# A Type-2 Fuzzy Portfolio Selection Problem Considering Possibility Measure and Crisp Possibilistic Mean Value

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***Abstract***—*This paper considers a portfolio selection problem with type-2 fuzzy future returns involving ambiguous and subjectivity. Since this proposed problem is not well-defined due to fuzziness, introducing the fuzzy goal for the total future return and the degree of possibility, the main problem is transformed into the standard fuzzy programming problem including the secondary fuzzy numbers. Furthermore, using the hybrid solution approaches based on the linearity of the deterministic equivalent problem and the crisp possibilistic mean value, the efficient solution is constructed.*

***Keywords***— Portfolio selection, Type-2 fuzzy, Possibility measure, Crisp possibilistic mean value, Efficient solution method

## 1 Introduction

Portfolio selection problems are standard and most important problems in investment and financial research fields, and various studies have been so far performed. Furthermore, in recent investment fields, not only big companies and institutional investors but also individual investors called Day-Traders invest in stock, currency, land and property. Therefore, the role of investment theory called portfolio theory becomes more and more important. As for the research history on mathematical approach, Markowitz (Markowitz [18]) has proposed the mean-variance analysis model. It has been central to research activity in the real financial field and numerous researchers have contributed to the development of modern portfolio theory (cf. Elton and Gruber [4], Luenberger [17]). On the other hand, many researchers have proposed models of portfolio selection problems which extended Markowitz model; mean-absolute deviation model (Konno [13], Konno, et al. [14]), safety-first model [4], Value at Risk and conditional Value at Risk model (Rockafellar and Uryasev [19]), etc.. As a result, nowadays it is common practice to extend these classical economic models of financial investment to various types of portfolio models. In practice, after Markowitz's work, many researchers have been trying different mathematical approaches to develop the theory of portfolio selection.

Particularly, the prediction of future returns is one of the most important factors in theoretical and practical investment, and they have been treated as only random variables in many previous studies. Then, the expected returns and variances also have been assumed to be fixed values. However, investors receive effective or ineffective information from the real world and ambiguous factors usually exist in it. Furthermore, investors often have the subjective prediction for future returns which are not derived from the statistical analysis of historical data. Then, even if investors hold a lot of information from the investment field, it is difficult that the present or future random distribution of each asset is strictly set. Consequently, we need to consider not only random conditions but also ambiguous and subjective conditions for portfolio selection problems.

Recently, in the sense of mathematical programming, some researchers have proposed various types of portfolio models under randomness and fuzziness. These problems with probabilities and possibilities are generally called stochastic programming problems and fuzzy programming problems, respectively, and there are some basic studies using a stochastic programming approach, goal programming approach, etc.., and fuzzy programming approach to treat ambiguous factors as fuzzy sets (Inuiguchi and Ramik [8], Leon, et al. [15], Tanaka and Guo [20], Tanaka, et al. [21], Watada [23]). Furthermore, some researchers have proposed the mathematical programming problems with both randomness and fuzziness as fuzzy random variables (for instance, Katagiri et al. [11, 12]). In the studies [11, 12], fuzzy random variables were related with the ambiguity of the realization of a random variable and dealt with a fuzzy number that the center value occurs according to a random variable. Then, Yazenin considered some models for portfolio selection problems in the probabilistic-possibilistic environment, that profitabilities of financial assets are fuzzy random variables (Yazenin [25, 26]). On the other hand, future returns may be dealt with random variables derived form the statistical analysis, whose parameters are assumed to be fuzzy numbers due to the decision maker's subjectivity, i.e., random fuzzy variables which Liu (Liu [16]) defined. There are a few studies of random fuzzy programming problem (Katagiri et al. [9, 10], Huang [7]). Most recently, Hasuike et al. [6] proposed several portfolio selection models including random fuzzy variables and developed the analytical solution method.

However, in [6], the random distribution of each asset is assumed to be a normal distribution. From some practical studies with respect to the present practical market, it is not clear that price movements of assets occur according to normal distributions. In fact, considering the existence of various types of investors in the practical market and the subjectivity of investors, it is important that we need to develop a new portfolio selection problem to deal with a lot of subjectivity. In this paper, we assume future returns to be Type-2 fuzzy numbers which can be dealt with various types of membership functions, and propose a new Type-2 portfolio selection problem.

