Terrain Morphology Classification over Fuzzy Digital Elevation Models

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Abstract— *One way to incorporate uncertainty associated to the data and to the interpolation method in a digital elevation model (DEM) is to use Fuzzy Sets Theory. A fuzzy DEM can be generated from fuzzy data, where elevations are represented by fuzzy numbers. To use that type of terrain models in further applications, a generalization of known methods is needed. An example about the generalization of terrain morphology classification methods is proposed in this work. Given a fuzzy DEM, a procedure to find ridges and other morphological features will be analyzed. Two examples of this approach will be presented to illustrate the proposed methods.*

Keywords— Fuzzy DEM, fuzzy morphological feature, signal of a fuzzy number.

1 Introduction

Automated identification of objects representing terrain features (such as peaks, channels and ridges) from a digital elevation model is a well known procedure in Geomorphology. The development of those methods has important implications in terms of spatial analysis and for the generalization of spatial databases, for example. Another important application is the study of water flow over the terrain surface that represents a fundamental geomorphological process and it is closely related to its shape: it constraints the water route over the surface and it is mainly shaped by this flow.

The topographic surface is usually defined through a gridded DEM constructed by interpolation from a finite set of samples and it is a well-known problem in geographic modeling. The uncertainty associated to this process generates uncertain elevation values which can be modeled using fuzzy numbers [1, 2, 3]. Therefore, methods to identify morphological terrain features have to be generalized to fuzzy DEMs in order to express the impact of elevation uncertainty in terrain classification. In this work we will study one approach to this problems, generalizing known methods over fuzzy DEMs.

2 Morphology classification

To identify morphological objects over the terrain surface it is adopted here the strategy developed by Wood [4]. This problem can be seen as an image processing problem to classify each *pixel* (grid cell) in one class of *geomorphological feature* (usually, *peak*, *ridge*, *pass*, *plane*, *channel* or *pit*). The feature classification is based on table 1, where the signals [1] of the four morphological parameters: *slope* (S), *cross sectional curvature* ($crosc$), *maximum profile convexity* ($maxic$) and *minimum profile convexity* ($minic$), are given by

$$sign(S) = sign(\arctan\sqrt{d^2+e^2}), \qquad (1)$$

[1]We assume that $sign$ is a three-valued function in $\{-1, 0, 1\}$.

$$sign(crosc) = sign(cde - bd^2 - ae^2), \qquad (2)$$

$$sign(maxic) = -sign(a + b - \sqrt{(a-b)^2 - c^2}), \qquad (3)$$

$$sign(minic) = -sign(a + b + \sqrt{(a-b)^2 - c^2}). \qquad (4)$$

The values of a, b, c, d and e are evaluated from the first and second order derivatives of the quadric surface

$$z = ax^2 + by^2 + cxy + dx + ey + f, \qquad (5)$$

usually fitted to a 3x3 data window [4].

Table 1: Feature classification criteria [4].

Feature	$slope$	$crosc$	$maxic$	$minic$
Peak	0	all	+	+
Ridge	0	all	+	0
	+	+	all	all
Pass	0	all	+	-
Plane	0	all	0	0
	+	0	all	all
Channel	0	all	0	-
	+	-	all	all
Pit	0	all	-	-

3 Fuzzy morphology classification

The above procedures are based on a regular elevation grid. Considering that the elevations have some degree of uncertainty which is expressed by fuzzy numbers, we will study the generalization of those procedures to a fuzzy DEM. The first step is to evaluate the first and second order fuzzy derivatives. This is done using interval arithmetic over the fuzzy numbers $\alpha - levels$. Those fuzzy derivatives will be used in equations (1 to 4) to obtain fuzzy morphological parameters using again interval arithmetic [5].

3.1 Signal of a fuzzy number

To use the fuzzy version of the above *slope*, *cross sectional curvature*, *maximum profile curvature* and *minimum profile curvature* it is need now a generalization of the $signal$ function. It is proposed here to adapt the concept of *overtaking between fuzzy numbers* in [3] to decide how much a fuzzy value is negative, positive or zero. This concept is based on the notion of *overtaking between intervals* defined by

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$$\sigma(X,Y) = \begin{cases} 0, & X_u \leq Y_l \\ \frac{X_u - Y_l}{X_u - X_l}, & X_u > Y_l \wedge X_l \leq Y_l \\ 1, & X_l < Y_l \end{cases}, \quad (6)$$

where X and Y are intervals defined by $X = [X_l, X_u]$ and $Y = [Y_l, Y_u]$.

