Fuzzy Intersection and Difference Model for Topological Relations

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Abstract—*Topological relations have played important roles in spatial query, analysis and reasoning in Geographic Information Systems (GIS) and spatial databases. The topological relations between crisp and fuzzy spatial objects based upon the 9-intersections topological model have been identified. However the formalization of the topological relations between fuzzy regions needs more investigation. The paper provides a theoretical framework for modelling topological relations between fuzzy regions based upon a new fuzzy topological model called the Fuzzy Intersection and Difference (FID) Model. A novel topological model is formalized based on Fuzzy Topological Space (FTS). In order to derive all fuzzy topological relations between two fuzzy spatial objects, the fuzzy spatial object (A) is decomposed in four components: the Interior, the Interior's Boundary, the Object's Boundary, and the Exterior's Boundary of A. By use of this definition of fuzzy spatial object, new 4*4-Intersection and Fuzzy Intersection and Difference (FID) models are proposed as a qualitative model for the identification of all topological relations between two simple fuzzy regions. These two new models are compared with other fuzzy models studied in the literature. Examples are provided to illustrate the use of these two models presented in this paper with results which can be applied for modeling GIS and geospatial databases.*

Keywords— Fuzzy Intersection and Difference Model, 4*4-Intersection Matrix, Fuzzy Objects, Topological Relations, GIS

1 Introduction

Topological relations have an important significance in GIS modelling since they are the basis for spatial modelling, spatial query, analysis and reasoning. How to identify the topological relations between spatial objects is a critical point in GIS modelling.

During recent years, topological relations have been much investigated in the crisp and fuzzy topological space. The well-known *4-intersection* approach described in [1, 2], as well as the *9-intersection* approach as discussed in [3], and the *Intersection and Difference* (*ID*) model studied in [4, 5, 6], were proposed to formalize topological relations between two simple regions in the *Crisp Topological Space* (*CTS*).

The *4-intersection* model is extended in [7] to deal with the topological relations between spatial objects with holes.

Geographical phenomena in GIS with uncertain boundaries can be modelled by *Regions with Broad Boundaries* (*BBRs*) as in [8]. A region with broad boundary is an extension of a region with a crisp boundary (refer to simple regions with holes as in [7]). Objects with broad boundaries as defined in [8] are spatial objects, whose crisp boundaries are replaced by an area expressing the boundary's uncertainty. The *9-intersection model* is extended in [9] to describe topological relations between *BBRs* by replacing the crisp boundary in the *9-intersection* with the *broad boundary*. Another method called *4-tuple representation* of topological relations between *BBRs* is used in [10] to infer new topological information. The *4-tuple representation* can distinguish the same topological relations as identified by the *extended 9-intersection*. The *4-tuple*, however, can be applied to the reasoning of topological relations between *BBRs* [10], because it uses the composition of topological relations between crisp regions to determine those between uncertain and vague regions.

More recently fuzzy spatial objects have been emphasized since there are spatial features which are not always crisp. Fuzzy spatial objects are those with indeterminate boundaries. For fuzzy boundaries, that is, boundaries that are by nature not crisp, the broad boundary represents their minimum and maximum extent.

In order to derive the topological relations between fuzzy spatial objects, the *9-intersection approach* was updated into the *3*3-intersection approach* in the fuzzy topological space [11, 12]. Furthermore, in these two works, a *4*4-intersection matrix* was built up by using the topological properties of fuzzy sets, and then a *5*5-intersection matrix* can be built up based on certain conditions.

In the next section, we compare these models.

2 Related Work about Topological Relation Models and Spatial Objects

Crisp spatial objects have been formally defined in GIS. Point, line and polygon are three primitives in GIS. Fig. 1 represents the closure, interior and boundary of a closed disk as crisp spatial objects [2]. The *4-intersection* and *9-intersection matrix* are well-known approaches to identifying topological relation models between these two crisp spatial objects using the concept of the interior, boundary and exterior.

