On the information provided by uncertainty measures in the classification of remote sensing images

Luísa M S Gonçalves [1,2] *Cidália C Fonte* [2,3] *Eduardo N B S Júlio* [4] *Mario Caetano* [5,6]

1 Polytechnic Institute of Leiria, Civil Engineering Department, Portugal
2 Institute for Systems and Computers Engineering at Coimbra, Portugal
3 Department of Mathematics, University of Coimbra, Portugal
4 ISISE, Civil Engineering Department, University of Coimbra, Portugal
5 Portuguese Geographic Institute (IGP), Remote Sensing Unit (RSU), Lisboa, Portugal
6 CEGI, Instituto Superior de Estatística e Gestão de Informação, ISEGI, Universidade Nova de Lisboa, Portugal
Email: luisag@estg.ipleiria.pt, cfonte@mat.uc.pt, ejulio@dec.uc.pt, mario.caetano@igeo.pt

Abstract*—This paper investigates the potential information provided to the user by the uncertainty measures applied to the possibility distributions associated with the spatial units of an IKONOS satellite image, generated by two fuzzy classifiers, based, respectively, on the Nearest Neighbour Classifier and the Minimum Distance to Means Classifier. The deviation of the geographic unit characteristics from the prototype of the class to which the geographic unit is assigned is evaluated with the Un non-specificity uncertainty measures proposed by [1] and the exaggeration uncertainty measure proposed by [2]. The classifications were evaluated using accuracy and uncertainty indexes to determine their compatibility. Both classifications generated medium to high levels of uncertainty for almost all classes, and the global accuracy indexes computed were 70% for the Nearest Neighbour Classifier and 53% for the Minimum Distance to Means Classifier. The results show that similar conclusions can be obtained with accuracy and uncertainty indexes and the latter, along with the analysis of the possibility distributions, may be used as indicators of the classification performance and may therefore be very useful tools. Since the uncertainty indexes may be computed to all spatial units, the spatial distribution of the uncertainty was also analysed. It's visualization shows that regions where less reliability is expected present a great amount of detail that may be potentially useful to the user.*

Keywords—Accuracy assessment, Minimum Distance to Mean Classifier, Nearest Neighbour Classifier, Non-specificity measures, Remote Sensing Images, Uncertainty.

1 Introduction

Some classifiers used in Remote Sensing allow the assignment of each spatial unit (pixel or object) to several classes through the computation of degrees of possibility, probability or membership associated with each class, as opposed to the traditional classifiers where each spatial unit is only assigned to one class [3, 4, 5, 6]. This additional information may be interpreted as degrees of membership of the spatial unit to the classes, and in this case are usually referred to as soft classifiers [7]. Even though the use of soft classifiers is increasing, most applications still require a hard classification into disjoint classes of interest. For these cases, the spatial units may be assigned to the class presenting the larger degree of possibility or probability. The additional information provided by these degrees of possibility of probability may be used as indicators of the classifier difficulty to assign only one class to the spatial unit and, together with the application of uncertainty measures, may provide valuable information to the user [2, 8, 9, 10].

In [1] it is shown that, for a fuzzy classifier which assigns to each spatial unit degrees of membership or possibility associated with the classes, the non-specificity measures may be used to estimate the classification accuracy. They can therefore be a useful tool, since, on one hand, they can give an estimation of the classification accuracy prior to the final accuracy evaluation, enabling the classification improvement before the final classification accuracy is assessed (which is a time consuming and expensive process), and on the other hand enable the spatialization of the uncertainty [11]. In this paper a very high resolution image is classified with two fuzzy classifiers and the uncertainty measures (two non-specificity measures and an exaggeration measure) are applied to both classifications, to determine the usefulness of the information provided by them. A comparison between the information provided by the non-specificity measures and the producer's and user's accuracy indexes is made to determine their compatibility.