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In the sense of mathematical programming, since the proposed model is not formulated as a well-defined problem due to fuzziness, we need to set some certain optimization criterion so as to transform into well-defined problems. In this paper, introducing a fuzzy goal and the degree of possibility, we transform the main problem into the possibility maximization problem. However, this problem includes the fuzzy number and it is also not a well-defined. Therefore, in order to solve possibility maximization problem analytically, we introduce the crisp possibilistic mean value proposed by Carlsson and R. Fullér [1] and develop an efficient solution method to find a global optimal solution of deterministic equivalent problem.

This paper is organized in the following way. In Section 2, we introduce mathematical concepts of type-2 fuzzy set and crisp possibilistic mean value. In Section 3, we propose a type-2 fuzzy portfolio selection problem maximizing the total future return. Then, introducing the degree of possibility and the crisp possibilistic mean value, we transform the proposed model into the deterministic equivalent problem. In Section 4, in order to compare our proposed models with other models for portfolio selection problems, we provide a numerical example derived from current practical market data. Finally, in Section 5, we conclude this paper.

## 2 Mathematical concept

*2.1 Type-2 fuzzy set*

A type-2 fuzzy set is a set in which we also have uncertainty about the membership function, i.e., a type-2 fuzzy set is characterized by a fuzzy membership function whose grade for each element is a fuzzy set [0,1] (Castillo and Melin [3]). In the real world, there are many problems where the decision maker cannot determine the exact form of the membership function such as in time series prediction because of noise in the data. Therefore, it is important that the type-2 fuzzy set is introduced into real world problems.

**Example 1** (Castillo and Melin [3])
Consider the case of a fuzzy set characterized by a Gaussian membership function with mean $m$ and a standard deviation that can take values in $[\sigma_1, \sigma_2]$, i.e.,

$$\mu(x) = \exp\left\{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2\right\},\ \sigma \in [\sigma_1, \sigma_2]$$

In this paper, with respect to future returns we treat the following *L*-shape fuzzy numbers as fuzzy numbers based on the type-2 fuzzy set:

$$\mu_{\tilde{r}_j}(\omega) = \max\left\{0, L\left(\frac{\omega - \bar{r}_j}{\alpha_j}\right)\right\},\ \alpha_j \in \left[\alpha_j^L, \alpha_j^U\right] \quad (1)$$

where $L(\omega)$ is a shape function from $\mathbb{R}$ to $\mathbb{R}$ satisfying the following conditions:

1. $L(-\omega) = L(\omega)$ for $\forall \omega \in \mathbb{R}$
2. $L(0) = 1$
3. $L(\cdot)$ is nonincreasing on $[0, \infty)$
4. Let $t_0 = \inf\{t > 0 \mid L(t) = 0\}$. Then $0 < t_0 < \infty$

The *L*-shape fuzzy number includes the more general membership function than the Gaussian in Example 1. Therefore, by using the *L*-shape fuzzy number, it is possible that we represent more versatile social problems.

*2.2 Crisp possibilistic mean value*

Carlsson and Fullér [1] introduced the notation of crisp possibilistic mean value of continuous possibility distribution, which are consistent with the extension principle. Let $A$ a fuzzy number. Then, $[A]^\gamma$ denote the $\gamma$ -level set of $A$ as the following form:

$$[A]^\gamma = [a_L(\gamma),\ a_U(\gamma)]$$

Using this $\gamma$ -level set of $A$, the crisp possibilistic mean value of $A$ is introduced as follows:

$$E(A) = \int_0^1 \gamma\left(a_L(\gamma) + a_U(\gamma)\right) d\gamma$$

**Example 2**
In the case that a fuzzy number $A$ is a trapezoidal fuzzy number with tolerance interval $[a, b]$, left spread $\alpha$ and right spread $\beta$. Then, the crisp possibilistic mean value of $A$ is

$$\begin{aligned} E(A) &= \int_0^1 \gamma\left(a - (1-\gamma)\alpha + b + (1-\gamma)\beta\right) d\gamma \\ &= \frac{a+b}{2} + \frac{\beta - \alpha}{6} \end{aligned}$$

This crisp possibilistic mean value has been applied to various types of portfolio selection problems with fuzzy numbers (in detail, see [5]), and it becomes one of the most useful tool in the sense of fuzzy programming problem.