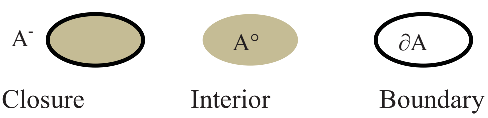

Figure 1: Closure, Interior and Boundary of a Crisp Object

By using some topological invariants of the intersection such as the empty/non empty contents, the topological

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relations between two crisp spatial objects can be identified. This approach implies the following facts in *CTS*: (1) the interior, boundary and the exterior of a subset are topological invariants; (2) these topological invariants are mutually disjoint in *CTS*; and (3) the empty/non-empty contents of the intersections between these three topological parts of two subsets are topological invariants. Then the *4-intersection* and *9-intersection models* are defined as:

$$I_4(A,B)=\begin{bmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B \\ \partial A \cap B^\circ & \partial A \cap \partial B \end{bmatrix}$$

$$I_9(A,B)=\begin{bmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B & A^\circ \cap B^- \\ \partial A \cap B^\circ & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^\circ & A^- \cap \partial B & A^- \cap B^- \end{bmatrix}$$

Eight topological relations of the spatial reasoning system (Region Connection Calculus) *RCC8* (*DC*, *EC*, *EQ*, *PO*, *TPP*, *TPPi*, *NTPP* and *NTPPi*) have been identified between two simple regions by using these two models in [2, 3].

In *ID model* [4, 5, 6], a crisp spatial object is defined by its interior and boundary; two intersection sets are $A^\circ \cap B^\circ$ and $\partial A \cap \partial B$; two difference sets are $A - B$ and $B - A$. This model can also distinguish the eight topological relations. This crisp topological model is represented by:

$$ID(A,B)=\begin{bmatrix} A^\circ \cap B^\circ & A-B \\ B-A & \partial A \cap \partial B \end{bmatrix}$$

The main difference between the *4-intersection model* and *ID model* is that both intersection sets $A^\circ \cap \partial B$ and $\partial A \cap B^\circ$ of 4-intersection model are replaced by two differences $A - B$ and $B - A$.

However, the fact (2) in CTS cannot hold in Fuzzy Topological Space (FTS). That means the interior; the boundary and the exterior of a fuzzy set may not be disjoint with each other. Therefore the *4-intersection, 9-intersection* and *ID models* cannot be directly applied for the identification of relations between two fuzzy sets.

The *9-intersection model* is extended in [8, 9] to spatial objects with broad boundary as simple fuzzy regions. It's expressed by the following matrix:

$$M = I_9(A,B)=\begin{bmatrix} A^\circ \cap B^\circ & A^\circ \cap \Delta B & A^\circ \cap B^- \\ \Delta A \cap B^\circ & \Delta A \cap \Delta B & \Delta A \cap B^- \\ A^- \cap B^\circ & A^- \cap \Delta B & A^- \cap B^- \end{bmatrix}$$

Using this model, *44 relations* between two simple fuzzy regions by using the *3*3-intersection matrix* are possible. For composite regions with broad boundaries, there are *14 additional* topological relations [8]. Fig. 2 represents a region with broad boundary as simple fuzzy spatial object.

Figure 2: Region with a Broad Boundary

In [14], they investigated a special space and formalized the 9-intersection in *Crisp FTS*. They proved that the *Crisp FTS* whose open sets are crisp is able to meet the above conditions.

The *9-intersection matrix* can be formalized as:

$$I_9(A,B)=\begin{bmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B & A^\circ \cap B^e \\ \partial A \cap B^\circ & \partial A \cap \partial B & \partial A \cap B^e \\ A^e \cap B^\circ & A^e \cap \partial B & A^e \cap B^e \end{bmatrix}$$

The above *3*3-intersection matrix* is derived based on the interior, boundary, and exterior of a simple fuzzy spatial object. By use of *9-intersection matrix*, *44 topological relations* are identified between two simple fuzzy regions (See Appendix 1. in [14]). Fig. 3 represents a region with indeterminate boundary as simple fuzzy spatial object.

Figure 3: Two Fuzzy Spatial Objects

In [14], it was shown that fuzzy spatial objects can be decomposed into four parts: the interior, the boundary of the boundary $\partial(\partial A)$, the interior of the boundary $(\partial A)^\circ$ and the exterior, which are mutually disjoint as in Fig. 4.

