A Multi-objective Evolutionary Algorithm for Tuning Fuzzy Rule-Based Systems with Measures for Preserving Interpretability.

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Abstract— *In this contribution we propose a multi-objective evolutionary algorithm for Tuning Fuzzy Rule-Based Systems by considering two objectives, accuracy and interpretability. To this aim we define a new objective that allows preserving the interpretability of the system. This new objective is an interpretability index which is the union of three metrics to preserve the original shapes of the membership functions as much as possible while a tuning of the membership function parameters is performed. The proposed method has been compared to a single objective accuracy-guided algorithm in two real problems showing that many solutions in the Pareto front dominate to those obtained by the single objective-based one.*

Keywords— Fuzzy Rule-Based Systems, Tuning, Interpretability, Multi-Objective Evolutionary Algorithms.

1 Introduction

Fuzzy modeling usually tries to improve the accuracy of the system without inclusion of any interpretability measure, an essential aspect of Fuzzy Rule-Based Systems (FRBSs). However, the problem of finding the right trade-off between accuracy and interpretability has achieved a growing interest [1].

One of the most widely-used approaches to enhance the performance of FRBSs is the *tuning* of the Membership Functions (MFs). It consists of refining a previous definition of the Data Base (DB) once the Rule Base (RB) has been obtained. Generally, tuning is a variation in the shape of the MFs that improves their global interaction. Classically, the tuning methods refine the parameters that identify the MFs associated to the labels comprising the DB [2].

For this model, to take into account interpretability issues it is necessary to use a measure to quantify the interpretability of the tuned DB. This kind of measure could be used as an additional objective to maintain the interpretability of the fuzzy partitions when the tuning is carried out.

In the literature, many authors improve the difficult trade-off between accuracy and interpretability of FRBSs, obtaining linguistic models not only accurate but also interpretable. We can distinguish two kinds of approaches for managing the interpretability:

1. The complexity of the model [3, 4, 5, 6] (usually measured as number of rules, variables, labels per rule, etc.)
2. Measuring the interpretability of the fuzzy partitions [4, 7, 8] by means of a semantic interpretability measure.

A way for optimizing both objectives (accuracy and interpretability) is the use of the Multi-Objective Evolutionary Algorithms (MOEAs) [9, 10]. Since this problem presents a multi-objective nature the use of MOEAs to obtain a set of solutions with different degrees of accuracy and interpretability is an interesting way to work [3, 5, 6, 8].

In this work, we focus our attention in measuring the interpretability of the fuzzy partitions. We propose a semantics interpretability index for maintaining the interpretability of the system by means of the aggregation of several metrics, with the aim of preserving the original form of the MFs while a tuning is performed. These metrics try to minimize the displacement of the central point of the MFs, besides maintaining the symmetry and the area of the original MFs associated to the linguistic labels. To this end, we apply a specific MOEA to obtain accurate and interpretable linguistic fuzzy models by performing a tuning of the MFs parameters with two main objectives:

- The system error
- The proposed interpretability index.

This algorithm is based on the well known $SPEA2$ [11] and is called SPEA2$_{SI}$ ($SPEA2$ for Semantic Interpretability). In order to improve its search ability, SPEA2$_{SI}$ implements some concepts as incest prevention and restarting [12]. Moreover, it is focused on the most accurate solutions to finally form a wide Pareto front. Thus, SPEA2$_{SI}$ is aimed at generating a complete set of Pareto-optimum solutions with different trade-offs between accuracy and interpretability. We have not considered the well-known NSGA-II [13] algorithm since in [3], approaches based on SPEA2 were shown to be more effective when performing a tuning of the MFs.

To show the good performance of the proposed method it is compared with a single objective accuracy-guided tuning algorithm [14] by applying both of them to initial linguistics models obtained from automatic learning methods. Two real-world problems with different complexities have been considered showing that the solutions of the accuracy based algorithm are dominated by those obtained by SPEA2$_{SI}$. We can see as both objectives required are certainly contradictory as the obtained Pareto fronts moves clearly from the most accurate solutions to the most interpretable ones.