Following, the *overtaking between fuzzy numbers* is

$$\sigma(\tilde{X}, \tilde{Y}) = \int_0^1 \sigma([\tilde{X}]_\alpha, [\tilde{Y}]_\alpha) w(\alpha) d\alpha, \quad (7)$$

where $[\tilde{X}]_\alpha$ and $[\tilde{Y}]_\alpha$ are the $\alpha - levels$ of fuzzy numbers $\tilde{X}$ and $\tilde{Y}$ and $w(\alpha)$ is a weight function $w : [0,1] \rightarrow \Re$ such that

$$\begin{aligned} & w(\alpha) \geq 0 \\ & \alpha < \alpha' \Rightarrow w(\alpha) < w(\alpha') \\ & \int_0^1 w(\alpha) d\alpha = 1. \end{aligned} \quad (8)$$

.

To apply this strategy, it is needed to define a fuzzy zero value. When fuzzy numbers like the fuzzy morphological parameters have very different support widths, ranging from 10 to 0.01, approximately, scaling problems may arise if comparison is needed. To prevent those problems, the zero value will depend on the fuzzy value to be evaluated. The fuzzy zero will have the same membership of the respective fuzzy value but there will be a shift over the real line such that the $\alpha - level$ of maximum membership will be centered in the zero of real line. For triangular fuzzy numbers it will be $\tilde{0} = (X_l - X_m / 0 / X_u - X_m)$ for fuzzy number $\tilde{X} = (X_l / X_m / X_u)$, see figure 1. This type of fuzzy zero value will be called here *twin zero* of $\tilde{X}$.

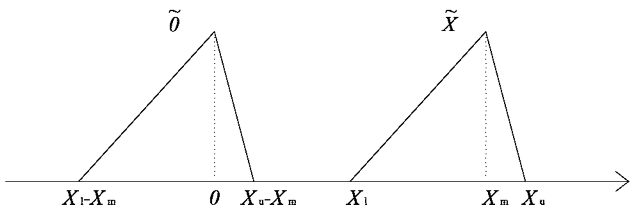

Figure 1: Twin zero of a triangular fuzzy number $\tilde{X}$.

First, to evaluate how much an interval X is positive, it is used a named $positive$ function defined by

$$pos(X) = \begin{cases} 0, & X_l \leq 0_l \\ \frac{X_u - 0_u}{X_u - X_l}, & 0_l < X_l < 0_u \\ 1, & X_l \geq 0_u \end{cases}, \quad (9)$$

where $[0_l, 0_u]$ is the twin zero of $X = [X_l, X_u]$, that is, it has the same width and it is centered in the real zero value. In the same way, $negative$ and $zero$ functions for intervals can be defined by

$$neg(X) = \begin{cases} 0, & X_u \geq 0_u \\ \frac{0_l - X_l}{X_u - X_l}, & 0_l < X_u < 0_u \\ 1, & X_u \leq 0_l \end{cases}, \quad (10)$$

and

$$zer(X) = \begin{cases} 0, & X_l \geq 0_u \\ \frac{0_u - X_l}{X_u - X_l}, & 0_l < X_l < 0_u \\ 1, & X_l = 0_l \\ \frac{X_u - 0_l}{X_u - X_l}, & 0_l < X_u < 0_u \\ 0, & X_u \leq 0_l \end{cases}. \quad (11)$$

Since the width of the zero interval and of X are the same, it is easy to see that[2]: $pos(X) + neg(X) + zer(X) = 1$ and $zer(X) = 1 - pos(X) \dot{\vee} zer(X) = 1 - neg(X)$. To extend those functions to fuzzy numbers it is used the same approach of (7):

$$pos(\tilde{X}) = \int_0^1 pos([\tilde{X}]_\alpha) w(\alpha) d\alpha, \quad (12)$$

where $w(\alpha)$ is the weight function in (8). The same can be done for $negative$ and $zero$ functions.