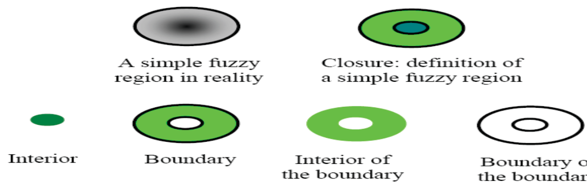

Figure 4: Interior, Boundary, Interior of the Boundary and Boundary of the Boundary of a Simple Fuzzy Region (After X. Tang and W. Kainz in [14])

Therefore, they introduced a *4*4-Intersection matrix* between two simple fuzzy spatial objects as follows:

$$I_{4*4}=\begin{bmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B & A^\circ \cap (\partial B)^\circ & A^\circ \cap B^e \\ \partial(\partial A) \cap B^\circ & \partial(\partial A) \cap \partial B & \partial(\partial A) \cap (\partial B)^\circ & \partial(\partial A) \cap B^e \\ (\partial A)^\circ \cap B^\circ & (\partial A)^\circ \cap \partial B & (\partial A)^\circ \cap (\partial B)^\circ & (\partial A)^\circ \cap B^e \\ A^e \cap B^\circ & A^e \cap \partial B & A^e \cap (\partial B)^\circ & A^e \cap B^e \end{bmatrix}$$

Under certain conditions, *152 relations* are identified by using the *4*4-intersection* approach (See Appendix 2. in [14] for more details).

After investigation about the topological relations between two simple fuzzy regions compared with these models studied in the literature, we can see that some topological relations can't be identified by these models. Here are some relations presented in Fig. 5.

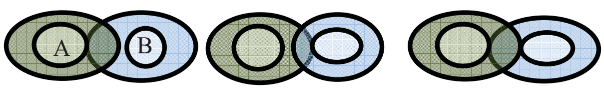

Figure 5: Some of Topological Relations to Identify

The question is how many topological relations there are exactly between two simple fuzzy objects? To answer this question, we will extend the *4-intersection* and the *ID models* with a new definition of the fuzzy boundary for fuzzy regions.

The disadvantage of the *3*3-intersection* and *4*4-intersection* models is that the *intersection operator* (∩) is the most expensive one in terms of computation. In order to reduce the computational cost of the *3*3-intersection* and *4*4-intersection* models; and to reduce the computational complexity by avoiding spatial operations between

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topological components with different dimensions (1-D and 2-D), we will try to reduce the number of intersections by introducing the *difference operator* (-).
Finally, the motivation of the paper is trying to build a topological model for identification of all fuzzy topological relations between two fuzzy regions.
The structure of the paper is as follows. The fuzzy topological space is defined in section 3. Section 4 is our contribution which consists of a new definition of simple fuzzy regions, fuzzy boundaries and their properties. The novel major contribution of our study, proposed in section 5, is a new form of *4*4- intersection model* and *the Fuzzy Intersection and Difference (FID) model*. Section 6 shows the identification of fuzzy topological relations between two simple fuzzy regions by using two models (*4*4- intersection and FID*) based on empty/non-empty contents. We end with a discussion about results and a conclusion.

3 Fuzzy Topological Space (FTS)

Fuzzy topology is constructed based on fuzzy sets. It is an extension of general (crisp) topology.
Let A be a fuzzy subset of an ordinary (crisp) set X, and $\wp(X)=[0,1]^X$ be the fuzzy power set of X. $[0,1]^X$ can be viewed as a lattice in which a supremum (or *join*) is denoted by $\vee$ and an infimum (or *meet*) by $\wedge$, conventionally. They correspond to the *union* and the *intersection*, respectively. $\forall\delta\in\wp(X)$ if (1) $\Phi, X\subseteq\delta$, (2) $\forall A_i\in\delta$, $\vee A_i\in\delta$, (3) $\forall U,V\in\delta$, $U\wedge V\in\delta$, then δ is called a *fuzzy topology* on *X* (*i*∈*I* is an index set). (*X, δ*) is called a *Fuzzy Topological Space (FTS)* as defined in [15, 16]. Every element of δ is called an open (fuzzy) set in (X, δ). A set A is a closed (fuzzy) set if its complement A^c is open. The union of all open sets contained in A is the interior of A, denoted by A°. The intersection of all closed sets containing A is called the closure of A, denoted by A^-. The exterior of A is the complement of A^- and is denoted by A^e. Obviously, it is an open set. The boundary ∂A of a subset A is the intersection of the closure of A with the closure of the complement of A.
The boundary of a subset may also have its interior and its boundary of the boundary. On the other hand, the interior and the closure of a subset also have their boundaries. For example, the boundary of the boundary of a fuzzy set A is the union of the boundary of the closure and the boundary of the interior of a fuzzy set [14].
Based on this information, one can define a maximum of four other different areas from that defined in [14] for each object: Interior, Boundary of the Object, Interior of the Boundary, and Exterior of the Boundary in the next section.