To evaluate the usefulness of the information provided by the spatialization of uncertainty, an approach similar to the one proposed by [2] is considered, where it is proposed that, when the spatial units are assigned to the best degree of similarity, errors of omission and commission are committed. The former occurs because, when assigning the spatial unit only to the best class, its similarity to the other classes is omitted. The latter occurs because, when the spatial unit is assigned to the best class and the degree of similarity to that class is smaller that one, a commission error is introduced, since a higher similarity is considered. To estimate omission errors [2] used an entropy measure. Since the Shannon entropy should only be applied to probability measures, the U_n uncertainty measure was used instead. To estimate the commission errors the measure of exaggeration proposed in [2] was used. The results are shown and conclusions drawn.

2 Data and methods

2.1 Data

The study was conducted in a rural area with a smooth topographic relief, situated in a transition zone between the

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centre and south of Portugal featuring diverse landscapes representing Mediterranean environments. The area is occupied mainly by agriculture, pastures, forest and agro-forestry areas where the dominant forest species in the region are eucalyptus, coniferous and cork trees. An image obtained by the IKONOS sensor was used, with a spatial resolution of respectively 1m in the panchromatic mode and 4m in the multi-spectral mode (XS) and a dimension of 11 884 m by 14 432m. The geometric correction of the multi-spectral image consisted of its orthorectification. The average quadratic error obtained for the geometric correction was 1.39 m, inferior to half the pixel size, which guarantees an accurate geo-referencing.

2.2 Classification

Two fuzzy classifiers were used in this application to obtain the elementary entities that are the basic units of landscape, like crown trees and parts of buildings, called Surface Elements (SE), to produce a Surface Elements Map (SEM). Both classifiers compute degrees of similarity [12] between the values observed at each spatial unit and a set of ideal values, which are, in this case, the spectral response observed in each spatial unit and what is considered to be the ideal characteristics of the class in terms of spectral values, respectively. Even though both classifiers use the same theoretical tools to derive the degrees of similarity, the definition of the ideal characteristics of each class is different for each of them.

The first classification method used is an object–oriented supervised Fuzzy classifier based on the Nearest Neighbor Classifier (FNNC) available on the software eCognition. In the supervised classification methods, a training set has to be chosen for each class. The spectral responses of the pixels inside this set are the signature files used to characterize each class. This classification method assigns to each pixel a degree of membership to each class, depending on the distance between its spectral response and the closest spectral response of the pixels used in the training set for each class (see Fig. 1).

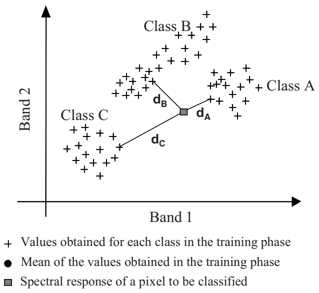

Figure 1: Nearest Neighbor Classifier.

The degrees of membership are computed using a membership function, where the ideal value, corresponding to a degree of membership equal to one, is obtained when the distance between the spectral response of the pixel to classify to the nearest reflectance value of the training set of the class is equal to zero (see Fig. 2).

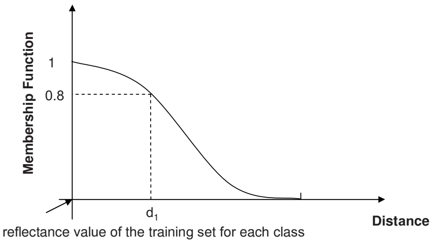

Figure 2: Membership function used in the fuzzy classifier based on the Nearest Neighbor Classifier.

The second classification method used is a pixel-based supervised Fuzzy classifier based on the underlying logic of Minimum-Distance-to-Means Classifier (FMDMC), available in the commercial software IDRISI. With this classification method, the image is classified based on the information contained in the signature files and a standard deviation unit (Z-score distance) introduced by the user. The fuzzy set membership is calculated based on a standardized Euclidean distance from each pixel reflectance, on each band, to the mean reflectance for each class signature (see Fig. 3), using a sigmoidal membership function (see Fig. 4).

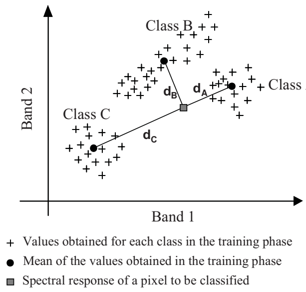

Figure 3: Minimum-Distance-to-Means Classifier.

