On interval and fuzzy calculations of economic uncertainty

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Abstract*—This paper emphasizes that numerically correct calculation of economic uncertainty with intervals and fuzzy numbers requires implementation of global optimization techniques in contrast to straightforward application of interval arithmetic. In general, the latter approach produces incorrect results in the sense that the degree of uncertainty is severely overestimated with the risk of improper decision making. This is demonstrated by a simple case from managerial economics as well as a cost estimation case from a real life railway reconstruction project.*

Keywords—Economic uncertainty, fuzzy numbers, intervals, possibility, probability.

1 Introduction

Criticism has been raised towards probability theory as being a too normative framework to take all the aspects of uncertain judgement into account, Dubois and Prade [1]. In this paper we will focus on alternative methods of modelling of economic uncertainty like the interval representation and the fuzzy number representation.

The interval representation, Moore [2], [3] is particularly well suited to a situation where the knowledge of an uncertain parameter is limited to knowing its minimum and maximum value whereas nothing else is known. Based upon a mathematical theory of interval analysis this approach has shown to be useful in keeping track of worst and best cases in economic analyses and thus contribute to improved decision processes, see e.g. Schjær-Jacobsen [4], [5]. The introduction of fuzzy sets supports the concept of possibility rather than probability and translates natural language expressions into the mathematical formalism of possibility measures. It is generally recognized that possibility is distinct from probability. Probabilities can be interpreted as relative frequencies or, more generally, uncertain knowledge or belief of a statistical nature. In contrast, possibility relates to the degree of feasibility and ease of attainment or imprecise knowledge. Particularly, this paper will deal with some computational aspects of uncertainty representation by intervals and fuzzy numbers. It will be demonstrated that correct results are obtained in the general case by applying global optimization. It is the author's experience that these aspects are widely neglected, probably due to lack of communication and interaction between professional communities each one advocating either concept.

2 A simple case from managerial economics

In order to demonstrate the principles of numerical modelling of uncertainty by intervals and fuzzy numbers presented in this paper we introduce a simple yet instructive referential case from managerial economics. Consider a company selling one product at price p and quantity q into a market. For the sake of simplicity we may assume fixed cost to be zero. The turnover TR is

$$TR = TR(p,q) = p \cdot q, \qquad (1)$$

and the variable cost VC is given by

$$VC = VC(q) = 20 \cdot q. \qquad (2)$$

Then the profit π is

$$\pi = \pi(p,q) = TR(p,q) - VC(q) = p \cdot q - 20 \cdot q. \qquad (3)$$

We want to find the price p* and quantity q* that gives the maximum profit π*. It is seen from (3) that the profit is a monotone function of q and p which means that no maximum of π exists unless variables are constrained. If for example $p \le 65$ and $q \le 450$, then p* = 65, q* = 450, and π* = 20.250.

Next we suppose that price and quantity are interdependent variables due to market conditions. Assuming that they are connected by the demand function

$$p = p(q) = 100 - 0{,}1 \cdot q \qquad (4)$$

we get for the turnover

$$TR = TR(p) = -10 \cdot p^2 + 1.000 \cdot p, \qquad (5)$$

the variable cost

$$VC = VC(p) = -200 \cdot p + 20.000, \qquad (6)$$

and the profit margin

$$\pi = \pi(p) = -10 \cdot p^2 + 1.200 \cdot p - 20.000. \qquad (7)$$

The profit of (7) is maximized by the optimality criterion that marginal cost MC be equal to marginal turnover MR which gives the results

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$$p^* = 60,\ q^* = 400,\ \text{and}\ \pi^* = 16.000. \tag{8}$$

In the following, we shall refer to the above examples for explanations of computational details and special features.

3 Uncertainty modelling using intervals

2.1 Basics of interval analysis

Following Moore [3] and Caprani, Madsen, and Nielsen [6] we define a real interval number as an ordered pair [a; b] of real numbers with $a \le b$. It may also be defined as an ordinary set of real numbers x such that $a \le x \le b$, or

$$[a; b] = \{x \mid a \le x \le b\}. \tag{9}$$

If the basic arithmetic operations addition, subtraction, multiplication, and division are denoted by the symbol #, we can define operations on two intervals $\mathbf{I}_1 = [a_1; b_1]$ and $\mathbf{I}_2 = [a_2; b_2]$ based on the set-theoretic formulation:

$$\mathbf{I}_1 \# \mathbf{I}_2 = \{x \# y \mid a_1 \le x \le b_1,\ a_2 \le y \le b_2\}. \tag{10}$$

