impact sets of two goals, the less cooperative the interaction between them. The higher the degree of non-inclusion between the positive impact set of one goal and the negative impact set of the second, the less competitive the relationship. The pair (S_g, D_g) represents the whole known impact of alternatives on the goal g. Then S_g is the fuzzy set of alternatives which satisfy the goal g. D_g is the fuzzy set of alternatives which are rather not recommendable from the point of view of satisfying the goal g.

Based on the inclusion and non-inclusion between the impact sets of the goals as described above, 8 basic fuzzy types of interaction between goals are defined. The different types of interaction describe the spectrum from a high confluence between goals (analogy) to a strict competition (trade-off) [1].

Def. 2) Let S_{g_1}, D_{g_1}, S_{g_2} and D_{g_2} be fuzzy sets given by the corresponding membership functions as defined in Def. 1). For simplicity we write S_1 instead of S_{g_1} etc.. Let $g_1, g_2 \in$ G where G is a set of goals. *T* is a t-norm.

The fuzzy types of interaction between two goals are defined as relations which are fuzzy subsets of $G \times G$ as follows*:*

1. g_1 *is independent of* g_2*:* <=> $T(\mathrm{N}(S_1, S_2), \mathrm{N}(S_1, D_2), \mathrm{N}(S_2, D_1), \mathrm{N}(D_1, D_2))$
2. g_1 *assists* g_2*:* <=> $T(\mathrm{I}(S_1 S_2), \mathrm{N}(S_1, D_2))$
3. g_1 *cooperates with* g_2*:* <=> $T(\mathrm{I}(S_1, S_2), \mathrm{N}(S_1, D_2), \mathrm{N}(S_2, D_1))$
4. g_1 *is analogous to* g_2*:* <=> $T(\mathrm{I}(S_1, S_2), \mathrm{N}(S_1, D_2), \mathrm{N}(S_2, D_1), \mathrm{I}(D_1, D_2))$
5. g_1 *hinders* g_2*:* <=> $T(\mathrm{N}(S_1, S_2), \mathrm{I}(S_1, D_2))$
6. g_1 *competes with* g_2*:* <=> $T(\mathrm{N}(S_1, S_2), \mathrm{I}(S_1, D_2), \mathrm{I}(S_2, D_1))$
7. g_1 *is in trade-off to* g_2*:* <=> $T(\mathrm{N}(S_1, S_2), \mathrm{I}(S_1, D_2), \mathrm{I}(S_2, D_1), \mathrm{N}(D_1, D_2))$
8. g_1 *is unspecified dependent from* g_2*:* <=> $T(\mathrm{I}(S_1, S_2), \mathrm{I}(S_1, D_2), \mathrm{I}(S_2, D_1), \mathrm{I}(D_1, D_2))$

The interactions between goals are crucial for an adequate orientation during the decision making process because they reflect the way the goals depend on each other and describe the pros and cons of the decision alternatives with respect to the goals. For example, for cooperative goals a conjunctive aggregation is appropriate. If the goals are rather competitive, then an aggregation based on an exclusive disjunction is appropriate.

Note that the complexity of the calculation of every type of interaction between two goals is O(card(A) * card(A)) = O((card(A))^2) [4].

2.3 Two Goals Aggregation based on the Type of their Interaction

The assumption, that cooperative types of interaction between goals imply conjunctive aggregation and conflicting types of interaction between goals rather lead to exclusive disjunctive aggregation, is easy to accept from the intuitive point of view. It is also easy to accept that in case of independent or unspecified dependent goals a disjunctive aggregation is appropriate. For a more detailed formal discussion see for instance [1],[2]. Knowing the type of interaction between two goals means to recognize for which goals rather a conjunctive aggregation is appropriate and for which goals rather a disjunctive or even exclusively disjunctive aggregation is appropriate. This knowledge then in connection with information about goal priorities is used in order to apply interaction dependent aggregation policies which describe the way of aggregation for each type of interaction. The aggregation policies define which kind of aggregation operation is the appropriate one for each pair of goals. The aggregation of two goals gi and gj leads to the so called local decision set $L_{i,j.}$. For each pair of goals there is a local decision set $L_{i,j.} \in$ P(A), where A is the set of decision alternatives (see Def 1 a)) and P(A) the power set upon A. For conflicting goals, for instance, the following aggregation policy which deduces the appropriate decision set is given:

if (g_1 is in trade-off to g_2) and (g_1 is slightly more important than g_2) then $L_{1,2} := S_1 / D_2$.

In case of very similar goals (analogous or cooperative goals) the priority information even is not necessary:
if (g_1 cooperates with g_2) then $L_{1,2} := S_1 \cap S_2$ because $S_1 \cap S_2$ surely satisfies both goals.
if (g_1 is independent of g_2) then $L_{1,2} := S_1 \cup S_2$ because

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$S_1 \cup S_2$ surely do not interact neither positively nor negatively and we may and want to pursue both goals.