Next section describes the classic tuning used in this work. Section 3 presents the proposed index to control the interpretability of the MFs. Section 4 presents the SPEA2$_{SI}$ algorithm describing its main characteristics and the genetic operators considered. Section 5 shows the experimental study

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and the results obtained. Finally, in section 6 we point out some conclusions.

2 Preliminaries: Tuning of MFs

This approach, usually called DB tuning, involves refining the MF shapes from a previous definition once the remaining FRBS components have been obtained [15, 16, 17, 18, 19, 20]. The classic way to refine the MFs is to change their definition parameters. For example, if the following triangular-shape MF is considered:

$$\mu(x) = \begin{cases} \frac{x-a}{b-a}, & \text{if } a \leq x < b \\ \frac{c-x}{c-b}, & \text{if } b \leq x \leq c \\ 0, & \text{otherwise} \end{cases}$$

changing the basic parameters — a, b, and c — will vary the shape of the fuzzy set associated to the MF, thus influencing the FRBS performance (See Figure 1). The same yields for other shapes of MFs (trapezoidal, gaussian, sigmoid, etc.).

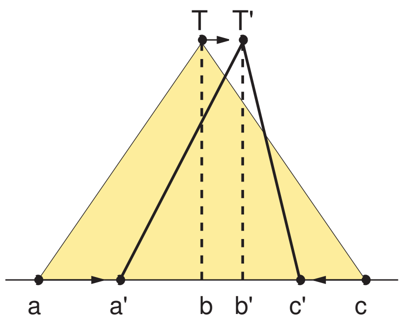

Figure 1: Tuning by changing the basic MF parameters

Tuning involves fitting the characterization of the MFs associated to the primary linguistic terms considered in the system. Thus, the meaning of the linguistic terms is changed from a previous definition (an initial DB). In order to ensure the semantics integrity through the MFs optimization process [1, 21, 22], some researchers have proposed several properties. Considering one or more of these properties several constraints can be applied in the design process in order to obtain a DB maintaining the linguistic model comprehensibility to the higher possible level [15, 23, 24, 25].

In this work we consider strong fuzzy partitions to define the initial MFs. In order to maintain semantics integrity we consider also these constraints by defining the variation intervals for each MF parameter. For each $MF_j = (a_j, b_j, c_j)$ where j=(1,...m) and m is the number of MFs in a given DB, the variation intervals are calculated in the following way:

$$[I^l_{a_j}, I^r_{a_j}] = [a_j - (b_j - a_j)/2, a_j + (b_j - a_j)/2] \ ,$$
$$[I^l_{b_j}, I^r_{b_j}] = [b_j - (b_j - a_j)/2, b_j + (c_j - b_j)/2] \ ,$$
$$[I^l_{c_j}, I^r_{c_j}] = [c_j - (c_j - b_j)/2, c_j + (c_j - b_j)/2] \ .$$

Thanks to these restrictions it is possible to maintain the comprehensibility of MFs to a reasonable level. In any case, it would be very interesting to have a measure for the quality of the tuned MFs. We propose three metrics trying to preserve the original form of the MFs, so improving if possible the trade-off between accuracy and interpretability.

3 An Interpretability Index based on Three Interpretability Metrics

In this section, we propose several metrics to measure the interpretability when a tuning is performed on the DB. At this point, we should remark that these metrics are based on the existence of the variation intervals (integrity constraints) defined in the previous section and, therefore, on the assumption that the initial DB is comprised of triangular uniformly distributed MFs (strong fuzzy partitions) with an associated linguistic meaning. Based on these assumptions, significant changes in the DB can influence negatively in the interpretability. In this way, each metric is proposed to control how good are different desirable aspects of the tuned MFs with respect to the original ones. The metrics proposed are:

- MFs displacements (δ): This metric measures the proximity of the central points of the MFs to the original corresponding ones. It should be higher as closer to the original point.
- MFs symmetry (γ): This metric measures the symmetry of the MFs. It should be higher when the two parts of the support of the MFs (left and right) present more similar lengths.
- MFs area similarity (ρ): This metric measures the similarity of the area of the tuned MFs to the original one. It should be higher when the tuned area and the original area are closer.