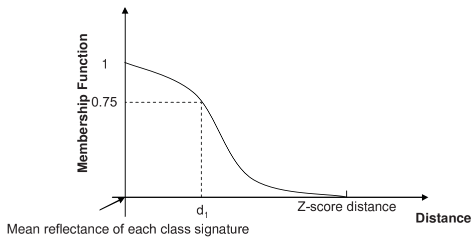

Figure 4: Membership function used in the fuzzy classifier based on the Minimum-Distance-to-Means Classifier

The underlying logic is that the mean of a signature represents the ideal point for the class, where fuzzy set membership is one. When distance increases, fuzzy set membership decreases, until it reaches the user-defined Z-score distance where fuzzy set membership decreases to zero. To determine the value to use for the standard deviation unit, the information of the training data set was used to study the spectral separability of the classes and determine their average separability measure.

The most representative surface elements of the study area are Eucalyptus Trees (ET), Coniferous Trees (CFT), Cork Trees (CKT), Shadows (S), Shallow Water (SW), Deep Water (DW), Herbaceous Vegetation (HV), Sparse Herbaceous Vegetation (SHV) and Non-Vegetated Area (NVA), and therefore these were the considered classes.

2.3 Uncertainty measures

To evaluate the uncertainty of the possibilistic classifier the measures of nonspecificity were used. The uncertainty measure *NSp* proposed by [13] and given by

$$NSp(\Pi) = 1 - \sum_{i=1}^{n} \left[\Pi\left(x_i\right) - \Pi\left(x_{i+1}\right) \right] \frac{1}{i}$$

and a normalized version proposed by [1] of the U-uncertainty measure developed by [14], given by

$$U_n(\Pi) = \frac{[1 - \Pi(x_1)] \log_2 n + \sum_{i=2}^{n} [\Pi(x_i) - \Pi(x_{i+1})] \log_2 i}{\log_2 n}$$

In both equations Π is an ordered possibility distribution defined over a universal set X, that is, a possibility distribution such that $\Pi\left(x_1\right) \geq \Pi\left(x_2\right) \geq \ldots \geq \Pi\left(x_n\right)$, n is the number of elements of the universal set and $\Pi\left(x_{n+1}\right)$ takes the value zero. A normalized version of the U-uncertainty measure was used to normalize the range of the measure, so that both non-specificity measures vary within the interval [0,1]. The non-specificity measures are appropriate to evaluate the uncertainty resulting from possibilistic classifications, since they quantify the ambiguity in specifying an exact solution [15, 16].

An additional uncertainty measure of exaggeration E, proposed by [2], was also used to quantify the commission errors at each spatial unit. This measure is given by

$$E = 1 - \Pi\left(x_1\right)$$

where $\Pi\left(x_1\right)$ is the largest degree of possibility.

2.4 Classification evaluation with the accuracy indexes

To evaluate the classifications accuracy a testing set was used. A random sampling of 900 pixels was selected considering the entire image scene. The sample unit was the pixel. The accuracy assessments was made with error matrixes, where the (i,j) entry is the number of pixels that is class i in the map and class j in the reference. The reference data were obtained from aerial images with larger resolution. The Global Accuracy was computed for both classifications. The User Accuracy (UA) and the Producer Accuracy (PA) indexes were also computed for all classes.

2.5 Comparison between uncertainty and accuracy indexes

To determine if the information given by the uncertainty measures may be used as indicators of the classifier's performance and if they are correlated with the results obtained with the classification quality assessment, two accuracy indexes were considered, namely the normalized user's and producer's accuracy; and one index for each uncertainty measure based on the mean, corresponding to the complement of the mean uncertainty per class ($I_{1-\overline{NSp}}$ and $I_{1-\overline{U_n}}$). Since high uncertainty values are expected to correspond to low accuracy values, and vice-versa, the complement of the uncertainty measures is considered to allow an easy comparison with the accuracy indexes. The correlation coefficient between the several accuracy and uncertainty indexes was also computed.