In this way for every pair of goals g_i and g_j, i,j ∈ {1,…,n} decision sets are aggregated. The importance of goals is expressed by the so called priorities. A priority of a goal g_i is a real number $P_i \in [0,1]$. The comparison of the priorities is modeled based on the linear ordering of the real interval [0,1]. The statements like g_i *slightly more important than* g_j are defined as linguistic labels that simply express the extend of the difference between P_i and P_j.

2.4 Multiple Goal Aggregation as Final Aggregation based on the Local Decision Sets

The next step of the aggregation process is the final aggregation. The final aggregation is performed based on a sorting procedure of all local decision sets $L_{i,j}$. Again the priority information is used to build a semi-linear hierarchy of the local decision sets by sorting them. The sorting process sorts the local decision sets with respect to the priorities of the goals. Subsequently an intersection set of all local decision sets is built. If this intersection set is empty then the intersection of all local decision sets except the last one in the hierarchy is built. If the resulting intersection set again is empty then the second last local decision set is excluded from the intersection process. The process iterates until the intersection is not empty (or more generally speaking until its fuzzy cardinality is big enough with respect to a given threshold). The first nonempty intersection in the iteration process is the final decision set and the membership values of this set give a ranking of the decision alternatives that is the result of the aggregation process (for more details see [2]).

2.5 Complexity Analysis of the Aggregation Process

As already discussed for instance in [4] the complexity of the aggregation process is $O((card(A))^2 * (card(G))^2)$ and the complexity of the information required for the description of both the positive and the negative impact functions is $O(card(A) * card(G))$.

3 Application of the Aggregation in Case of Reduced Preference Relations

In preference based decision making the input preference information has to be defined upon the power set of the decision alternatives [6]. This means that the complexity of the input information required is exponential with respect to the cardinality of the set of decision alternatives. This also means that the input information required is very difficult to obtain. Especially if the number of decision goals increases and the goals are partly conflicting the required preference information has to be multidimensional and the provider of the preference information has to express all the multidimensional interactions between the goals through the preference relation. With increasing number of goals and interactions between them the complexity of the required input preference relation possesses the same complexity as the decision problem itself. But, if the complexity of the required input is the same as the solution of the underlying decision problem itself then the subsequent aggregation of the input does not really help to solve the problem and is rather obsolete.

Therefore we propose to reduce the complexity of the input preference relation. Instead of requiring a preference relation defined upon the power set of the set of the decision alternatives which expresses the multidimensionality of the impacts of the decision alternatives on the goals for every single decision goal a linear preference ranking of the decision alternatives with respect to that goal is required. This means that for every goal a preference ranking defined on the set of decision alternatives is required instead of a ranking defined on the power set of the alternatives. The multidimensionality of the goals is then computed from all the single goal preference rankings using the concept of interactions between the goals as defined in section X.2.

In the sequent we extend the definition Def. 1. The extension defines how a single goal preference ranking defined on the set of the decision alternatives is transformed into positive and negative impact sets:

Def. 1c) Let A be a non-empty and finite set of decision alternatives, G a non-empty, finite set of goals as defined in Def. 1) a),

$A \cap G = \emptyset, a \in A, g \in G, \delta \in (0,1]$.

Let $>p_g$ be a preference ranking defined upon A with respect to *g* defining a total order upon A with respect to g, such that

$a_{i1} >p_g a_{i2} >p_g a_{i3} >p_g \dots >p_g a_{im}$, where m=card(A) and $\forall a_{ij}, a_{ik} \in A$, $a_{ij} >p_g a_{ik} :\Leftrightarrow a_{ij}$ is preferred to a_{ik} with respect to the goal g. The preference relation $>p_g$ is called the *reduced single goal preference relation* of the goal g.

For simplicity, instead of $>p_g$ we also equivalently write RSPR of the goal g. All the RSPRs for all $g \in G$ are called the *reduced preference relation* RPR for the whole set of goals *G*. In order to avoid complete redundancy within the RPR we additionally define that the RSPRs of all the goals are different.

Let us assume that there is a decision situation with *n* decision goals where *n=card(G)* and *m* decision alternatives where *m=card(A)*. In the subsequent we propose an additional extension of the original definition Def. 1b with the aim to transform the single goal preference relations RSPR of every goals $g \in G$ into the positive and negative impact sets *Sg* and *Dg*:

Def. 1d) Let again A be a non-empty and finite set of decision alternatives, G a non-empty, finite set of goals as defined in Def. 1) a), $A \cap B \neq \emptyset$, $a_i \in A$, $m=card(A)$, $g \in G$, i, c∈ {1, …, *m*}. For any goal g we obtain both the positive and the negative impact sets Sg and Dg by defining the values of δ according to Def. 1a) and 1b) as follows:

Def. 1d1) For the positive impact set:

$Sg(a_i)=\delta:=1/i$ iff $i \in [1,c-1]$,

$Sg(a_i)=\delta:=0$ iff $i \in [c,m]$.