## 3 Type-2 fuzzy portfolio selection problem

In this paper, we deal with the following portfolio selection problem with future returns based on the type-2 fuzzy set maximizing total future returns.

$$\begin{aligned} &\text{Maximize} \quad \sum_{j=1}^n \tilde{r}_j x_j \\ &\text{subject to} \quad \sum_{j=1}^n x_j = 1,\quad 0 \le x_j \le p_j,\quad j = 1, 2, \cdots, n \end{aligned} \quad (2)$$

where $\boldsymbol{x} = (x_1, \cdots, x_n)$ is a composition of the portfolio and $p_j$ is a limited upper rate of *j*th asset. In this problem, the objective function $\tilde{Z} = \sum_{j=1}^n \tilde{r}_j x_j$ is also a fuzzy number characterized by the following membership function:

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$$\mu_{\tilde{Z}}(\omega) = \max\left\{0, L\left(\frac{\omega - \sum_{j=1}^{n}\bar{r}_j x_j}{\sum_{j=1}^{n}\alpha_j x_j}\right)\right\},\ \alpha_j \in \left[\alpha_j^L, \alpha_j^U\right] \quad (3)$$

Due to the fuzziness of objective function, problem (2) is not a well-defined problem, and so we need to set some certain optimization criterion. Until now, researchers have proposed some solution approaches to solve fuzzy programming problem based on not only strict solution methods such as linear and nonlinear programming but also approximate solution methods such as GA and NN. In this paper, we focus on the strict solution method in the sense of mathematical programming, and so we introduce the possibility measure as follows:

$$\text{Pos}\left\{\sum_{j=1}^{n}\tilde{r}_j x_j \geq f\right\} = \sup_{\omega}\left\{\mu_{\tilde{Z}}(\omega) \middle| \omega \geq f\right\}$$

where $f$ is the target value to total future return. The possibility measure means that the total future return is more than $f$ as much as possible in the aspiration level of each investor.

### *3.1 Possibility maximization model for the proposed type-2 fuzzy portfolio selection problem*

On the other hand, in practical situations, the investor considers increasing the goal of total future return and that of possibility, simultaneously. Furthermore, considering many real decision cases and taking account of the vagueness of human judgment and flexibility for the execution of a plan, the investor often has subjective and ambiguous goals with respect to the target return $f$ such as "Total future return $\sum_{j=1}^{n}\tilde{r}_j x_j$ is approximately larger than $f_1$ .". In this subsection, we propose the more flexible model considering the aspiration level to the goal for the total future return. We represent the subjective and ambiguous goals with respect to $f$ as a fuzzy goal characterized by the following membership function:

$$\mu_{\tilde{G}}(f) = \begin{cases} 1 & f_1 \leq f \\ g(f) & f_0 \leq p < f_1 \\ 0 & f < f_0 \end{cases} \quad (4)$$

where $g(f)$ is the strict increasing function. Furthermore, using a concept of possibility measure, we introduce the degree of possibility as follows:

$$\prod_{\tilde{Z}}\left(\tilde{G}\right) = \sup_{f}\min\left\{\mu_{\tilde{Z}}(f), \mu_{\tilde{G}}(f)\right\} \quad (5)$$

Using this degree of possibility, we formulate a possibility maximization model for the proposed portfolio selection problem as the following form:

$$\begin{aligned} &\text{Maximize} \quad \prod_{\tilde{Z}}\left(\tilde{G}\right) \\ &\text{subject to} \quad \sum_{j=1}^{n} x_j = 1,\quad 0 \leq x_j \leq p_j,\quad j = 1, 2, \cdots, n \end{aligned} \quad (6)$$

Then, this problem is equivalently transformed into the following problem introducing a parameter $h$:

$$\begin{aligned} &\text{Maximize} \quad h \\ &\text{subject to} \quad \prod_{\tilde{Z}}\left(\tilde{G}\right) \geq h, \\ &\qquad \sum_{j=1}^{n} x_j = 1,\quad 0 \leq x_j \leq p_j,\quad j = 1, 2, \cdots, n \end{aligned} \quad (7)$$