3.2 Fuzzy morphological features

From table 1, we can evaluate how much a grid cell belongs to a feature class using the above functions to evaluate the signals of the fuzzy morphological parameters and the standard fuzzy conjunction and disjunction defined by min and max operators [6]. This procedure will generate the fuzzy morphological classification, providing a method to quantify the membership of every terrain grid cell to the *fuzzy peak*, *fuzzy ridge*, *fuzzy pass*, *fuzzy plane*, *fuzzy channel* or *fuzzy pit* classes.

4 Examples

Two examples will be presented in this section to illustrate the methods discussed above. In the first case it will be used a digital elevation model of Coimbra region, crossed by Mondego river with narrow valleys and steep slopes (see figure 2). The second example is Kilimanjaro region, where a big volcanic cone lies down on the middle of a plain (see figure 5). In the following figures, all distance units are in meters and angles in degrees.

4.1 Coimbra region

The study region is a 2.0 by 2.1 kilometers rectangle, with elevations ranging from 20 do almost 300 meters. The elevation uncertainty model here is just the root mean square error (RMSE) provided by the map producer. It was introduced a small random gaussian variation added to that RMSE to define the support of symmetrical triangular fuzzy number to express elevation uncertainty. The modal values are taken from the elevations of the crisp DEM. Some results can be seen in figures 3 and 4.

4.2 Kilimanjaro mountain

It was used a rectangle with about 85 by 80 kilometers to enclose Kilimanjaro mountain, with elevations going from 800 to almost 6000 meters. The DEM for this region was taken from SRTM mission, version 4, with original gaps filled by interpolation and with other elevation data available available for that region. To quantify the distribution of DEM elevation uncertainty, about 100 GPS control points were measured around the mountain and in the top. The difference between the elevations of those points and the DEM were calculated

[2]The symbol $\dot{\vee}$ represents xor operator

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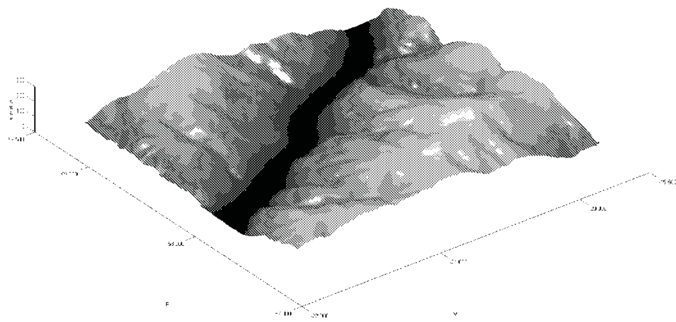

Figure 2: Coimbra digital elevation model.

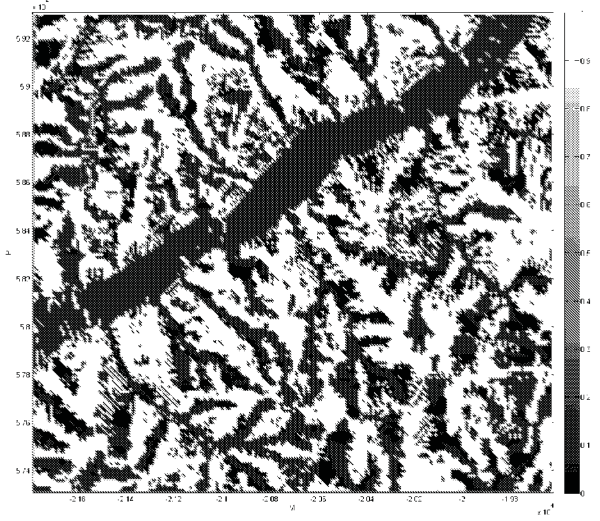

Figure 3: Fuzzy ridge membership for Coimbra study area.