Def. 1d2) For the negative impact set:

$Dg(a_i)=\delta:=0$ iff $i \in [1,c-1]$,

$Sg(a_i) = \delta:= 1/(m-i+1)$ iff $i \in [c, m]$.

Using the definition Def. 1d1 and Def. 1d2 for any goal $g \in G$ we obtain a transformation of the RPR into positive

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and negative impact sets of all the goals and can evaluate the interactions of the goals using Def. 2 that are implied by the RPR.

Compared to classical preference based decision models this transformation helps to reduce the complexity of the input preference information required without losing modeling power for complex real world problems. The advantage is that using Def. 2. the interactions between goals that are implied by the RPR expose the incompatibilities and compatibilities that may be hidden in the RPR. The exposed incompatibilities and compatibilities are used adequately during the further calculation of the decision sets. Note that the exposition is calculated with a polynomial number of calculation steps with degree 2. The only additional information required from the decision maker is the priority information for each goal which has to be expressed as a weight with a value between 0 and 1. Many real world applications show that despite the reduced input complexity there is no substantial loss of decision quality [3].

Statement 1: *In particular this means that it is not necessary to have classical preference relations defined upon the power set of the decision alternatives in order to handle complex decision problems with both positively and negatively interacting decision goals.*

Another interesting question is how the decision making based on interactions between decision goals is formally related to aggregation methods based on weighted sums. In order to investigate this we introduce the notion of r-consistency of RPRs and will consider the question under which conditions the weighted sum aggregation may be appropriate from the point of view of the application of the decision making based on interactions between goals if we have an RPR as input. For this we define the following:

Def.3) Given a discrete and finite set A of decision alternatives. Given a discrete and finite set G of goals. Let r $\in$ (0,1]. The reduced preference relation RPR is called r-consistent $:\Leftrightarrow \exists$ c $\in$ $\{1, \ldots ,m\}$, $m=card(A)$ such that $\forall$ $(g_i, g_j) \in G \times G$, $i,j \in \{1, \ldots ,n\}$, $n=card(G)$, *(*g_i *cooperates with* g_j*)* $\geq r$.

Let us now consider an important consequence of the interaction between goals using the notion of r-consistency. This notion will imply a condition under which an aggregation based on a weighted sum may lead to an appropriate final decision.

For this let us assume that the quite sophisticated final aggregation process of the iterative intersections as described in section 2.4 is replaced by the following rather intuitive straight forward consideration of how to obtain an optimal final decision.

Again let us identify the local decision sets $L_{i,j}$ obtained after the application of the local decision policies for each pair of goals (g_i, g_j) as the first type of decision subsets of the set of all decision alternatives A which, according to the decision model are expected to contain an optimal decision alternative a_k. Thus we define the set of sets

T1:= $\{ L_{i,j} \mid i,j \in \{1, .. n\}, n=card(G)\}$ and expect that the optimal decision alternative has a positive membership in at least one of the sets of T1. Since we want to consider multiple goals we may also expect that an optimal decision alternative may have a positive membership in at least one of the intersections of pairs of the local $L_{i,j}$. Thus we define T2:= $\{ L_{k,l} \cap L_{p,q} \mid k,l,p,q \in \{1, .. n\}, n=card(G)\}$. In order to simplify the subsequent explanation we concentrate on the crisp case and replace all membership values > 0 in all $L_{i,j}$, and $L_{k,l} \cap L_{p,q}$ by the membership value 1. Now we define the system of these crisp sets GDS as follows:

Def.4) GDS:=$\{\emptyset$,T1,T2$\}$, T1,T2 are the sets of crisp sets that we construct as described above by replacing all membership values > 0 in all $L_{i,j}$, and $L_{k,l} \cap L_{p,q}$ by the membership value 1.

With this definition we are able to formulate the following theorem that describes a property of the crisp GDS in the case that the underlying decision situation stems from a reduced preference relation RPR to which the decision model based on interactions between decision goals is applied. This property will enable us to relate this decision making to the calculation of optimal decisions by concepts based on weighted sums that are strongly connected with the notion of a matroid [7] and optimal decisions obtained by Greedy algorithms.

Theorem 1: If the reduced preference relation RPR is r-consistent then the system ($\mathcal{P}(A)$,GDS) is a matroid.