In this problem, the possibility constraint $\prod_{\tilde{Z}}\left(\tilde{G}\right) \geq h$ is transformed into the following form:

$$\begin{aligned} &\prod_{\tilde{Z}}\left(\tilde{G}\right) \geq h \\ &\Leftrightarrow \sup_{f}\min\left\{\mu_{\tilde{Z}}(f), \mu_{\tilde{G}}(f)\right\} \geq h \\ &\Leftrightarrow \mu_{\tilde{Z}}(f) \geq h,\ \mu_{\tilde{G}}(f) \geq h \\ &\Leftrightarrow L\left(\frac{f - \sum_{j=1}^{n}\bar{r}_j x_j}{\sum_{j=1}^{n}\alpha_j x_j}\right) \geq h, f \geq g^{-1}(h) \\ &\Leftrightarrow f \leq \sum_{j=1}^{n}\bar{r}_j x_j + L^*(h)\sum_{j=1}^{n}\alpha_j x_j, f \geq g^{-1}(h) \\ &\Leftrightarrow \sum_{j=1}^{n}\bar{r}_j x_j + L^*(h)\sum_{j=1}^{n}\alpha_j x_j \geq g^{-1}(h) \end{aligned} \quad (8)$$

Using these inequalities, the main problem (6) is equivalently transformed into the following problem:

$$\begin{aligned} &\text{Maximize} \quad h \\ &\text{subject to} \quad \sum_{j=1}^{n}\bar{r}_j x_j + L^*(h)\sum_{j=1}^{n}\alpha_j x_j \geq g^{-1}(h), \\ &\qquad \sum_{j=1}^{n} x_j = 1,\quad 0 \leq x_j \leq p_j,\quad j = 1, 2, \cdots, n \end{aligned} \quad (9)$$

In this problem, if each spread of fuzzy number $\alpha_j$ is fixed, this problem is analytically solved by using the hybrid solution method of bisection algorithm on parameter $h$ and the linear programming problem. However, in the case that $\alpha_j$ is not fixed but a fuzzy number, it is difficult that this problem is analytically solved by standard linear programming approaches due to including secondary fuzzy numbers.

### *3.2 Solution method based on the crisp possibilistic mean value*

In order to solve problem (9) in the sense of mathematical programming, we introduce the crisp possibilistic mean value. Subsequently, we assume that each spread $\tilde{\alpha}_j$ is the following trapezoidal fuzzy number:

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$$\mu_{\tilde{\alpha}_j}(\omega)=\begin{cases}\dfrac{\omega-\alpha_j^L}{\underline{\alpha}_j-\alpha_j^L} & \left(\alpha_j^L\le\omega<\underline{\alpha}_j\right)\\ 1 & \left(\underline{\alpha}_j\le\omega\le\bar{\alpha}_j\right)\\ \dfrac{\alpha_j^U-\omega}{\alpha_j^U-\bar{\alpha}_j} & \left(\bar{\alpha}_j<\omega\le\alpha_j^U\right)\\ 0 & \text{otherwise}\end{cases},\ j=1,2,...,n \quad (10)$$

Using these trapezoidal fuzzy numbers and the fuzzy extension principle, the membership function of fuzzy number $\tilde{\alpha}(\boldsymbol{x})=\sum_{j=1}^{n}\tilde{\alpha}_j x_j$ is also the following trapezoidal fuzzy number:

$$\mu_{\tilde{\alpha}(\boldsymbol{x})}(\omega)=\begin{cases}\dfrac{\omega-\sum_{j=1}^{n}\alpha_j^L x_j}{\sum_{j=1}^{n}\underline{\alpha}_j x_j-\sum_{j=1}^{n}\alpha_j^L x_j} & \left(\sum_{j=1}^{n}\alpha_j^L x_j\le\omega<\sum_{j=1}^{n}\underline{\alpha}_j x_j\right)\\ 1 & \left(\sum_{j=1}^{n}\underline{\alpha}_j x_j\le\omega\le\sum_{j=1}^{n}\bar{\alpha}_j x_j\right)\\ \dfrac{\sum_{j=1}^{n}\alpha_j^U x_j-\omega}{\sum_{j=1}^{n}\alpha_j^U x_j-\sum_{j=1}^{n}\bar{\alpha}_j x_j} & \left(\sum_{j=1}^{n}\bar{\alpha}_j x_j<\omega\le\sum_{j=1}^{n}\alpha_j^U x_j\right)\\ 0 & \text{otherwise}\end{cases},\ j=1,2,...,n \quad (11)$$