4 Definition of Simple Fuzzy Regions and a Fuzzy Boundary

In this section, we will develop a definition of simple fuzzy region, fuzzy boundary and their properties.

4.1 Definition of Simple Fuzzy Region

A crisp region is defined in *CTS*. Correspondingly, a fuzzy region should be defined in *FTS*. We now define a simple fuzzy region in *FTS*.
A simple fuzzy region is made up of two regions A_1 and A_2 with $A_1\subset A_2$ (see Fig. 6), where: (1) the interior of A is the interior of A_1 $A^\circ=(A_1)^\circ$ and A° is an open subset and connected; (2) the interior's boundary of A is the boundary of A_1 as $A^i=\partial(A^\circ)=\partial A_1$, and A^i is a closed subset and connected; (3) the boundary of A is ∂A defined as the interior of the difference between A_1 and A_2 as $\partial A=(A_2-A_1)^\circ$, and ∂A is an open subset and connected; (4) the exterior's boundary of A is the boundary of ∂A_2 as $A^e=\partial^e(\partial A)=\partial A_2$, and A^e is a closed subset and connected; and (5) the intersection of all closed sets containing A is called the closure of A, denoted by A^-. Fig. 6 shows the four components of simple fuzzy regions.

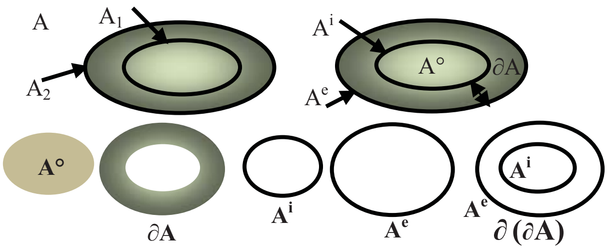

Figure 6: Interior, Boundary, Interior's Boundary, and Exterior's Boundary of a Simple Fuzzy Region

This definition is considered as the de decomposition of the boundary in [14, 15, 16] into disjoint subsets such as the interior boundary of the boundary, the exterior boundary of the boundary and the interior of the boundary with condition that is the interior of the boundary couldn't be a non-empty set.
We called the boundary (∂A) of A by the fuzzy boundary. This definition of fuzzy regions is very interesting to identify all topological relations between two simple fuzzy regions that other models [8, 9, 14] can't identify. We will prove that in the next section by developing a *4*4-intersection* matrix. In the next part, we will define the properties of the fuzzy boundary.

4.2 Properties of Fuzzy Boundary

Let A be a fuzzy set in *FTS* (X, δ). Based on the definition of simple fuzzy regions, we find the properties of the fuzzy boundary (∂A) as follows: (1) ∂A is an open subset of A; (2) ∂A = ¬ Ø is a non-empty subset; (3) the boundary of A (∂A) is the interior of the difference $\partial A=(A^e-A^i)^\circ$ between the exterior's boundary (A^e) and the interior's boundary (A^i) of A; (4) the union $A^i\cup\partial A\cup A^e$ of the interior's boundary (A^i), the boundary (∂A) and the exterior's boundary (A^e) of A is a closed subset of A; (5) the interior boundary of the boundary (∂A) of A $\partial^i(\partial A)=A^i=\partial(A^\circ)$ is the interior's boundary (A^i); and (6) the exterior boundary of the boundary (∂A) of A $\partial^e(\partial A)=A^e=\partial^e(A)$ is the exterior's boundary (A^e).
We also find that the intersections between A°, A^i, ∂A, A^e are, respectively, always empty, and the union of these parts

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is equal to A as follows: $A^{\circ} \cap A^{i} = \emptyset$; $A^{i} \cap \partial A = \emptyset$; $\partial A \cap A^{e} = \emptyset$; $A^{\circ} \cap A^{i} \cap \partial A \cap A^{e} = \emptyset$;

and $A^{\circ} \cup A^{i} \cup \partial A \cup A^{e} = A$.

It also can be easily proven by the above intersections that the interior (A^{o}), the interior's boundary (A^{i}), the boundary ∂A and the exterior's boundary (A^{e}) of a simple fuzzy region (A) are mutually disjoint.

In order to identify all possible topological relations, the condition of the mutual disjointness of these four parts of simple fuzzy regions is important to propose and construct a new method to form the intersection matrix. In the next section, a new *4*4-intersection matrix* and *fuzzy intersection and difference model* are proposed based upon this definition of simple fuzzy region.