Sketch of the Proof:

The proof will show that the following matroid conditions [7] hold: 1. $\emptyset \in$ GDS, 2. $X \subseteq Y$, $Y \in$ GDS $\Rightarrow$ $X \in$ GDS and 3. $X,Y \in$ GDS card(X)<card(Y) $\Rightarrow \exists x \in Y\backslash X$ and $X \cup \{x\} \in$ GDS.

Ad1.: Holds by definition of GDS. Ad2.: If X is one of the $L_{i,j}$s then Y must be one of the another $L_{k,l}$ with $i \neq k$, $j \neq l$. If X is one of the $L_{k,l} \cap L_{p,q}$ then Y must be $L_{k,l}$ or $L_{k,l}$. Ad3.: This condition holds because the RPR is r-consistent and therefore we have $\forall$ k,l,p,q $\in \{1, .. n\}$, $n=card(G)$, card($L_{i,j}$)=card($L_{k,l} \cap L_{p,q}$)=c.

With this result we obtain the following corollary that relates decision making based on interactions between decision goals to calculating optimal decisions by concepts based on weighted sums.

Corollary 1: If the reduced preference relation RPR is r-consistent then there exists a greedy algorithm based on the maximization of the weighted sum $\Sigma_{i \in \{1,\ldots,n\}, k \in \{1,\ldots,m\}} P_i * S_i(a_k)$, *where* S_i *is defined as in Def. 1d1 and* $a_k \in A$ *being a decision alternative,* $n=card(G)$, $m=card(A)$. P_i *is the priority of the goal* g_i *as used for the calculation of both the local and the global decision sets (see sections 4.3 and 4.4).*

Sketch of the Proof:

The proof is an immediate result from the Theorem 1 and the theory of matroids used in the field of combinatorial optimization [7].

4 Discussion of the Consequences

As already mentioned, this corollary relates the decision making based on interactions between decision goals to the calculation of optimal decisions by concepts based on weighted sums. It shows that weighted sums both as aggregation and optimization concept are rather appropriate if the goals or criteria are cooperative e.g. if they interact

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positively (or at least do not interact at all being independent). In contrast to this the decision making based on interactions between decision goals is more general and it reflects both positive and negative interactions between the goals. This is important in the context of real world decision making and optimization problems which usually possess partly conflicting goal structures. If the aggregation is performed by weighted sums, like for instance Choquet integrals, then one can suppose that the aggregation will only work if the choices to be made are free of conflicts. The hypothesis formulated in [6] postulating that there exists a preference relation on subsets of decision alternatives which is compatible with a weak order of decision alternatives belonging to different subsets of decision alternatives seems not to be that natural as postulated (especially in the field of optimization problems where partly contradicting goals are daily business).

Even if for a given application field the postulated preference relation exists, in case the number of decision alternatives increases it becomes almost impossible to provide the exponentially increasing number of required preference statements. If additionally, as in many real world applications, the number of decision goals increases, it is almost impossible for a human decision maker to keep all the interactions between the goals in mind and supervise the consistency of the preference statements with respect to the interactions between these decision goals. We see that the adapted decision making approach based on interactions between goals helps reducing the complexity of the required preference information without reducing neither the quality of the decision results [3] nor the complexity of decision situations that are modeled [4]: The required input preference information is not defined upon the power set of the decision alternatives. Instead of defining the required preferences upon the power set of the decision alternatives with respect to all the goals, for every decision goal the preference of the decision alternatives for this goal is defined upon the set of decision alternatives by a simple linearly ordered preference ranking of the decision alternatives (for each particular goal). This means that it is not necessary to require from the decision maker that he or she keeps in mind all the interactions between the goals. We are able to calculate the interactions as implication of linearly ordered single goal preference rankings by applying Def. 2. accordingly as shown in section 3. The only additional information that we require from the decision maker are assertions about the importance of the goals, namely again only a simple (linear) ordering of the so-called priorities of goals which are real numbers from the interval [0,1] and express for every goal its importance in a particular decision situation (see sections 2.3 and 2.4).

5 Conclusions

In this paper it is discussed in which way a decision making approach based on interactions between goals is applied in order to use preference information that is not defined upon the power set of the set decision alternatives but simply on the decision set itself. Instead of defining the preferences upon the power set of the decision alternatives with respect to all the goals, for every decision goal the preference of the decision alternatives for this goal is defined upon the set of decision alternatives by a linear preference ranking of the decision alternatives. Although the input complexity is reduced, the decision quality is not. Therefore we conclude that classical preference relations defined upon the power set of the decision alternatives are not necessary in order to adequately handle complex decision problems. We have also discussed in which way the decision model based on interaction between decision goals can be related to decision making methods based on weighted sums only. Through a link to the theory of matroids it is shown that the model is more general in the sense that an adequate aggregation is possible even if the goals are partly conflicting whereas weighted sums are in this kind of situations rather restricted.

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