Then, the $\gamma$ -level set of $\tilde{\alpha}(\boldsymbol{x})$ is as follows:

$$\left[\tilde{\alpha}(\boldsymbol{x})\right]^{\gamma}=\left[\gamma\sum_{j=1}^{n}\underline{\alpha}_j x_j+(1-\gamma)\sum_{j=1}^{n}\alpha_j^L x_j,\ \gamma\sum_{j=1}^{n}\bar{\alpha}_j x_j+(1-\gamma)\sum_{j=1}^{n}\alpha_j^U x_j\right] \quad (12)$$

Therefore, from the mathematical concept in Subsection 2.2, the crisp possibilistic mean value of $\tilde{\alpha}(\boldsymbol{x})$ is as follows:

$$\begin{aligned}E\left(\tilde{\alpha}(\boldsymbol{x})\right)&=\int_0^1\gamma\left(\gamma\sum_{j=1}^{n}\left(\underline{\alpha}_j+\bar{\alpha}_j-\alpha_j^L-\alpha_j^U\right)x_j+\sum_{j=1}^{n}\left(\alpha_j^L+\alpha_j^U\right)x_j\right)d\gamma\\&=\frac{1}{3}\sum_{j=1}^{n}\left(\underline{\alpha}_j+\bar{\alpha}_j\right)x_j+\frac{1}{6}\sum_{j=1}^{n}\left(\alpha_j^L+\alpha_j^U\right)x_j\end{aligned} \quad (13)$$

Consequently, problem ( ) is transformed into the following problem from the viewpoint of crisp possibilistic mean value:

$$\begin{aligned}&\text{Maximize}\quad h\\&\text{subject to}\quad \sum_{j=1}^{n}\bar{r}_j x_j+\frac{L^*(h)}{3}\sum_{j=1}^{n}\left(\underline{\alpha}_j+\bar{\alpha}_j+\frac{1}{2}\left(\alpha_j^L+\alpha_j^U\right)\right)x_j\ge g^{-1}(h),\\&\qquad\sum_{j=1}^{n}x_j=1,\quad 0\le x_j\le p_j,\quad j=1,2,\cdots,n\end{aligned} \quad (14)$$

In this problem, coefficients $\underline{\alpha}_j+\bar{\alpha}_j+\frac{1}{2}\left(\alpha_j^L+\alpha_j^U\right)$ are initially set as fixed values, and so this problem is similar to problem (9), which is one of the standard possibility programming problem. Then, it should be noted here that problem (14) is a nonconvex programming problem and it is not directly solved by the linear programming techniques or convex programming techniques. However, a decision variable $h$ is involved only in first constraint. Therefore, we introduce the following subproblem involving a parameter $q$ :

$$\begin{aligned}&\text{Maximize}\quad \sum_{j=1}^{n}\bar{r}_j x_j+\frac{L^*(q)}{3}\sum_{j=1}^{n}\left(\underline{\alpha}_j+\bar{\alpha}_j+\frac{1}{2}\left(\alpha_j^L+\alpha_j^U\right)\right)x_j\\&\text{subject to}\quad \sum_{j=1}^{n}x_j=1,\quad 0\le x_j\le p_j,\quad j=1,2,\cdots,n\end{aligned} \quad (15)$$

In the case that parameter $q$ is fixed, problem (15) is degenerated to a linear programming problem. Therefore, in a way similar to the standard possibility maximization model, we construct the following efficient solution method using the hybrid approach of bisection algorithm and linear programming based on the study [6].

**Solution algorithm**

STEP1: Elicit the membership function of a fuzzy goal for the total future return with respect to the objective function value.

STEP2: Set $q\leftarrow 1$ and solve problem (14). If the optimal objective value $Z(q)$ of problem (14) satisfies $Z(q)>g^{-1}(q)$ , then terminate. In this case, the obtained current solution is an optimal solution of main problem.