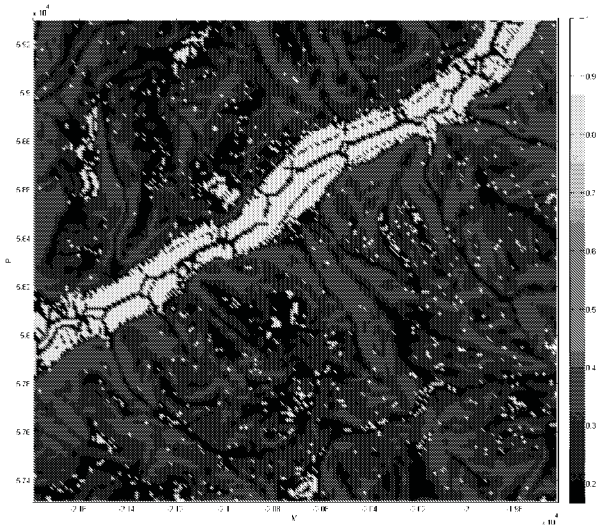

Figure 4: Fuzzy plane membership for Coimbra.

and an error surface were interpolated from those error samples. The error surface in figure 6 was used to estimate the support of symmetric triangular fuzzy numbers that will express the fuzzy elevations. The modal values of those fuzzy numbers are given by the crisp DEM. Some results can be seen in figures 7 and 8. Instead of crisp 0 or 1 membership values, now there are partial membership values to the fuzzy morphological objects, reflecting the uncertainty of the terrain model. Comparing figures 7 and 8, it is possible to detect visually some correlation between the error surface and the fuzzy membership values.

5 Conclusions

This work gave an example that methods applicable to crisp digital elevation models can be generalized to be also applicable to fuzzy DEMs or to similar gridded data expressed by fuzzy numbers. This approach has the advantage to allow the inclusion of uncertainty modeled by Fuzzy Set Theory in geographic analysis. The examples applications presented here are just a first step to develop more robust and advanced terrain classification methods to apply over fuzzy DEMs. Fuzzy values of morphological parameters, like slope and curvatures, have sometimes support intervals too wide. Therefore, those numerical operations are very sensitive to the input values and maybe more robust algorithms are needed here. These methods can also be combined with multi-scale classification methods used in [7, 8, 9, 10] or others approaches like in [11], for example, in order to improve the automated identification of morphological objects.

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Appendix

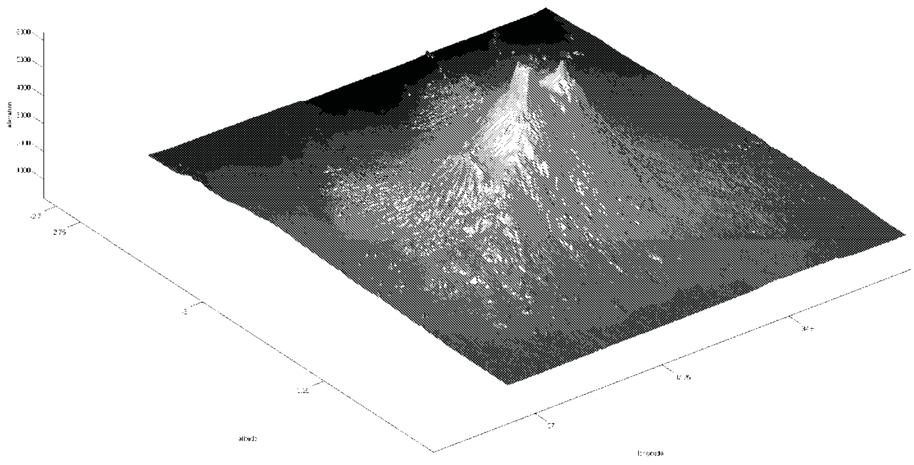

Figure 5: Kilimanjaro digital elevation model.