5 Contributions: 4*4 - Intersection and Fuzzy Intersection and Difference (FID) Models

In this section, we will develop two models to identify the topological relations between two simple fuzzy regions.

Supposing there are two simple fuzzy objects A and B in the *FTS*, we adopt the interior, boundary, interior's boundary, and exterior's boundary to formalize two new topological models as in the next.

*5.1 Contribution 1: 4*4 - Intersection Model*

In the first contribution, the first model is a new *4*4 - intersection matrix* which uses the operator (∩) of intersection. Between these two simple fuzzy spatial regions A and B, the *4*4-intersection matrix* will be as presented in Table 1:

Table 1: 4*4 - Intersection Matrix

∩	B°	B^i	∂B	B^e
A°	A°∩ B°	A°∩ B^i	A°∩ ∂B	A°∩ B^e
A^i	A^i ∩B°	A^i ∩ B^i	A^i ∩∂B	A^i ∩ B^e
∂A	∂A ∩B°	∂A ∩ B^i	∂A ∩∂B	∂A ∩ B^e
A^e	A^e ∩B°	A^e ∩ B^i	A^e ∩ ∂B	A^e ∩ B^e

And the *4*4-inetrsection model* applied to simple fuzzy objects is expressed by the following expression:

$$I_{4*4}(A,B) = \begin{bmatrix} A^{\circ} \cap B^{\circ} & A^{\circ} \cap B^{i} & A^{\circ} \cap \partial B & A^{\circ} \cap B^{e} \\ A^{i} \cap B^{\circ} & A^{i} \cap B^{i} & A^{i} \cap \partial B & A^{i} \cap B^{e} \\ \partial A \cap B^{\circ} & \partial A \cap B^{i} & \partial A \cap \partial B & \partial A \cap B^{e} \\ A^{e} \cap B^{\circ} & A^{e} \cap B^{i} & A^{e} \cap \partial B & A^{e} \cap B^{e} \end{bmatrix}$$

This new *4*4- intersection matrix* (I_{4*4}) is considered as an extension of the *4-intersection* model [1, 2] for simple fuzzy spatial regions.

The *intersection operator* (∩) is perhaps the most expensive one in terms of computation. In order to reduce the computational cost of this *4*4-intersection* model, we will extend this model to the fuzzy intersection and difference model in the next part.

5.2 Contribution 2: Fuzzy Intersection and Difference (FID) Model

In this model, we will introduce the *difference operator* (-). In order to avoid spatial operations between topological components with different dimensions (A°, B°, ∂A, ∂B as 2-D; and A^i, B^i, A^e, B^e as 1-D), we will replace the intersection between the terms $A^{\circ} \cap B^{i}$, $A^{\circ} \cap B^{e}$, $A^{i} \cap B^{\circ}$, $A^{i} \cap \partial B$, $\partial A \cap B^{i}$, $\partial A \cap B^{e}$, $A^{e} \cap B^{\circ}$ and $A^{e} \cap \partial B$ in the *4*4-intersection* model by the differences as in the matrix (see Table 2).

The four intersections ($A^{\circ} \cap B^{\circ}$, $A^{\circ} \cap \partial B$, $\partial A \cap B^{\circ}$, $\partial A \cap \partial B$) with topological components with dimension 2-D and the four intersections ($A^{i} \cap B^{i}$, $A^{i} \cap B^{e}$, $A^{e} \cap B^{i}$, $A^{e} \cap B^{e}$) with topological components with dimension 1-D remain unchanged as similar as in [4, 5, 6] for the ID model.

Table 2: 4*4 - Intersection and Difference Matrix

∩	B°	B^i	∂B	B^e	-
A°	A°∩ B°	A^i - B^i	A°∩ ∂B	A^i - B^e	A°
A^i	B^i - A^i	A^i ∩ B^i	B^i - A^i B^e – A^i	A^i ∩ B^e	A^i
∂A	∂A ∩B°	A^e - B^i A^i - B^i	∂A ∩∂B	A^e - B^e A^i - B^e	∂A
A^e	B^i - A^e	A^e ∩ B^i	B^i – A^e B^e - A^e	A^e ∩ B^e	A^e

By simplification and arrangement of this *4*4 - Intersection and Difference matrix*, we obtain two intersection matrices and two difference matrices (Table 3):