For basic operations on the intervals $\mathbf{I}_1$ and $\mathbf{I}_2$ we get the resulting interval $\mathbf{I} = [a; b]$ by the formulas

$$\begin{aligned}
\mathbf{I} &= \mathbf{I}_1 + \mathbf{I}_2 = [a_1+a_2;\ b_1+b_2],\\
\mathbf{I} &= \mathbf{I}_1 - \mathbf{I}_2 = [a_1-b_2;\ b_1-a_2],\\
\mathbf{I} &= \mathbf{I}_1 \cdot \mathbf{I}_2 = [\min(a_1\cdot a_2, a_1\cdot b_2, b_1\cdot a_2, b_1\cdot b_2);\\
&\qquad \max(a_1\cdot a_2, a_1\cdot b_2, b_1\cdot a_2, b_1\cdot b_2)],\\
\mathbf{I} &= \mathbf{I}_1 / \mathbf{I}_2 = [\min(a_1/a_2, a_1/b_2, b_1/a_2, b_1/b_2);\\
&\qquad \max(a_1/a_2, a_1/b_2, b_1/a_2, b_1/b_2)],\ 0 \notin [a_2; b_2].
\end{aligned} \tag{11}$$

It can be shown that the four basic interval operations are inclusion monotonic, commutative, and associative. However, the distributive rule is not valid in general. Instead, the so-called sub-distributivity holds, but only for addition and multiplication [6]

$$\mathbf{I}_1\cdot(\mathbf{I}_2 + \mathbf{I}_3) \subseteq \mathbf{I}_1\cdot\mathbf{I}_2 + \mathbf{I}_1\cdot\mathbf{I}_3. \tag{12}$$

From a rational real valued function F of n real valued variables

$$F = F(x_1, x_2, \ldots, x_n) \tag{13}$$

we can create the interval extension function as an interval function $\mathbf{F}^I$ of n intervals

$$\mathbf{F}^I = \mathbf{F}^I(\mathbf{I}_1, \mathbf{I}_2, \ldots, \mathbf{I}_n) \tag{14}$$

simply by replacing the real operators by interval operators and the real variables by intervals.

A rational function can be formulated in many ways whereas the same reformulations cannot be done for interval expressions due to the invalidity of the distributive rule. This implies that different formulations of a rational function will lead to different interval extension functions and thus to different interval results [6]. In the case of F being a monotonic function within the entire range of the input variables the minimum and maximum of $\mathbf{F}^I$ as an interval can simply be found among the function values F at the extreme points of the variables. In the general case of F being non-monotonic or variables appearing more then once, the calculation of $\mathbf{F}^I$ as an interval is non-trivial, which is demonstrated in the following example.

Example: Based on the real valued function $F = x\cdot(1 - x)$ the interval function $\mathbf{F}^I = \mathbf{I}\cdot(1 - \mathbf{I})$, $\mathbf{I} = [0; 1]$ is calculated. Straightforward application of formulas from (11) gives the result $\mathbf{F}^I = [0; 1]$ whereas the correct result is [0; 0,25].

In this paper the term "correct" is used to indicate the narrowest possible interval that can be calculated for an uncertain variable. Generally, to obtain this, iterative global optimization methods have to be used, see e.g. Hansen [7] and Kjøller *et al.* [8]. In order to obtain correct results (as in the above example) to an accuracy specified by the user, interval calculations in this paper are carried out using the Interval Solver 2000 program, Hyvönen and De Pascale [9], [10], as an add-in module to MS-Excel 2000. An overall absolute and relative precision of 10^{-6} has been applied.

Correct calculation of interval functions allows for strong statements about the uncertainties involved. *Firstly*, you can say that provided all uncertain input variables stay within their minimum and maximum values, the uncertain output function will stay within its minimum and maximum values. *Secondly*, the uncertain output function will not attain any value that is not a function value of some combination of the uncertain input values (within their minimum and maximum values).