STEP 3: Set $q\leftarrow 0$ and solve problem (14). If the optimal objective value $Z(q)$ of problem (14) satisfies $Z(q)\le g^{-1}(q)$, then terminate. In this case, there is no feasible solution and it is necessary to reset a fuzzy goal for the probability or the aspiration level $f$ .

STEP 4: Set $U_q\leftarrow 1$ and $L_q\leftarrow 0$ .

STEP 5: Set $\gamma\leftarrow\left(U_q+L_q\right)/2$ .

STEP 6: Solve problem (14) and calculate the optimal objective value $Z(q)$ of problem (14). If $Z(q)>g^{-1}(q)$, then set $L_q\leftarrow q$ and return to Step 5. If $Z(q)\le g^{-1}(q)$, then set $U_q\leftarrow q$ and return to Step 5. If $Z(q)=g^{-1}(q)$ , then terminate the algorithm. In this case, $\boldsymbol{x}^*(q)$ is equal to a global optimal solution of main problem.

## 4 Numerical example

In order to compare our proposed models with other models for portfolio selection problems, let us consider a numerical example based on the data of securities on the Tokyo Stock Exchange. In this paper, we compare our proposed model (14) in Section 3 with Carlsson et al. model [2] and Vercher et al. model [22]. These models are type-1 fuzzy portfolio models and include the possibilistic mean value and variance. These problems are formulated as the following form:

(Carlsson et al. model)

$$\begin{aligned}&\text{Maximize}\quad \sum_{j=1}^{n}\frac{1}{2}\left[a_j+b_j+\frac{1}{3}\left(\beta_j-\alpha_j\right)\right]x_j\\&\qquad-\frac{0.0123}{4}\left(\sum_{j=1}^{n}\frac{1}{2}\left[b_j-a_j+\frac{1}{3}\left(\alpha_j+\beta_j\right)\right]x_j\right)^2-\frac{0.0123}{72}\left(\sum_{j=1}^{n}\left(\alpha_j+\beta_j\right)x_j\right)^2\\&\text{subject to}\quad \sum_{j=1}^{n}x_j=1,\quad 0\le x_j\le p_j,\quad j=1,2,\cdots,n\end{aligned}$$

(Vercher et al. model)

$$\begin{aligned} &\text{Minimize} && \sum_{j=1}^{n}\frac{1}{2}\left[b_j-a_j+\frac{1}{3}(\alpha_j+\beta_j)\right]x_j \\ &\text{subject to} && \sum_{j=1}^{n}\frac{1}{2}\left[a_j+b_j+\frac{1}{3}(\beta_j-\alpha_j)\right]x_j \ge \rho, \\ &&& \sum_{j=1}^{n}x_j=1,\quad 0\le x_j\le p_j,\quad j=1,2,\cdots,n \end{aligned}$$

Let us consider ten securities shown in Table 1, whose mean values and standard deviations are based on historical data in the decade between 1995 and 2004. Then, we introduce the asset allocation rate $x_j,\ (j=1,2,...,10)$ to each security, and its upper value is assumed to be 0.2.

Table 1: Sample data from Tokyo Stock Exchange

| Returns | Sample mean | SD |
|---|---|---|
| R1 | 0.055 | 0.445 |
| R2 | 0.046 | 0.289 |
| R3 | 0.015 | 0.306 |
| R4 | 0.114 | 0.208 |
| R5 | 0.043 | 0.253 |
| R6 | 0.034 | 0.269 |
| R7 | 0.018 | 0.230 |
| R8 | 0.171 | 0.297 |
| R9 | 0.087 | 0.388 |
| R10 | 0.090 | 0.318 |

With respect to our proposed type-2 fuzzy portfolio model, parameter $\overline{r}_j$ of the primary *L*-fuzzy number is assumed to be sample mean in Table 1. Then, in two previous models parameters, parameters of trapezoidal fuzzy number $a_j,\ b_j,\ \alpha_j,\ \beta_j,\ (j=1,2,...,10)$ based on study [22] are shown in Table 2 using the historical data in Table 1.