In the following subsections the three proposed metrics will be explained in depth.

3.1 *MFs displacements (δ)*

This metric can control the displacements in the central point of the MFs. It is based on computing the normalized distance between the central point of the tuned MF and the central point of the original MF, and is calculated through keeping the maximum displacement obtained on all the MFs. For each MF_j: $\delta_j = |b_j - b'_j|/I$, where $I = (I^r_{b_j} - I^l_{b_j})/2$ represents the maximum variation for each central parameter. Thus δ^* is defined as: $\delta^* = max_j\{\delta_j\}$.

The δ^* metric takes values between 0 and 1, thereby value near to 1 represent that the MFs present a great displacement. The following transformation is made so this metric represents proximity (maximization):

$$\delta = 1 - \delta^* \ ,$$

3.2 *MFs symmetry (γ)*

This metric can be used to control the symmetry of the MF shapes. It is based on relating the two parts of the support of the tuned MFs. Let us define that $leftS'_j = |a'_j - b'_j|$ is the length of the left part of the MF support and that $rightS'_j = |b'_j - c'_j|$ is the right part. γ_j is calculated using the following equation:

$$\gamma_j = \frac{min\{leftS'_j, rightS'_j\}}{max\{leftS'_j, rightS'_j\}} \ .$$

Values near to 1 mean that the two parts of the support of the MFs are more similar (higher symmetry). Finally γ is calculated by keeping the minimum value obtained.

$$\gamma = min_j\{\gamma_j\} \ .$$

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3.3 MFs area similarity (ρ)

This metric can be used to control the area of the MF shapes. It is based on relating the areas of the original and the tuned MFs. Let us define that A_j is the area of the triangle representing the original MF_j, and A'_j is the new area. ρ_j is calculated using the following equation for each MF:

$$\rho_j = \frac{min\{A_j, A'_j\}}{max\{A_j, A'_j\}} \ .$$

Values near to 1 mean that the original area and the tuned area of the MFs are more similar (less changes). The ρ metric is calculated through keeping the minimum value obtained:

$$\rho = min_j\{\rho_j\} \ .$$

3.4 A global Semantics Interpretability index based on the aggregation of the three metrics

We propose an aggregation of the metrics in a global index based on the geometric mean, that is denoted as GM3M index. It is clear that if only one of the proposed metrics has very low values a problem in the interpretability will arise. The aggregation operator should consider this fact:

$$GM3M = \sqrt[3]{\delta * \gamma * \rho}$$

The value of $GM3M$ ranges between 0 (the lowest level of interpretability) and 1 (the highest level of interpretability).

4 Proposed Multi-objective Evolutionary Algorithm

The proposed algorithm performs a parametric tuning to improve the system accuracy as a first objective and uses the $GM3M$ index to try to preserve the interpretability. It is called $SPEA2$ for Semantic Interpretability ($SPEA2_{SI}$) and is based on the well-known $SPEA2$[11] algorithm. In the next subsections the main components of this algorithm are described and the specific characteristics are presented.

4.1 Coding Scheme and Initial Gene Pool

We consider a real coding scheme, being m^i the number of labels of each of the n variables comprising the DB,

$$\begin{aligned} &C_i = (a^i_1, b^i_1, c^i_1, \ldots, a^i_{m^i}, b^i_{m^i}, c^i_{m^i}), \\ &\ i = 1, \ldots, n \ , \\ &C = C_1 C_2 \ldots C_n \ . \end{aligned}$$

The initial DB is included as first individual of the initial population. The remaining individuals are generated at random within the corresponding variation intervals defined in the previous section.