Table 3: Intersection and Difference Matrices

∩	B°	∂B
A°	A°∩ B°	A°∩ ∂B
∂A	∂A ∩B°	∂A ∩∂B

∩	B^i	B^e
A^i	A^i ∩ B^i	A^i ∩ B^e
A^e	A^e ∩ B^i	A^e ∩ B^e

-	B^i	B^e
A^i	A^i - B^i	A^i - B^e
A^e	A^e - B^i	A^e - B^e

-	A^i	A^e
B^i	B^i - A^i	B^i - A^e
B^e	B^e - A^i	B^e - A^e

At the end, the *Fuzzy Intersection and Difference (FID) model* is written as follows:

$$FID_{4*4}(A,B) = \begin{bmatrix} A^{\circ} \cap B^{\circ} & A^{\circ} \cap \partial B & A^{i} - B^{i} & A^{i} - B^{e} \\ \partial A \cap B^{\circ} & \partial A \cap \partial B & A^{e} - B^{i} & A^{e} - B^{e} \\ A^{i} \cap B^{i} & A^{i} \cap B^{e} & B^{i} - A^{i} & B^{i} - A^{e} \\ A^{e} \cap B^{i} & A^{e} \cap B^{e} & B^{e} - A^{i} & B^{e} - A^{e} \end{bmatrix}$$

The *FID* model combines two different operators (intersection and difference). The *FID* has two advantages: first, it reduces the computational complexity by avoiding spatial operations between topological components with different dimensions, e.g., $A^{\circ} \cap B^{i}$, $A^{\circ} \cap B^{e}$, $A^{i} \cap B^{\circ}$, $A^{i} \cap \partial B$, $\partial A \cap B^{i}$, $\partial A \cap B^{e}$, $A^{e} \cap B^{\circ}$ and $A^{e} \cap \partial B$, with A°, B°, ∂A, ∂B as 2-D, and A^i, B^i, A^e, B^e as 1-D; and second, it reduces the computational cost due to only eight intersections in the matrix of *FID* model.

This *FID* model is considered as an extension of the *ID* model [4, 5, 6] for simple fuzzy spatial regions.

In general, there are 2^{16}= 65536 relations between two fuzzy regions by using the *4*4-intersection* matrix and *FID* model. For GIS applications, some conditions will limit the number of these relations as in [7, 8, 14]. However, how to find all possible topological relations between two simple fuzzy regions needs more investigation. It's done in the next section for the *4*4-intersection* matrix and *FID* model.

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6 Identification of Topological Relations based upon 4*4 - Intersection and FID Models

In this section, we focus on the identification of all fuzzy topological relations between two simple fuzzy regions by *4*4-intersection* and *FID* models.

Let A and B be two simple fuzzy regions. For relation identification, each intersection or difference in *4*4-intersection* and *FID matrices* takes value of either empty ($\emptyset$) or non-empty ($\neg\emptyset$). Every different set of *4*4-intersection* and *FID* matrices describes a different topological relation. Some values of these two matrices have no sense on the topological relation.

*6.1 Identification by 4*4 - Intersection Model*

For identification by *4*4-intersection model*, by respecting the definition in section 3, we scan all possible configurations for A and B in two different steps as following: (1) If the exterior's boundary of A intersects with the exterior's boundary of B ($A^e \cap B^e = \neg\emptyset$), then, we search all possible topological relations between A and B, we find *105 relations*; (2) If the exterior's boundary of A doesn't intersect with the exterior's boundary of B ($A^e \cap B^e = \emptyset$), then, we search all topological relations between A and B, we find *47 relations*. The total number of topological relations identified between A and B is *152 relations*. Some of these topological relations are not identified and determined in [8, 9, 14]. Here are some examples in Fig .7 and Fig .8.

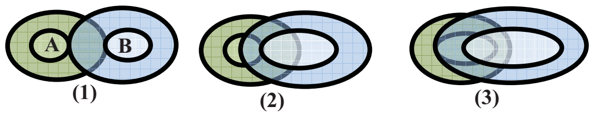

Figure 7: Examples for $A^e \cap B^e = \neg\emptyset$

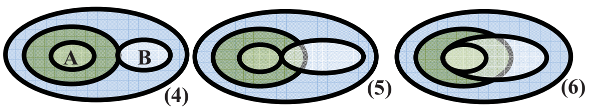

Figure 8: Examples for $A^e \cap B^e = \emptyset$

The *4*4-intersection matrices* correspondent for (1), (2) and (3) in Fig .7, and for (4), (5) and (6) in Fig .8 are given, respectively, by:

$$(1)\begin{bmatrix} \phi & \phi & \phi & \phi \\ \phi & \phi & \phi & \neg\phi \\ \phi & \phi & \neg\phi & \neg\phi \\ \phi & \neg\phi & \neg\phi & \neg\phi \end{bmatrix} (2)\begin{bmatrix} \phi & \phi & \neg\phi & \neg\phi \\ \phi & \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \end{bmatrix} (3)\begin{bmatrix} \neg\phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \end{bmatrix}$$

$$(4)\begin{bmatrix} \phi & \phi & \neg\phi & \phi \\ \phi & \phi & \neg\phi & \phi \\ \phi & \phi & \neg\phi & \phi \\ \phi & \neg\phi & \neg\phi & \phi \end{bmatrix} (5)\begin{bmatrix} \phi & \phi & \neg\phi & \phi \\ \phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \end{bmatrix} (6)\begin{bmatrix} \neg\phi & \phi & \phi & \phi \\ \neg\phi & \neg\phi & \phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \end{bmatrix}$$

We note that the separateness between these relations can't be realized by other models studied in [8, 14]. In the next part, we will identify all relations by the *FID model*.

6.2 Identification by FID Model

For identification by *FID model*, by respecting the definition in section 3, we apply the same two steps in previous part (6.1) for $A^e \cap B^e = \neg\emptyset$ and $A^e \cap B^e = \emptyset$. 152 topological relations can be identified by using the *FID model*. These relations identified by *FID model* is the same relations by *4*4-intersection model*. We give some examples in Fig .9 and Fig .10.

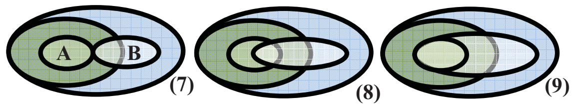

Figure 9: Examples for $A^e \cap B^e = \neg\emptyset$

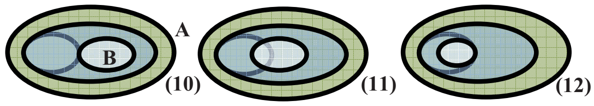

Figure 10: Examples for $A^e \cap B^e = \emptyset$

The *FID* matrices correspondent to (7), (8) and (9) in Fig .9, and to (10), (11) and (12) in Fig .10 are given, respectively, by:

$$(7)\begin{bmatrix} \phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \end{bmatrix} (8)\begin{bmatrix} \neg\phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \end{bmatrix} (9)\begin{bmatrix} \neg\phi & \phi & \phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \end{bmatrix}$$

$$(10)\begin{bmatrix} \phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \\ \phi & \phi & \neg\phi & \phi \end{bmatrix} (11)\begin{bmatrix} \neg\phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \\ \phi & \phi & \neg\phi & \phi \end{bmatrix} (12)\begin{bmatrix} \neg\phi & \neg\phi & \neg\phi & \phi \\ \phi & \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \phi & \phi \\ \phi & \phi & \neg\phi & \phi \end{bmatrix}$$

To find and extract these relations by the *4*4-intersection* and *FID models*, we have developed and implemented these two steps on MATLAB.

7 Applications and Discussion

For GIS applications, satellite images and spatial databases, these two models (*4*4-intersection* and *FID*) can determine the topological relations between simple fuzzy regions. For that, we need to know how to generate fuzzy spatial objects from satellite images and to find the four components of our definition (interior, interior's boundary, boundary and exterior's boundary) for satellite images and GIS objects. We can adopt processed data such as classification or segmentation results of satellite images. For example, Land Use and Land Cover (LULC), most of which is obtained from the classification results of satellite images, may be a good example of a fuzzy spatial object as in [17]. In principle, a fuzzy spatial object can also be generated by other methods as in [14, 18].

These two models can be used in order to evaluate the change detection process (for Land Cover changes)of geographical objects (Beach, Forest, Residential Area…) represented in GIS and satellite imagery (TM and SPOT images) databases as in [14, 17, 19, 20].

Comparing between the relations identified in [8, 9, 14] and our models (*4*4-intersection and FID*), we find that there are *100 new relations* which can't be discriminated and identified by other models. Some of these *100 new topological relations* identified by *4*4-intersection and FID* models are presented in the appendix of this paper.