Table 2. Each parameter value based on the historical data

| Returns | $a_j$ | $b_j$ | $\alpha_j$ | $\beta_j$ |
|---|---|---|---|---|
| R1 | -0.123 | 0.005 | 0.239 | 0.868 |
| R2 | -0.069 | 0.069 | 0.301 | 0.467 |
| R3 | -0.129 | 0.025 | 0.200 | 0.713 |
| R4 | 0.005 | 0.177 | 0.198 | 0.235 |
| R5 | -0.082 | 0.114 | 0.217 | 0.323 |
| R6 | -0.052 | 0.108 | 0.290 | 0.382 |
| R7 | -0.056 | 0.067 | 0.236 | 0.369 |
| R8 | 0.060 | 0.193 | 0.310 | 0.456 |
| R9 | -0.093 | 0.130 | 0.312 | 0.626 |
| R10 | 0.009 | 0.236 | 0.563 | 0.264 |

The secondary membership functions for $\tilde{\alpha}_j$ of type-2 fuzzy numbers are assumed to be triangle fuzzy numbers, whose $\underline{\alpha}_j=\overline{\alpha}_j$ are variances based on the SD in Table1 and the spreads $\underline{\alpha}_j-\alpha_j^L$ and $\alpha_j^U-\overline{\alpha}_j$ become $\alpha_j$ and $\beta_j$ in Table 2, respectively. Therefore, using these data and introducing the following fuzzy goals for our proposed model;

$$\mu_G(f)=\begin{cases} 1 & (0.15\le f) \\ \dfrac{f-0.12}{0.03} & (0.12\le f<0.15) \\ 0 & (f<0.12) \end{cases},$$

we set the parameter $\rho$ in the Vercher et al. model as $\rho=0.06$, and obtain optimal portfolios for three models shown in Table 3.

Subsequently, we consider the case where an investor purchases securities at the end of 2004 according to each portfolio shown in Table 3. Then, the total return of three models at term ends of 2005, 2006 and 2007 become the following values shown in Table 4, respectively.

Table 3. Each optimal solution with respect to three models

| | Proposed model | Carlsson model | Vercher model |
|---|---|---|---|
| R1 | 0.200 | 0 | 0 |
| R2 | 0 | 0 | 0 |
| R3 | 0 | 0 | 0 |
| R4 | 0.200 | 0.200 | 0.200 |
| R5 | 0 | 0 | 0.200 |
| R6 | 0 | 0.200 | 0.200 |
| R7 | 0 | 0 | 0.200 |
| R8 | 0.200 | 0.200 | 0.200 |
| R9 | 0.200 | 0.200 | 0.200 |
| R10 | 0.200 | 0.200 | 0.200 |

Table 4. Total return to the portfolio of each model

| Term end | Proposed | Carlsson | Vercher |
|---|---|---|---|
| 2005 | 0.2524 | 0.2194 | 0.2852 |
| 2006 | 0.4050 | 0.3575 | 0.3257 |

From the result in Table 4, we find that our proposed model obtains the larger total return than the Carlsson et al. model dealt with the only crisp possibilistic mean value. Furthermore, from the total return at the term end of 2006, in the case that the investment period is longer, our proposed model obtains much larger total return.

## 5 Conclusion

In this paper, we have proposed a new portfolio selection problem based on the type-2 fuzzy set. Our proposed model has been initially not a well-defined problem due to primary and secondary fuzzy numbers. Therefore, in order to solve analytically in the sense of mathematical programming, we have introduced the possibility maximization model with fuzzy goal for the object and degree of possibility. Furthermore, by using the concept of crisp possibilistic mean value for the secondary fuzzy numbers, the main problem has been equivalently transformed into the parametric linear programming problem. Consequently, we have developed the efficient solution method to combine the bisection algorithm with the standard linear programming approach. Since this proposed model includes various types of investor's subjectivity in the form of type-2 fuzzy number, it is more versatile than previous models. Our proposed model is dealt with various practical situations in the investment such as the case putting the whole policy of investors including a lot of subjectivity by using the possibilistic mean value and the case

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indicating the sensible decision making to the investor with shaken investment policy due to uncertain but attractive information.

In this paper, as the criterion for the object, we introduced the possibility measure and the crisp possibilistic mean value. On the other hand, there are various types of criterions with respect to fuzzy object and constraints such as necessity and credibility measure, fuzzy interval approach, etc.. Therefore, as the future works, we need to consider more general and versatile type-2 fuzzy portfolio selection problems with such criterions.

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