4.2 Objectives

In this algorithm we use two objectives. They are:

- The interpretability index ($GM3M$) that is a maximization objective representing the geometric mean of the three proposed metrics (interpretability).
- The Mean Squared Error (MSE) that is a minimization objective (accuracy):

$$\text{MSE} = \frac{1}{2 \cdot |E|} \sum_{l=1}^{|E|} (F(x^l) - y^l)^2,$$

with $|E|$ being the data set size, $F(x^l)$ being the output obtained from the FRBS decoded from such chromosome when the l-th example is considered and y^l being the known desired output. The fuzzy inference system considered to obtain $F(x^l)$ is the *center of gravity weighted by the matching* strategy as defuzzification operator and the *minimum t-norm* as implication and conjunctive operators.

4.3 Main Characteristics of SPEA2$_{SI}$

The proposed algorithm makes use of the $SPEA2$ selection mechanism. However in order to improve the search ability of the algorithm the following changes are considered:

- The proposed algorithm includes a mechanism for incest prevention based on the concepts of CHC [12], for maintaining population diversity. This mechanism avoids premature convergence. Only those parents whose hamming distance divided by 4 is higher than a threshold are crossed. The well-known BLX-0.5 [26] crossover is applied to obtain the offspring.

  Since we consider a real coding scheme, we have to transform each gene considering a Gray Code with a fixed number of bits per gene ($BGene$) determined by the system expert. In this way, the threshold value is initialized as:

  $$L = (\#C * BGene)/4,$$

  where $\#C$ is the number of genes of the chromosome.

  At each generation of the algorithm, the threshold value is decremented by one which allows crossing closer solutions. Following the concept of the CHC algorithm that does not used mutation, in our algorithm we do not include mutation operator.

- A restarting operator is applied by maintaining the most accurate individual, and the most interpretable individual as a part of the new population (external population is forced to be empty) and obtaining the remaining individuals with the tuning parameters generated at random within the corresponding variation intervals. This way preserves both the most accurate and the most interpretable solutions obtained.

  Restarting should be applied when we detect that all crossovers are allowed. However in order to avoid premature convergence we apply the first restart if 50 percent of crossovers are detected at any generation. This condition is updated each time restarting is performed as $\%_{Required} = (100{+}\%_{Required})/2$. Moreover, the most accurate solution should be improved before each restarting. To preserve a well formed Pareto front at the end, the restarting is not applied in the last evaluations. The number of evaluations without restart can be estimated as the number of evaluations needed to apply the first restart multiplied by 10. Additionally, restart is disabled if it was never applied before reaching the mid of the total number of evaluations.

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- In each stage of the algorithm (between restarting points), the number of solutions in the external population ($\overline{P}_{t+1}$) considered to form the mating pool is progressively reduced, by focusing only on those with the best accuracy. To do that, the solutions are sorted from the best to the worst (considering accuracy as sorting criterion) and the number of solutions considered for selection is reduced progressively from 100% at the beginning to 50% at the end of each stage by taking into account the value of L.

  In the last evaluations when restart is disabled. This mechanism, whose main objective is focusing on the most accurate solutions, is also disabled in order to obtain a wide well formed Pareto front.

5 Experiments

To evaluate the goodness of the proposed approach, two real-world problems with different complexities (different number of variables and available data) are considered to be solved (these data sets are available at, *http://www.keel.es/*) [27]:

- Estimating the maintenance costs of medium voltage lines in a town (ELE).
- Predicting the Abalone Age (ABA).

In both cases, the well-known *ad hoc* data-driven learning algorithm of Wang and Mendel [28] is applied to obtain an initial set of candidate linguistic rules. To do so, we will consider strong fuzzy partitions of triangular-shaped MFs. Once the initial RB is generated, the proposed post-processing algorithm can be applied.

Methods considered for the experiments are:

- T method performs a classic MFs parameter tuning by only considering the accuracy of the model as the sole objective [14].
- $SPEA2_{SI}$ is the proposed MOEA method for the classic tuning considering two objectives, precision and the semantics interpretability index (GM3M).