2.2 Independent and interdependent variables

Independent variables using interval arithmetic

To calculate the uncertain profit we first use the formulas of interval arithmetic (11) by three different ways of calculation. As an example look at the independent and uncertain quantity and price

$$\mathbf{p} = [55; 65],\ \mathbf{q} = [350; 450], \tag{15}$$

Firstly, we use the turnover and variable cost as intermediate variables. By (1) we get for the uncertain turnover **TR** and by (2) for the uncertain variable cost **VC**

$$\begin{aligned}
\mathbf{TR} &= [55; 65]\cdot[350; 450] = [19.250; 29.250],\\
\mathbf{VC} &= 20\cdot[350; 450] = [7.000; 9.000].
\end{aligned} \tag{16}$$

Then by (3) we get for the uncertain profit

$$\boldsymbol{\pi} = \mathbf{TR} - \mathbf{VC} = [10.250; 22.250]. \tag{17}$$

However, the above calculation produces a too wide interval (17) for the profit. The reason for this is that in the expression (3) the variable q appears twice thus allowing the quantity q used to calculate **TR** to be different from the quantity used to calculate **VC**.

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Secondly, the rightmost form of (3) is used, giving

$$\boldsymbol{\pi} = [350; 450]\cdot[55; 65] - 20\cdot[350; 450] = [10.250; 22.250], \quad (18)$$

which is identical to (17) (and thus incorrect) because the arithmetic operations are identical.

Thirdly, (3) is rearranged before the interval calculations are carried out:

$$\pi = \pi(p,q) = q\cdot(p - 20). \quad (19)$$

We then get for the uncertain profit

$$\boldsymbol{\pi} = [350; 450]\cdot([55; 65] - 20) = [350; 450]\cdot[35; 45] = [12.250; 20.250], \quad (20)$$

which is a somewhat narrower interval than (17) and (18) because each variable is appearing only once in (19). Actually, (20) is the correct result. This can easily be verified simply by closer numerical inspection of the profit function (3) for various combinations of the variables p and q. The results are summarised in Table 1.

Independent variables using global optimization

Next, we calculate the uncertain profit by global optimization using Interval Solver 2000. With the same input variables (15) and intermediate variables **TR** and **VC** we get from (3)

$$\mathbf{TR} = [19.250; 29.250],\ \mathbf{VC} = [7.000; 9.000],$$
$$\boldsymbol{\pi} = \mathbf{TR} - \mathbf{VC} = [10.250; 22.250], \quad (21)$$

which is identical to (17) and (18) for the same reason as mentioned above. With the same input variables and by way of formulas (18) and (19) we get the result

$$\mathbf{p} = [55; 65],\ \mathbf{q} = [350; 450],$$
$$\boldsymbol{\pi} = [12.250; 20.250], \quad (22)$$

which is seen to be the correct result identical to (20). The results are summarised in Table 1.

Interdependent variables using interval arithmetic

To perform an uncertainty analysis around a given price p = 60 we set

$$\mathbf{p} = [55; 65]. \quad (23)$$

Obviously, since the quantity variable q can be eliminated by using the demand function (4), the number of variables is reduced from two to one, namely the price. Even in this simple case, as it turns out, the results obtained by interval arithmetic can be dramatically incorrect due to the fact that the profit function is now non-monotonic.

The profit is calculated in two different ways using interval arithmetic (11). *Firstly*, the profit is calculated by

Table 1. Summary of uncertainty analyses, independent price **p** = [55; 65] and quantity **q** = [350; 450]. Correct results shown in *italics*.

Calculation Method	$\boldsymbol{\pi} = \mathbf{TR} - \mathbf{VC}$	$\boldsymbol{\pi} = \mathbf{p}\cdot\mathbf{q} - 20\cdot\mathbf{q}$	$\boldsymbol{\pi} = \mathbf{q}\cdot(\mathbf{p} - 20)$
Interval Arithmetic	[10.250; 22.250]	[10.250; 22.250]	*[12.250; 20.250]*
Global Optimization	[10.250; 22.250]	*[12.250; 20.250]*	*[12.250; 20.250]*

intermediate variables **TR** and **VC** according to (5) and (6), respectively

$$\mathbf{TR} = -10\cdot[55; 65]^2 + 1.000\cdot[55; 65] = [12.750; 34.750] \quad (24)$$

and

$$\mathbf{VC} = -200\cdot[55; 65] + 20.000 = [7.000; 9.000], \quad (25)$$

which gives

$$\boldsymbol{\pi} = \mathbf{TR} - \mathbf{VC} = [3.750; 27.750]. \quad (26)$$

Secondly, the profit is calculated by the above formula

$$\boldsymbol{\pi} = -10\cdot\mathbf{p}^2 + 1.200\cdot\mathbf{p} - 20.000 \quad (27)$$

yielding

$$\boldsymbol{\pi} = -10\cdot[55; 65]^2 + 1.200\cdot[55; 65] - 20.000 = [3.750; 27.750]. \quad (28)$$

The results found above are identical but way out of order, since the correct result of the uncertain profit is [15.750; 16.000]. This is easily verified by closer numerical inspection of (7) with varying p.