Even though uncertainty measures can be computed to the whole image, since the objective was to compare the results given by the uncertainty measures with the ones given by the error matrix, for this comparison only the sample pixels were used, so that the results were not influenced by the sample representativeness.

2.6 Spatial variation of uncertainty

Since there is a possibility distribution associated with all pixels or objects of the image, the uncertainty indexes may be computed for the whole image. This enables the visualization of the spatial distribution of uncertainty, and its spatial relation with the classes assigned to each spatial unit. As an estimator of the spatial units omission errors the U_n uncertainty measure was used, since it is more sensitive to dispersion than NSp (see [1]) and to estimate the commission errors the exaggeration E measure was used.

3 Results and discussion

3.1 Classification evaluation with accuracy indexes

The error matrix computed for the classification with the FNNC is presented in Fig. 5. The global accuracy obtained with this classification method was 70%.

	Error matrix of the classification with the fuzzy KNN									User's Accuracy (%)
	DW	SW	NVA	ET	S	HV	CKT	CFT	SHV	
DW	102									100.0
SW	3	92								96.8
NVA			76				2	1	8	87.4
ET				42	4	6	7	18	11	47.7
S	2				77		5			91.7
HV			2	1		90			7	90.0
CKT		1	11	8	18		50	6	26	41.7
CFT				3		43	10	42	9	39.3
SHV			23	3		22	16	3	73	52.1
Producer's Accuracy (%)	95.3	98.9	67.9	73.7	77.8	55.9	55.6	60.0	54.5	**69.77%**

Figure 5: Error matrix of the image classification with the method based on the Nearest Neighbour Classifier.

The error matrix shows that water classes (DW and SW) were well identified. Forestry species were often confused between each other and with other classes, such as Sparse Herbaceous Vegetation (SHV) and Herbaceous Vegetation (HV). Significant confusion was observed between Herbaceous Vegetation (HV) and Coniferous Trees (CFT). The class with the smaller value of PA is SHV (54.5%), which means it is the class with more omission error. And the class with smaller UA is CFT (39.3%) and therefore the class with more commission errors.

The global accuracy obtained with the classification with the FMDMC was 53% and the error matrix computed is presented in Fig. 6. The error matrix shows that Deep Water (DW) was the class best identified. Significant confusion was observed between Cork Trees (CKT), Eucalyptus Trees (ET) and Sparse Herbaceous Vegetation (SHV) and between ET and Coniferous Trees (CFT). The class with the smaller value of PA is SHV (14.8%), which means this is the class with more omission error. The pixels which are erroneously not included in this class are included mainly in the class Non Vegetated Areas (NVA), which is the class with smaller UA (27%) and therefore the class with more commission

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error, which receives pixels from all classes. These results show that this classifier presents great difficulty in classifying the class NVA.

	Error matrix of the classification with the fuzzy MDM									User's Accuracy (%)
	DW	SW	NVA	ET	S	HV	CKT	CFT	SHV	
DW	89				1					98.9
SW	3	54	1							93.1
NVA	16	40	104	8	19	79	15	15	89	27.0
ET				37		5	8	17	4	52.1
S	1				35		7			81.4
HV						65			1	98.5
CKT			7	13	3		50	11	18	49.0
CFT				5		16	6	27	3	47.4
SHV			3	1		2	6		20	62.5
Producer's Accuracy (%)	81.7	57.4	90.4	57.8	60.3	38.9	54.3	38.6	14.8	**53.21%**

Figure 6: Error matrix of the image classification with the method based on the Minimum Distance to Mean Classifier.

3.2 Comparison between uncertainty and accuracy indexes

For the classification made with the FNNC, the uncertainty indexes $I_{1-\overline{NSp}}$ and $I_{1-\overline{U_n}}$ along with the user's and producer's accuracy, ordered with increasing values of $I_{1-\overline{NSp}}$, are shown in Fig. 7.