5.1 Experimental Set-up

We consider a *5-fold cross-validation model*, i.e., 5 random partitions of data each with 20%, and the combination of 4 of them (80%) as training and the remaining one as test. For each one of the 5 data partitions, the tuning methods have been run 6 times, showing for each problem the averaged results of a total of 30 runs.

In the case of $SPEA2_{SI}$ the averaged values are calculated considering the most accurate solution from each Pareto obtained. In this way, $SPEA2_{SI}$ can be compared with the single objective method T.

The values of the input parameters considered by T are: population size of 61, 100000 evaluations, 0.6 as crossover probability and 0.2 as mutation probability per chromosome. The values of the input parameters considered by $SPEA2_{SI}$ are: population size of 200, external population size of 61, 100000 evaluations and 30 bits per gene for the Gray codification.

5.2 Results and Analysis

Table 1 shows the results obtained with WM, where $\#R$ stands for the number of rules, MSE$_{tra/tst}$ for the averaged error obtained over the training/test data, σ for their respective standard deviations and $GM3M$ for the interpretability index. This method obtains the initial knowledge bases that will be tuned by T and $SPEA2_{SI}$. For the WM method the interpretability index takes value 1 that is the highest level of interpretability.

Table 1: Results obtained with WM method

Dataset	**#R**	**MSE$_{tra}$**	σ_{tra}	**MSE$_{tst}$**	σ_{tst}	**GM3M**
ELE	65	56136	1498	56359	4685	1
ABA	68	8.407	0.443	8.422	0.545	1

The results obtained by both post-processing methods are shown in Table 2. In addition we also show δ, γ and ρ that represent the individual values of the metrics comprising, and t represents the results of applying a *test t-student* (with 95 percent confidence) in order to ascertain whether differences in the performance of the best results are significant when compared with that of the other algorithm in the table. The interpretation of the t column is:

- $\star$ represents the best averaged result.
- $+$ means that the best result has better performance than that of the related row.

Analysing the results showed in Table 2, we can highlight the following facts:

- The proposed method obtains the best results in training and test with respect to T in both problems. In the case of the electrical problem, SPEA2$_{SI}$ improves among 7% and 8.5% in training and test respectively and in the abalone problem it obtains an improvement of 1.5%.
- The most accurate solutions from SPEA2$_{SI}$ improve the accuracy and obtain more interpretable models, with 29%(ELE) and 52%(ABA) of improvement in the interpretability index with respect to the T method.

Figure 2 shows the Pareto front obtained with SPEA2$_{SI}$ method, the solution obtained by T and the initial knowledge base obtained by WM in the same data partition and seed of ELE. We can observe that the obtained Pareto front is quite wide. In fact, the number of non dominated solutions is always equal to the external population size. Moreover, the WM solution coincides with the last point of the Pareto front and the solution obtained with T is dominated by several solutions from SPEA2$_{SI}$. Furthermore there is not overfitting in the results obtained with the proposed method.

The Pareto front obtained allows selecting solutions with different degrees of accuracy and interpretability. Figure 2 presents that an improvement in the interpretability produces lack of precision and an improvement in the precision produces lack of interpretability. This figure clearly shows that

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Table 2: Results obtained in both problems

Dataset	Method	MSE_{tra}	σ_{tra}	t-test	MSE_{tst}	σ_{tst}	t-test	GM3M	σ_{GM3M}	t-test	δ	γ	ρ
ELE	T	17020	1893	+	21027	4225	=	0.225	0.046	+	0.058	0.337	0.694
	$SPEA2_{SI}$	**15884**	1191	*	**19257**	2893	*	**0.319**	0.170	*	0.225	0.398	0.648
ABA	T	2.688	0.063	+	2.770	0.242	=	0.144	0.051	+	0.032	0.182	0.636
	$SPEA2_{SI}$	**2.648**	0.051	*	**2.744**	0.276	*	**0.298**	0.153	*	0.177	0.392	0.625