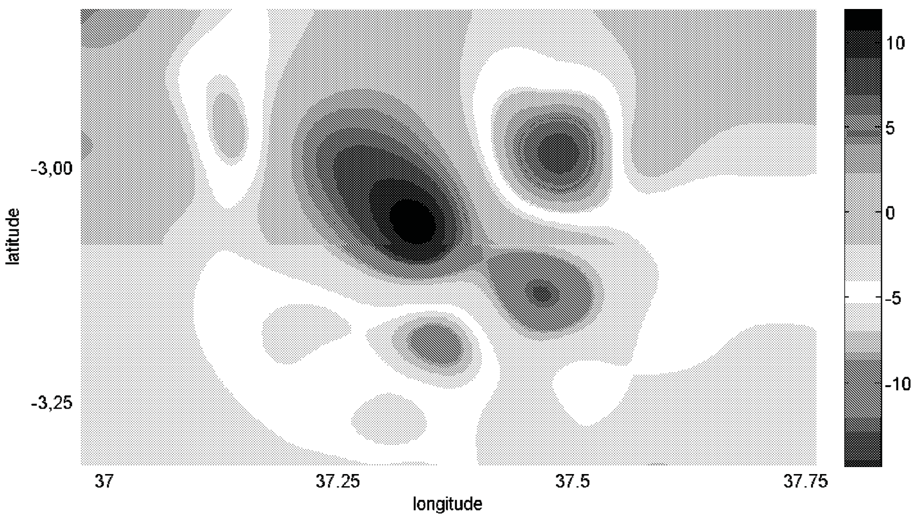

Figure 6: Kilimanjaro uncertainty spatial distribution.

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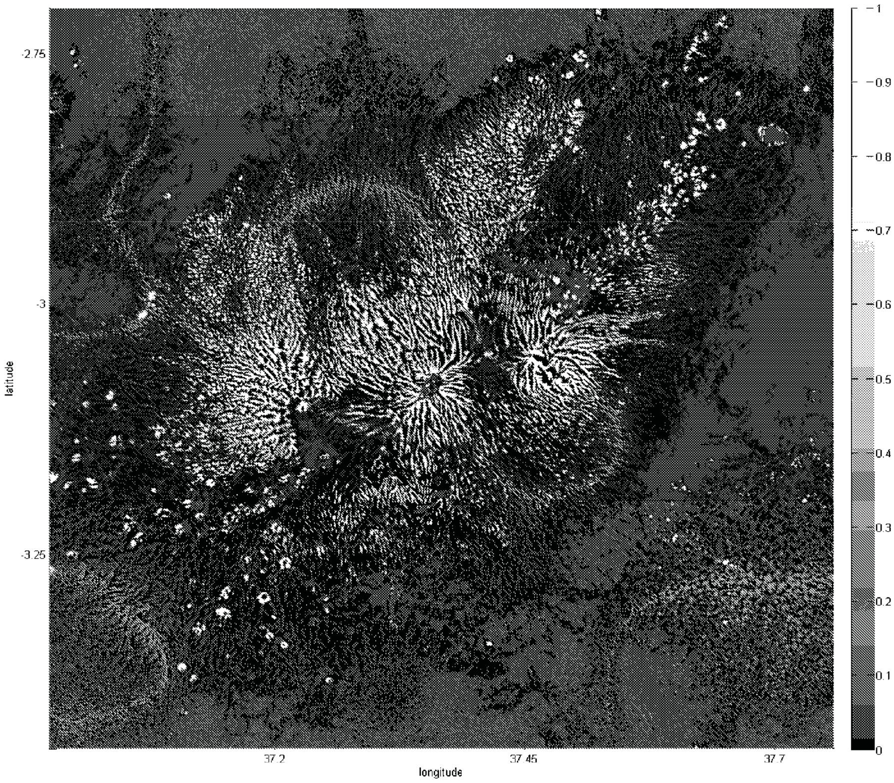

Figure 7: Fuzzy ridge membership for Kilimanjaro region.