8 Conclusion

In this paper, we have proposed a new definition of simple fuzzy spatial region by decomposing the spatial region into

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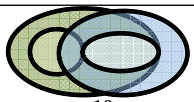
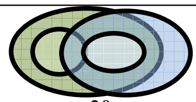
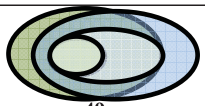
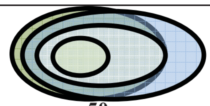
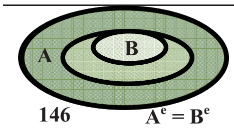
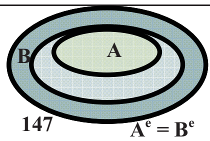
four components: interior, interior's boundary, boundary and exterior's boundary. Based upon these four components, a *4*4-intersection matrix* is introduced to identify all topological relations between two simple fuzzy regions. Then, in order to reduce the computational complexity of the *4*4-intersection model,* the *Fuzzy Intersection and Difference (FID) model* is developed based on the *4*4-intersection matrix*. The main contribution of this work is these two models *4*4-intersection and FID. 152* fuzzy topological relations can be identified by using the *4*4-intersection and FID models*. Among these *152 relations*, *100 new relations* can't be discriminated or identified as different relations by other models [8, 9, 14].

In our future work, we will try to find more topological relations which can be identified if we adopt in the definition $A^e \cap A^i = \neg\emptyset$, $\partial A = \neg\emptyset$ and A^i don't intersect with the exterior of A. Then, we will try to classify these fuzzy topological relations identified by I4*4 and FID models to be grouped into the eight relations DC, EC, PO, TPP and TPPi, NTPP and NTPPi, and EQ of the spatial reasoning system *RCC8*.

Appendix

Some Topological Relations between Two Simple Fuzzy Regions by Using the *4*4-Intersection Model (I_{4*4} Matrix)* and the *Fuzzy Intersection and Difference (FID Matrix) Model*

Illustration	I_{4*4} Matrix	FID Matrix	Illustration	I_{4*4} Matrix	FID Matrix
 19	(19) $\begin{bmatrix} \phi & \phi & \neg\phi & \neg\phi \\ \phi & \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \\ \phi & \neg\phi & \neg\phi & \neg\phi \end{bmatrix}$	(19) $\begin{bmatrix} \phi & \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \end{bmatrix}$	 20	(20) $\begin{bmatrix} \phi & \phi & \neg\phi & \neg\phi \\ \phi & \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \\ \phi & \phi & \neg\phi & \neg\phi \end{bmatrix}$	(20) $\begin{bmatrix} \phi & \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \\ \phi & \neg\phi & \neg\phi & \neg\phi \end{bmatrix}$
 49	(49) $\begin{bmatrix} \neg\phi & \phi & \phi & \phi \\ \neg\phi & \neg\phi & \phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \\ \phi & \neg\phi & \neg\phi & \neg\phi \end{bmatrix}$	(49) $\begin{bmatrix} \neg\phi & \phi & \phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \end{bmatrix}$	 50	(50) $\begin{bmatrix} \neg\phi & \phi & \phi & \phi \\ \neg\phi & \phi & \phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \\ \phi & \neg\phi & \neg\phi & \neg\phi \end{bmatrix}$	(50) $\begin{bmatrix} \neg\phi & \phi & \phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \\ \phi & \phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \neg\phi \end{bmatrix}$
 146 $A^e = B^e$	(146) $\begin{bmatrix} \neg\phi & \phi & \neg\phi & \phi \\ \phi & \neg\phi & \neg\phi & \phi \\ \phi & \phi & \neg\phi & \phi \\ \phi & \phi & \phi & \neg\phi \end{bmatrix}$	(146) $\begin{bmatrix} \neg\phi & \neg\phi & \neg\phi & \phi \\ \phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \phi & \phi & \phi \\ \phi & \neg\phi & \neg\phi & \phi \end{bmatrix}$	 147 $A^e = B^e$	(147) $\begin{bmatrix} \neg\phi & \phi & \phi & \phi \\ \neg\phi & \neg\phi & \phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \\ \phi & \phi & \phi & \neg\phi \end{bmatrix}$	(147) $\begin{bmatrix} \neg\phi & \phi & \phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \phi & \neg\phi & \phi \\ \phi & \neg\phi & \neg\phi & \phi \end{bmatrix}$

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