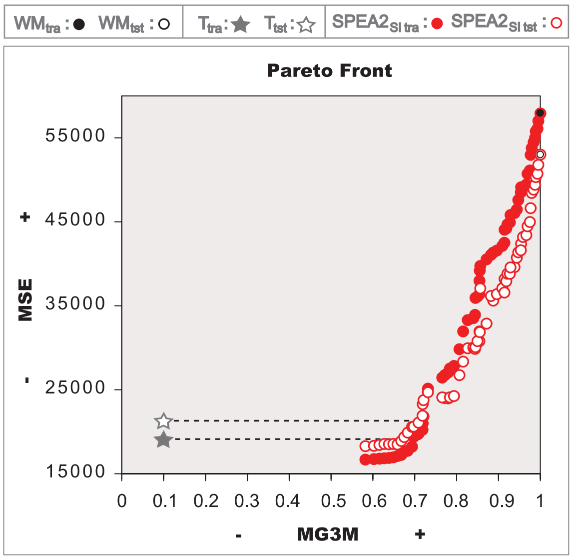

Figure 2: Pareto Front obtained in ELE

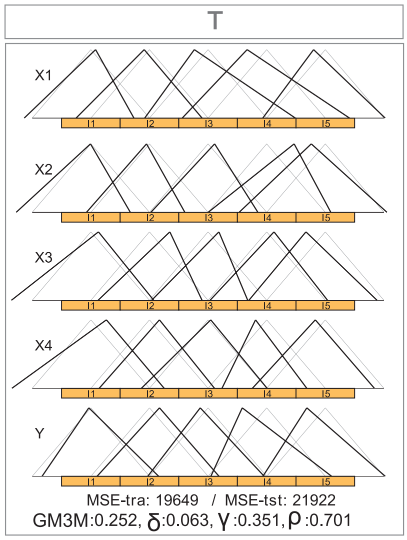

Figure 3: A DB obtained with T method in ELE

both objectives are actually contradictory. In the extremes of the Pareto front an improvement in one objective represents small lost in the other objective. On the contrary in the mid part of the Pareto front improvements in one objective deteriorates the other objective.

Figure 3 presents an illustrative DB obtained with T. Some DBs obtained with $SPEA2_{SI}$, in the same data partition and seed than this shown for T, are shown in Figure 4. This figure includes three DBs, one with the most accurate solution, other with a solution not only accurate but also interpretable and another highly interpretable DB, that obtains 35% of improvement with respect to the WM method with a value of the interpretability index near to 1.

6 Conclusions

The proposed index is useful to preserve the original shape of the MFs, in order to maintain the interpretability and it is a measure of the quality of the DB.

The proposed method obtains wide well formed Pareto fronts that provide a large variety of solutions to select from more accurate solutions to more interpretable ones.

Finally, we can stand out that $SPEA2_{SI}$ algorithm is very competitive and efficient since it is able to maintain the DB interpretability at a better level while accuracy is greatly improved.

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References

[1] J. Casillas, O. Cordón, F. Herrera, and L. Magdalena, editors. *Interpretability issues in fuzzy modeling*, volume 128 of *Studies in Fuzziness and Soft Computing*. Springer, 2003.

[2] O. Cordón, F. Herrera, F. Hoffmann, and L. Magdalena. *Genetic Fuzzy Systems. Evolutionary tuning and learning of fuzzy knowledge bases*, volume 19 of *Advances in Fuzzy Systems - Applications and Theory*. World Scientific, 2001.

[3] R. Alcalá, M. J. Gacto, F. Herrera, and J. Alcalá-Fdez. A multi-objective genetic algorithm for tuning and rule selection to obtain accurate and compact linguistic fuzzy rule-based systems. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 15(5):539–557, 2007.