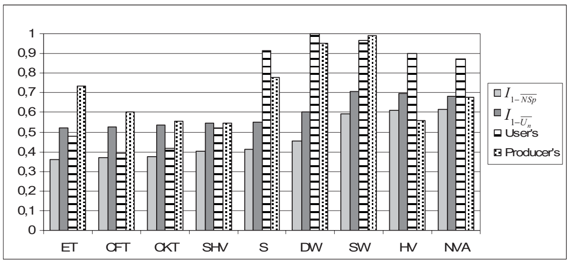

Figure 7: $I_{1-\overline{NSp}}$ and $I_{1-\overline{U_n}}$ uncertainty indexes along with the user's and producer's accuracy of the classification based on the Nearest Neighbour Classifier.

It can be observed that for all classes the uncertainty indexes present low to medium values. To determine the reason for these relatively low values of certainty, the observation of the possibility distribution was made, and it was observed that the largest degree of possibility is in general very high, but also high levels of possibility are obtained for the second and sometimes even for the third classes, which explains the high levels of uncertainty (see Table 1). The analysis of the possibility distributions and the classes associated with the second degrees of possibility for the several points enable the identification of classes that might be easily confused, which are mainly the forest classes and SHV. Similar conclusions can be taken from the confusion matrix, with the exception that considerable confusion was also expected between CFT and HV, and this is not expected from the analysis of the degrees of possibility.

From this analysis, worst results are expected for the classes ET, CFT, CKT and SHV, which are exactly the classes with low levels of UA, $I_{1-\overline{NSp}}$ and $I_{1-\overline{U_n}}$. This is an expected result, since in [1] it is shown that high correlation values are expected to occur between the uncertainty indexes and the UA. Similarly, the classes with higher values of $I_{1-\overline{NSp}}$ and $I_{1-\overline{U_n}}$ are in general the ones presenting higher UA, except for the classes S and DW, which have slightly lower value of $I_{1-\overline{NSp}}$ and $I_{1-\overline{U_n}}$.

Table 1: Mean values of the highest and second highest degrees of possibility of the possibility distributions associated with the sample points used to evaluate the accuracy of the classification with the FNNC, along with the most frequent second class.

Best class	$\overline{\Pi(x_1)}$	$\overline{\Pi(x_2)}$	Second class	% of pixels with the second class
SW	0.96	0.66	NVA	100
DW	0.99	0.87	NVA	51
NVA	0.84	0.37	SHV	52
ET	0.99	0.96	CKT	48
HV	0.93	0.55	SHV	50
SHV	0.94	0.83	CKT	52
CFT	0.98	0.94	ET	56
CKT	0.99	0.94	SHV	46
S	0.98	0.89	CKT	64

For the classification made with the FMDMC, the uncertainty indexes $I_{1-\overline{NSp}}$ and $I_{1-\overline{U_n}}$ along with the user's and producer's accuracy, ordered with increasing values of $I_{1-\overline{NSp}}$, are shown in Fig. 8. It can be seen that the values of certainty given by both NSp and U_n are low to medium for all classes, presenting $I_{1-\overline{U_n}}$ slightly larger values than $I_{1-\overline{NSp}}$.

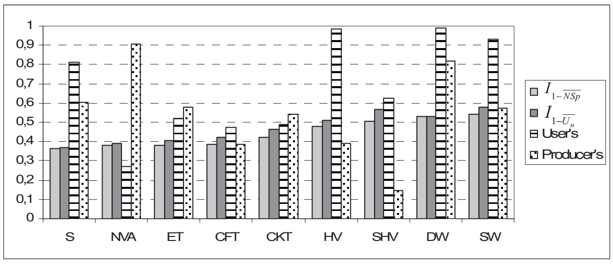

Figure 8: $I_{1-\overline{NSp}}$ and $I_{1-\overline{U_n}}$ uncertainty indexes along with the user's and producer's accuracy for the classification with the FMDMC.

To determine why such relatively high levels of uncertainty were obtained, an analysis of the possibility distributions was also made. This analysis showed that, for all classes, a high percentage of the pixels presented relatively low values for the highest degree of possibility, which explains the relatively low levels of certainty, and no point was classified with the highest degree of possibility equal to one (see Table 2). Further more, for all classes, the second highest degree of possibility for the great majority of testing sites was to the class Non Vegetated Areas (NVA) (except, of course, for the class NVA), in some cases with the degrees of possibility close to the higher degree, which means that there was great confusion between this class and all other classes.