Interdependent variables using global optimization

Global optimization is used to calculate the uncertain profit by means of (26) yielding [13.750; 17.750] and (27) yielding [15.750; 16.000]. It may be observed that correct results are produced in the latter case but not in the former, due to usage of intermediate variables **TR** and **VC**, which allows for a too wide resulting interval. The results are shown in Table 2.

Table 2. Summary of uncertainty analyses, interdependent price **p** = [55; 65] and quantity **q** = 1.000 – 10·**p**. Correct result shown in *italics*.

Calculation Method	$\boldsymbol{\pi} = \mathbf{TR} - \mathbf{VC}$	$\boldsymbol{\pi} = -10\cdot\mathbf{p}^2 + 1.200\cdot\mathbf{p} - 20.000$
Interval Arithmetic	[3.750; 27.750]	[3.750; 27.750]
Global Optimization	[13.750; 17.750]	*[15.750; 16.000]*

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The differences in Tables 1 and 2 between correct intervals and the more wide intervals are considerable. When an uncertain variable is appearing more than once, interval arithmetic will usually produce too wide results and global optimization must to be used. With non-monotonic functions, global optimization is necessary. In all cases, intermediate variables should be avoided, since both interval arithmetic and global optimization will produce incorrect results.

4 Uncertainty modelling using fuzzy numbers

4.1 Fuzzy numbers and intervals

A fuzzy set A in X where X is a space of points (objects) with a generic element of X denoted by x, i.e. X = {x}, is characterized by a membership function $f_A(x)$ which associates with each point in X a real number in the interval [0; 1]. The value of the membership function $f_A(x)$ at x represents the "grade of membership" of x in A. Thus the closer the value of $f_A(x)$ to unity, the higher the grade of membership of x in A. Note that when A is an ordinary set, i.e. non-fuzzy, the membership function can take only two values 0 and 1. In other words, a fuzzy set is a set of ordered pairs (x, $f_A(x)$)

$$A = \{(x, f_A(x)) \mid x \in X\}. \quad (34)$$

It is also useful to define the ordinary (non-fuzzy) set A_α as the α-cut of A:

$$A_\alpha = \{x \in X \mid f_A(x) \geq \alpha, 0 \leq \alpha \leq 1\}. \quad (35)$$

In this paper we are mainly interested in the concept of fuzzy numbers as a means of representing uncertain or fuzzy information, Dubois and Prade [11], [12]. In addition to the simplest fuzzy number, namely the interval, we also make use of the triangular fuzzy number, Chiu and Park [13], [a; c; b] where a ≤ c ≤ b, that can be defined by its membership function:

$$\begin{aligned} f(x) &= (x-a)/(c-a), && a \leq x \leq c, \\ &= (b-x)/(b-c), && c \leq x \leq b, \\ &= 0, && \text{otherwise.} \end{aligned} \quad (36)$$

Mathematical operations on triangular fuzzy numbers can be facilitated by introducing the left L(α) and right R(α) representation of a fuzzy triangular number $\mathbf{F}^T$, refer to the α-cut (35):

$$\mathbf{F}^T = [L(\alpha); R(\alpha)], \text{ where}$$
$$L(\alpha) = a + (c-a)\alpha \text{ and } R(\alpha) = b + (c-b)\alpha, \quad (37)$$
$$\alpha \in [0, 1].$$

Observe that in this notation a fuzzy number is written as an interval with upper and lower bounds depending on α. This means that addition, subtraction, multiplication, and division can be carried out by using interval methods for all values of α. Likewise, for any triangular function, the resulting triangular functional values can be calculated and represented by L and R functions using interval methods for all values of α.

Example: Based on the real valued function F = x·(1 – x) calculate the corresponding fuzzy function with triangular argument [0; 0,5; 1]. Correct results have been calculated with global optimization and are shown in Table 3. From the results in Table 3 it can be seen that the function F has a maximum of 0,250 at x = 0,5 corresponding to α = 1. Also note that the R(α) function has been correctly calculated to 0,250 for all values of α.