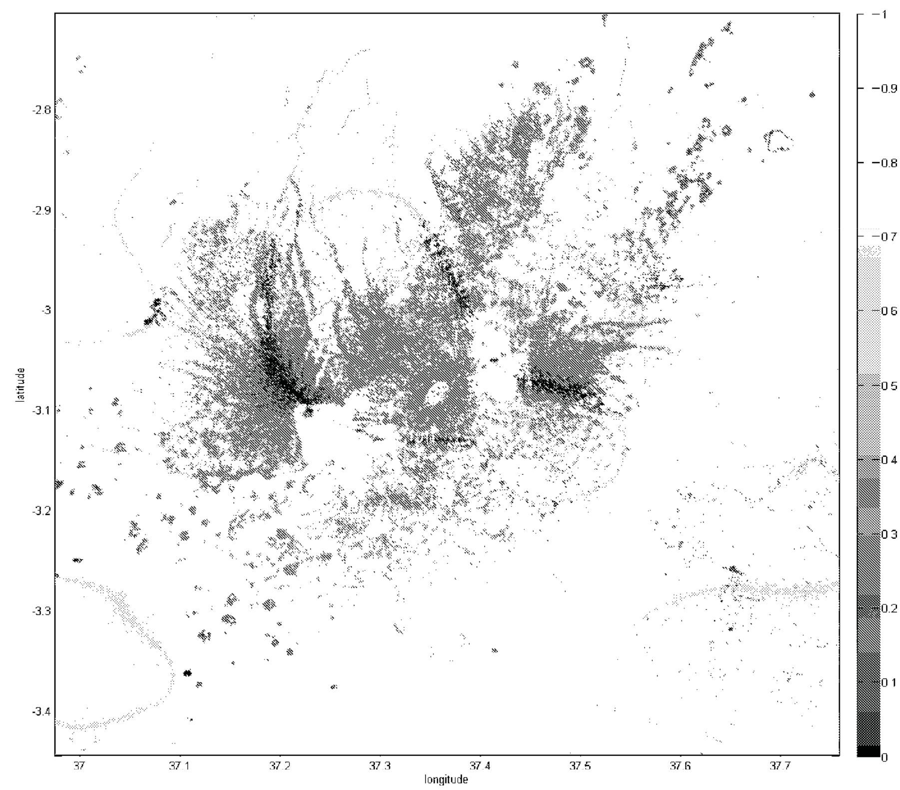

Figure 8: Fuzzy plane membership for Kilimanjaro region.

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References

[1] A. M. Anile, B. Falcidieno, G. Gallo, M. Spagnuolo, and S. Spinello. Modeling uncertain data with fuzzy b-splines. *Fuzzy Sets and Systems*, 113(3):397–410, 2000.

[2] W. Lodwick and J. Santos. Constructing consistent fuzzy surfaces from fuzzy data. *Fuzzy Sets and Systems*, 135(2):259–277, 2003.

[3] W. Lodwick, editor. *Fuzzy Surfaces in GIS and Geographical Analysis: Theory, Analytical Methods, Algorithms and Applications*. CRC Press, New York, 2008.

[4] J. Wood. *The geomorphological characterisation of digital elevation models*. PhD thesis, University of Leicester, UK, 1996.

[5] J. Santos. *Interpolação Difusa na Modelação de Fenómenos Geográficos*. PhD thesis, University of Coimbra, Portugal, 2004.

[6] G. J. Klir and B. Yuan. *Fuzzy Sets and Fuzzy Logic, Theory and Applications*. Prentice Hall Inc., New Jersey, 1995.

[7] P. Fisher, J. Wood, and T. Cheng. Where is helvellyn? fuzziness of multi-scale landscape morphometry. *Transactions of the Institute of British Geographers*, 29:106–128, 2004.

[8] P. Fisher, J. Wood, and T. Cheng. *Fuzzy Modelling with Spatial Information for Geographic Problems*, chapter Fuzziness and ambiguity in multi-scale analysis of landscape morphometry, pages 209–232. Springer, New York, 2005.

[9] P. Fisher, J. Wood, and T. Cheng. Higher order vagueness in geographical information: Empirical geographical population of type n fuzzy sets. *Geoinformatica*, 11:311–330, 2007.

[10] Y. Deng and J. P. Wilson. Multi-scale and multi-criteria mapping of mountain peaks as fuzzy entities. *International Journal of Geographical Information Science*, 22:205–218, 2007.

[11] P. A. Burrough, P. F. M. Van Gaans, and R. Hootsmans. High-resolution landform classification using fuzzy k-means. *Fuzzy Sets and Systems*, 113:37–52, 2000.

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