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Table 2: Mean values of the highest and second highest degrees of possibility of the possibility distributions associated with the sample points used to evaluate the accuracy of the classification with the FMDMC, along with the most frequent second class.

Best class	$\overline{\Pi(x_1)}$	$\overline{\Pi(x_2)}$	Second class	% of pixels with the second class
SW	0.62	0.19	NVA	100
DW	0.53	0.003	NVA	26
NVA	0.41	0.07	HV	16
ET	0.48	0.14	NVA	88
HV	0.57	0.16	NVA	95
SHV	0.63	0.38	NVA	100
CFT	0.52	0.22	NVA	82
CKT	0.54	0.22	NVA	90
S	0.38	0.03	NVA	100

This indicates that one major problem of this classification was mainly due to the difficulty in discriminating NVA from the other classes. This same conclusion can be obtained from the accuracy analysis. The class NVA presents very different values for the user and producer accuracy, respectively 27% and 90%. This means that this class presents a very high percentage of commission errors, and therefore, a great amount of sites that should have been assigned to other classes were assigned to NVA, and were therefore absent from the other classes, such as SHV and HV, increasing their omission errors.

The information provided by the uncertainty measures is therefore in accordance with the one provided by the accuracy indexes, even though the analysis of the possibility distributions is necessary to reach the obtained conclusions.

The most evident discrepancies between the uncertainty and accuracy indexes obtained with the classification with the FMDMC are mainly for the class DW and S. These classes present higher values of accuracy, meaning that they were well identified according to the accuracy indexes, but considerably low levels of classification certainty, corresponding to lower values of the uncertainty indexes. This can be explained by the relatively low level of the first degree of possibility and the confusion between classes, which reinforces the assumption that uncertainty indexes capture the classifier difficulty in determining the correct class. Apparently some discrepancies occur between uncertainty and accuracy indexes for the SW and HV classes due to the fact that they present higher values of UA (93,1 and 98,5 respectively). However, the PA results are lower (57,4% and 38,9 % respectively) which reveal that these classes present higher omission errors and are in accordance with the lower values of $I_{1-\overline{U_n}}$ index. Two main aspects seem to be responsible for the results obtained with this classifier: 1) the larger degrees of possibility for each pixel are in general relatively low, which results in high uncertainty levels; 2) there is considerable confusion between all classes and the class NVA.

To explain the low degrees of possibility obtained with this classifier, a closer look to its classification approach is required. The computed degrees of possibility reflect the closeness of the spectral response of the testing set to an ideal value, which is the mean of the spectral responses obtained for the training set. The computed degrees of possibility are then obtained considering the distance between the spectral response at each testing site and that mean value, and therefore, if the obtained value is relatively distant from the mean, even if its spectral response is very close to some values obtained for the training set, a low degree of possibility will be assigned to it. This classifier and the obtained degrees of possibility, have therefore a limited capability to translate the information provided by the training set. For example, if a pixel has a spectral response equal to the spectral response of one of the pixels used in the training set, but which is relatively far from the mean, it will not have a degree of possibility of belonging to that class equal to one, as should be expected.

The fuzzy classifier based on the FNNC has a better behavior on this aspect, since it translates the information contained in the training set in a more reliable way. For example, all points with a spectral response equal to the spectral response of a point included in the training set will have a degree of possibility equal to one. Furthermore, if the spectral response of pixels is located inside the regions populated by the spectral responses of a particular class, high degrees of similarity are expected to occur to that class.

3.3 Spatial variation of uncertainty

Fig. 10 shows on a), b) and c) respectively an extract of the classification with the FNNC and the values of the U_n and E uncertainty measures. Fig. 10 d), e) and f) show the corresponding images obtained with the FMDMC.