To obtain simpler representations and reduce the number of calculations, triple and quadruple representations of fuzzy variables corresponding to α-cuts 0 and 1 in (35) may be used. In the above example the result then is [0; 0,25; 0,25], where the extreme function values is obtained by global optimization on the interval [0; 1] and the interior point is obtained by conventional calculation at x = 0,5.

4.2 Simple case with triangular fuzzy numbers

Next we calculate the simple case from Section 2 using profit function (3) with independent triangular fuzzy input parameters **p** = [55; 60; 65], **q** = [350; 400; 450] defined by (36). The resulting membership function is shown in Table 4 for different values of α. It is easily seen that the value α = 0 corresponds to the correct profit interval previously found and α = 1 corresponds to the single point calculation of the profit. This is one of the important features of the triangular fuzzy number representation of uncertainty: It is easily communicated and understood that the ordinary single point calculation is extended to an interval around the single point representing the uncertain value of the input variable. The resulting triangular fuzzy profit is interpreted as follows: With the given uncertain input variables, the most possible value of the profit is 16.000 and profits outside the interval [12.250; 20.250] are impossible.

Calculations have also been carried out with interdependent triangular fuzzy input price parameter corresponding to **p** = [55; 60; 65] and the profit function (7), the results are shown in Table 5. For all values of α the correct maximum profit of 16.000 has been found. No profit values outside the interval [15.750; 16.000] are possible given the uncertain price input variable **p**.

Table 3. Fuzzy extension of x·(1-x) calculated with triangular argument [0; 0,5; 1].

α	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
L(α)	0,000	0,047	0,090	0,127	0,160	0,188	0,210	0,227	0,240	0,248	0,250
R(α)	0,250	0,250	0,250	0,250	0,250	0,250	0,250	0,250	0,250	0,250	0,250

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Table 4. Uncertain profit (3) by global optimization, triangular fuzzy input **p** = [55; 60; 65], **q** = [350; 400; 450].

α	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
L(α)	12.250	12.603	12.960	13.323	13.690	14.063	14.440	14.823	15.210	15.603	16.000
R(α)	20.250	19.803	19.360	18.923	18.490	18.063	17.640	17.223	16.810	16.403	16.000

Table 5. Uncertain profit (7) by global optimization, triangular fuzzy input **p** = [55; 60; 65].

α	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
L(α)	15.750	15.797	15.840	15.877	15.910	15.937	15.960	15.977	15.990	15.977	16.000
R(α)	16.000	16.000	16.000	16.000	16.000	16.000	16.000	16.000	16.000	16.000	16.000

Table 6. Cost estimation for railway reconstruction case by triangular fuzzy numbers.

Var	Code	Item	[a; c; b]
	0,00	Management and specs.	[1.732; 1.780; 1.884]
$\mathbf{X_1}$	0,10	Project management	[524; 540; 575]
$\mathbf{X_2}$	0,20	Construction management etc.	[975; 1.000; 1.050]
$\mathbf{X_3}$	0,30	Design specifications etc.	[233; 240; 259]
$\mathbf{X_4}$	10,00	Environmental and soil eng.	[864; 888; 950]
$\mathbf{X_5}$	20,00	Traffic tasks	[48; 50; 53]
	30,00	Renewal of tracks	[8.905; 9.190; 9.664]
$\mathbf{X_6}$	30,10	New outbound main track	[975; 1.000; 1.050]
$\mathbf{X_7}$	30,20	Track renewal at platform 3/5	[5.432; 5.600; 5.880]
$\mathbf{X_8}$	30,30	New platform edge	[1.533; 1.580; 1.643]
$\mathbf{X_9}$	30,40	Track renewal depot, West	[285; 300; 321]
$\mathbf{X_{10}}$	30,50	Track layout design	[682; 710; 770]
$\mathbf{X_{11}}$	40,00	Platform and station	[538; 560; 602]
$\mathbf{X_{12}}$	50,00	Safety and signal installations	[5.035; 5.245; 5.586]
	60,00	Informatics incl. power supply	[2.374; 2.417; 2.626]
$\mathbf{X_{13}}$	60,10	Phase 2-4	[78; 80; 86]
$\mathbf{X_{14}}$	60,20	Sub project management	[249; 259; 275]
$\mathbf{X_{15}}$	60,30	Passenger information	[1.009; 1.030; 1.123]
$\mathbf{X_{16}}$	60,40	Electrical power supply	[1.038; 1.048; 1.142]
	70,00	Overhead line incl. pylons	[3.507; 3.624; 3.787]
$\mathbf{X_{17}}$	70,10	Overhead cables	[3.021; 3.122; 3.262]
$\mathbf{X_{18}}$	70,20	Layout and planning	[486; 502; 525]
$\mathbf{Y_1}$		Total cost before corrections	[23.003; 23.754; 25.152]
$\mathbf{X_{19}}$	A	Internal decision process	[1,006; 1,032; 1,098]
$\mathbf{X_{20}}$	B	Design specifications etc.	[1,009; 1,040; 1,100]
$\mathbf{X_{21}}$	C	Working process	[1,021; 1,042; 1,084]
Y		Total cost after corrections	[23.842; 26.565; 32.930]