[4] José M. Alonso, Luis Magdalena, and Serge Guillaume. Hilk: A new methodology for designing highly interpretable linguistic knowledge bases using the fuzzy logic formalism. *International Journal of Intelligent Systems*, 23(7):761–794, 2008.

[5] A. Botta, B. Lazzerini, F. Marcelloni, and D. C. Stefanescu. Context adaptation of fuzzy systems through a multi-objective evolutionary approach based on a novel interpretability index. *Soft Computing*, 13(5):437–449, 2008.

[6] H. Ishibuchi and Y. Nojima. Analysis of interpretability-accuracy tradeoff of fuzzy systems by multiobjective fuzzy

ISBN: 978-989-95079-6-8

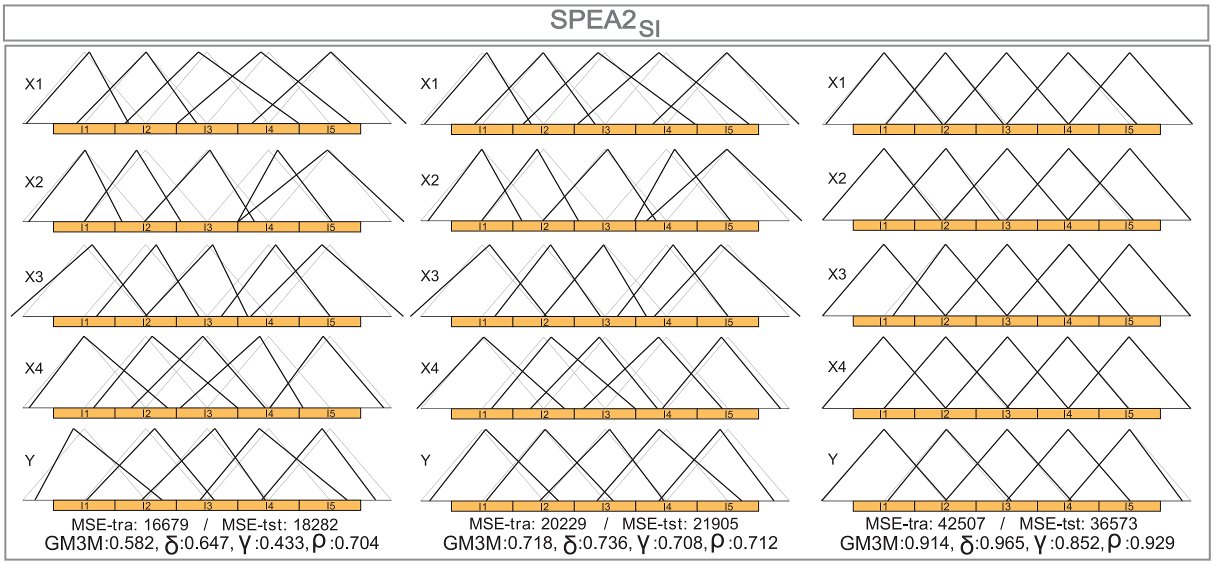

Figure 4: Three DBs obtained in a run of SPEA2$_{SI}$ in ELE

genetics-based machine learning. *International Journal of Approximate Reasoning*, 44(1):4–31, 2007.

[7] J.M. Alonso, O. Cordón, S. Guillaume, and L. Magdalena. Highly interpretable linguistic knowledge bases optimization: Genetic tuning versus solis-wetts. looking for a good interpretability-accuracy trade-off. In *Proceedings of the 2007 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2007)*, pages 901–906, 2007.

[8] P. Fazendeiro and J. V. de Oliveira. A working hypothesis on the semantics/accuracy synergy. In *EUSFLAT-LFA 2005*, pages 266–271, 2005.

[9] C. A. Coello, D. A. Van Veldhuizen, and G. B. Lamont, editors. *Evolutionary algorithms for solving multi-objective problems*. Kluwer Academic Publishers, 2002.

[10] K. Deb. *Multi-objective optimization using evolutionary algorithms*. John Wiley & Sons, NY, USA, 2001.