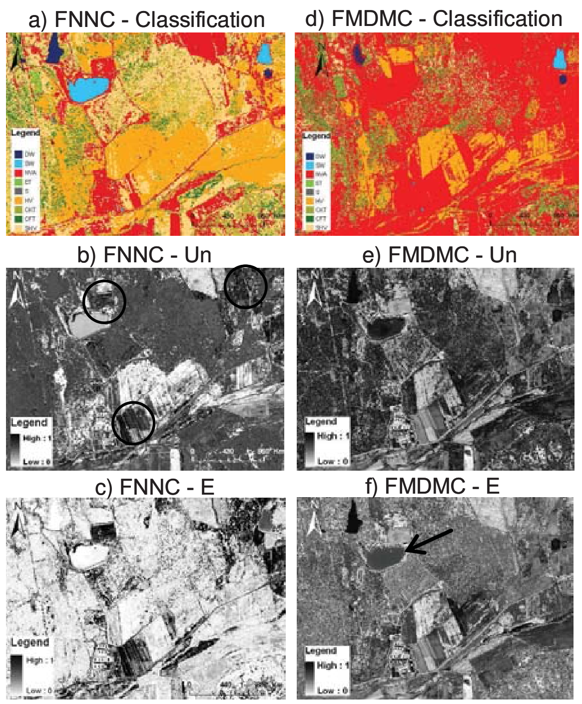

Figure 10: a), b) and c) show respectively the classification results, U_n and E uncertainty values obtained with the FNNC. Images d), e) and f) show the results corresponding to a), b) and c) obtained with the FMDMC.

It can be easily observed that the classification results are very different. Even though the accuracy evaluation and the uncertainty indexes already indicated that the classification with the FMDMC was worst, the visualization of the results clearly shows that it is bad. It can be seen that the FMDMC classified most of the image as NVA, while the FNNC manage to differentiate much more structures in the image.

Images shown in Fig. 10 b) and e) correspond to the spatial distribution of the U_n uncertainty measure of both classifications and may be used to illustrate the omission error committed when the pixel is assigned to the class corresponding to the largest degree of possibility. The regions with larger uncertainty are the ones where the degrees of possibility were more dispersed over other classes. Fig. 10b) shows that relatively large values of U_n were associated with regions that correspond to landscape units composed by different surface elements such as forest trees and Sparse Herbaceous Vegetation. Some region with higher U_n uncertainty can also be seen in the zones identified by the black circumferences. Fig. 10c) and f) show the commission errors, that is, the exaggeration committed when the best class is chosen. In general in Fig. 10c) low values are obtained for most of the image, higher values are mainly visible in some landscape boundaries and in the regions identified by the circumferences in Fig. 10b), which indicate, once again, that those regions may be problematic. Fig. 10f) shows that larger commission errors are obtained with the FMDMC than with the FNNC. One aspect clearly visible from Fig. 10e) and f) is that the spatial variation of uncertainty shows many structures invisible in the classification, such as the SW region shown in the region identified by the arrow.

4 Conclusions

The results obtained with the presented case study highlight that the classifiers FMDMC and FNNC presented possibility distributions with particular characteristics, which are a consequence of the method used to compute the degrees of possibility. The classification with the FNNC presented large possibility values for the best class, but also presented large values for the second and sometimes third class, and the classification with the FMDMC presented low values of the largest possibility. In both cases the presented characteristics increase uncertainty and are a signal of possible difficulties in the classification. In fact, neither of these classifications presented very good results, since Global Accuracies of only 70% and 53% were obtained.

It is also shown that when the classifications present relatively large and similar uncertainty values for all classes it is more difficult to take conclusions regarding the problematic classes only with these measures. Even though, an analysis of the possibility distributions may give valuable information. It is also very important to know how the degrees of possibility associated to the pixels are obtained, to understand what they represent.

In this study it was considered that the confusion matrix and the accuracy indexes were reliable, but in reality a human interpreted may have large difficulties in differentiating for example NVA from SHV, and this type of confusion between classes may influence greatly the accuracy indexes. In future analysis the uncertainty information present in the reference data, used to build the confusion matrixes, should be taken in consideration when comparing the results provided by the uncertainty and accuracy information.

The presented results also show that the spatialization of uncertainty may provide valuable information to the user. This will enable the identification of problematic regions, where the classification is probably less reliable. On the other hand, the spatial variation of uncertainty provides a great amount of information, which, if introduced in the classification itself, might be very useful.

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