Table 7. Total cost after corrections for railway reconstruction case by triangular fuzzy numbers.

α	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
L(α)	23.842	24.108	24.353	24.624	24.875	25.148	25.480	25.734	26.020	26.276	26.565
R(α)	32.930	32.244	31.592	30.895	30.269	29.626	28.983	28.387	27.743	27.161	26.565

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5 A railway reconstruction project

Consider the case of estimating the total cost incurred by a railway reconstruction project described by independent fuzzy input variables, namely 18 cost items $\mathbf{X}_1,..., \mathbf{X}_{18}$ and 3 correction factors $\mathbf{X}_{19},..., \mathbf{X}_{21}$. This case has been treated previously by using the concept of imprecise stochastic variables, Schjær-Jacobsen [14]. The correction factors are introduced in order to account for overall influences not accounted for by the individual cost items. The total cost before corrections is the sum

$$\mathbf{Y}_1 = \mathbf{X}_1 + \mathbf{X}_2 + ... + \mathbf{X}_{18}. \qquad (38)$$

The total cost after corrections $\mathbf{Y} = \mathbf{Y}(\mathbf{X})$ is a function of all 21 uncertain variables

$$\mathbf{Y} = (\mathbf{X}_1 + \mathbf{X}_2 + ... + \mathbf{X}_{18}) \cdot \mathbf{X}_{19} \cdot \mathbf{X}_{20} \cdot \mathbf{X}_{21}. \qquad (39)$$

The uncertain input variables should be estimated by railway experts with relevant project experience. Subsequently, the total cost **Y** is calculated by means of global optimization. The total cost estimation results are shown in Table 6 in the case of triangular fuzzy input variables. The term “code” refers to the cost structure hierarchy. For example the item carrying code 0,00 is the sum of cost items at lower levels carrying the codes 0,10, 0,20, and 0,30. Thus the latter are input variables whereas the former is an output variable. The cost item carrying the code 10,00 is an input variable itself because it does not have cost items at lower levels involved. In Table 7 the membership function of total cost **Y** after corrections is shown.

For α = 1 the single point calculation of 26.565 is shown corresponding to an ordinary calculation without uncertainty. This value is the most possible one and is attained when all input variables are attaining their most possible values. For α = 0 the total cost after corrections is within the interval [23.842; 32.930] and any value outside this interval is impossible considering the uncertain input variables.

It should be mentioned here that comparative studies of approaches involving probabilistic methods (including Monte Carlo simulation), trapezoidal fuzzy numbers, and imprecise stochastic variables have been made in [14] and [15] and is further investigated in a research project concerning economic uncertainty in mega projects.

6 Conclusion

In order to calculate correct results with intervals and fuzzy numbers global optimization methods must be implemented in contrast to straightforward application of interval arithmetic. However, the experimental calculations reported in the paper also show that further care should be taken not to introduce intermediate variables, which may also result in excess width intervals. When calculating fuzzy extensions of non-monotonic functions global optimization must be used in order to produce correct results. Otherwise, additional and unnecessary uncertainty will arise eventually leading to wrong decisions.

The results of the paper indicate that correct calculations of intervals (and fuzzy number membership functions) allow for rather strong statements pertaining to economic uncertainties. For example, it can be said that provided all uncertain input variables stay within their limits the uncertain output variables will stay within their limits. Obviously, it is an advantage of the interval and triangular (and trapezoidal) fuzzy representation of uncertainty that the meaning is easily communicated and understood.

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