[11] E. Zitzler, M. Laumanns, and L. Thiele. Spea2: Improving the strength pareto evolutionary algorithm for multiobjective optimization. In *Proc. Evolutionary Methods for Design, Optimization and Control with App. to Industrial Problems*, pages 95–100, Barcelona, Spain, 2001.

[12] L. J. Eshelman. The CHC adaptive search algorithm: How to have safe search when engaging in nontraditional genetic recombination. In G.J.E. Rawlin, editor, *Foundations of genetic Algorithms*, volume 1, pages 265–283. Morgan Kaufman, 1991.

[13] K. Deb, S. Agrawal, A. Pratab, and T. Meyarivan. A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Trans. Evolutionary Computation*, 6(2):182–197, 2002.

[14] J. Casillas, O. Cordón, M. J. Del Jesus, and F. Herrera. Genetic tuning of fuzzy rule deep structures preserving interpretability and its interaction with fuzzy rule set reduction. *IEEE Trans. Fuzzy Syst.*, 13(1):13–29, 2005.

[15] O. Cordón and F. Herrera. A three-stage evolutionary process for learning descriptive and approximate fuzzy logic controller knowledge bases from examples. *International Journal of Approximate Reasoning*, 17(4):369–407, 1997.

[16] H. B. Gürocak. A genetic-algorithm-based method for tuning fuzzy logic controllers. *Fuzzy Sets and Systems*, 108(1):39–47, 1999.

[17] F. Herrera, M. Lozano, and J. L. Verdegay. Tuning fuzzy logic controllers by genetic algorithms. *International J. of Approximate Reasoning*, 12:299–315, 1995.

[18] Y. Jin, W. von Seelen, and B. Sendhoff. On generating fc^3 fuzzy rule systems from data using evolution strategies. *IEEE Transactions on Systems, Man, and Cybernetics*, 29(6):829–845, 1999.

[19] C.L. Karr. Genetic algorithms for fuzzy controllers. *AI Expert*, 6(2):26–33, 1991.

[20] D. Nauck and R. Kruse. Neuro-fuzzy systems for function approximaton. *Fuzzy Sets and Systems*, 101(2):261–271, 1999.

[21] J. Valente de Oliveira. Semantic constraints for membership function optimization. *IEEE Trans. Syst., Man, Cybern. - Part A: Systems and Humans*, 29(1):128–138, 1999.

[22] J. Valente de Oliveira. Towards neuro-linguistic modeling: constraints for optimization of membership functions. *Fuzzy Sets and Systems*, 106(3):357–380, 1999.

[23] U. Bodenhofer and P. Bauer. A formal model of interpretability of linguistic variables. In Casillas et al. [1], pages 524–545.

[24] F. Cheong and R. Lai. Constraining the optimization of a fuzzy logic controller using an enhanced genetic algorithm. *IEEE Trans. Syst., Man, Cybern. - Part B: Cybernetics*, 30(1):31–46, 2000.

[25] J. Espinosa and J. Vandewalle. Constructing fuzzy models with linguistic integrity from numerical data-AFRELI algorithm. *IEEE Trans. Fuzzy Syst.*, 8(5):591–600, 2000.

[26] L. J. Eshelman and J. D. Schaffer. Real-coded genetic algorithms and interval-schemata. *Foundations of Genetic Algorithms*, 2:187–202, 1993.

[27] J. Alcalá-Fdez, L. Sánchez, S. García, M.J. del Jesus, S. Ventura, J.M. Garrell, J. Otero, C. Romero, J. Bacardit, V.M. Rivas, J.C. Fernández, and F. Herrera. Keel: A software tool to assess evolutionary algorithms for data mining problems. *Soft Computing*, 13(3):307–318, 2009.

[28] L. X. Wang and J. M. Mendel. Generating fuzzy rules by learning from examples. *IEEE Trans. Syst., Man, Cybern.*, 22(6):1414–1427, 1992.

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