Fuzzy concept lattice is made by proto-fuzzy concepts.

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Abstract— *An L-fuzzy context is a triple consisting of a set of objects, a set of attributes and an L-fuzzy binary relation between them. An l-cut is a classical context over the same sets with relation as a set of all object attribute pairs, which fuzzy relation assigns truth degree geater or equal than l. Proto-fuzzy concept is a triple made of a set of objects and a set of attributes, which form a concept in some cut of L-fuzzy context and a supremum of all degrees in which cuts this concept exists. Aim of the paper is to show the connection of the structure of proto-fuzzy concepts and fuzzy concept lattice constructed in the way of [1][5]. This connection can help to generate of all fuzzy concepts.*

Keywords— formal concept analysis, fuzzy concept lattice, fuzzy Galois connection

1 Preliminaries

Basic notions of Formal Concept Analysis(FCA) are *formal context* and *formal concept*.

Definition 1 *A formal context $\langle B, A, R\rangle$ consists of a set of objects B, a set of attributes A and a relation R between B and A.*

Definition 2 *Define the mappings $\uparrow$: ${}^B2 \to {}^A2$ and $\downarrow$: ${}^A2 \to {}^B2$. The first assigns to the set $X \subseteq B$ the set of all attributes common to all objects of the set X*

$$\uparrow(X) = \big\{a \in A : \big(\forall o \in X\big)(o, a) \in R\big\}$$

and the second assigns to the set $Y \subseteq A$ the set of all objects common to all attributes of the set Y

$$\downarrow(Y) = \big\{o \in B : \big(\forall a \in Y\big)(o, a) \in R\big\}.$$

Definition 3 *A formal concept of the context $\langle B, A, R\rangle$ is a pair $\langle X, Y\rangle$ such that $X \subseteq B$, $Y \subseteq A$, $\uparrow(X) = Y$ and $\downarrow(Y) = X$.*

Ganter and Wille in [3] showed that the pair of mappings $\big(\uparrow, \downarrow\big)$ is a Galois connection and the composite mappings $\uparrow\downarrow$: ${}^B2 \to {}^B2$ and $\downarrow\uparrow$: ${}^A2 \to {}^A2$ are closure operators. Authors prooved an important theorem in FCA well known as The Basic Theorem On Concept Lattices.

Theorem 1 *(The Basic Theorem on Concept Lattices)The Concept Lattice (lattice of concepts with ordering $\langle X_1, Y_1\rangle \le \langle X_2, Y_2\rangle$ iff $X_1 \subseteq X_2$ iff $Y_1 \supseteq Y_2$) is a complete lattice in which infimum and supremum are given by*

$$\bigwedge_{i\in I}\langle X_i, Y_i\rangle = \Big\langle \bigcap_{i\in I} X_i, \uparrow\downarrow\Big(\bigcup_{i\in I} Y_i\Big)\Big\rangle$$

$$\bigvee_{i\in I}\langle X_i, Y_i\rangle = \Big\langle \downarrow\uparrow\Big(\bigcup_{i\in I} X_i\Big), \bigcap_{i\in I} Y_i\Big\rangle.$$

A complete lattice V is isomorphic to the concept lattice of some context $\langle B, A, R\rangle$ if and only if there are mappings $\beta : B \to V$ and $\alpha : A \to V$, such that $\beta(B)$ is supremum-dense in V and $\alpha(A)$ is infimum-dense in V and $(o, a) \in R$ is equivalent to $\beta(o) \le \alpha(a)$ for all $o \in B$ and $a \in A$. In particular V is isomorphic to the concept lattice of context $\langle V, V, \le\rangle$.

Bělohlávek and Krajči in [1, 2, 5, 6] showed that above mentioned basic notions may be generalized by applying the fuzzy logic.

Everybody knows that reality provides situations where many of attributes are rather fuzzy than crisp. Answer of question “Does the object has the attribute?” is rather somewhere in the middle of false (0) and true (1).

Definition 4 An L-fuzzy formal context *is a triple $\langle B, A, r\rangle$ consists of a set of objects B, a set of attributes A and an L-fuzzy binary relation r, i.e. the L-fuzzy subset of $B \times A$ or mapping from $B \times A$ to L, where L is a complete residuated lattice.*

The class of all L-fuzzy sets in X will be denoted by ${}^XL$. If L is complete then the relation $\subseteq$ (defined by $f \subseteq g$ iff $f(x) \le g(x)$ for all $x \in X$) makes ${}^XL$ into a complete lattice.

Definition 5 *A* complete residuated lattice *is an algebra $\mathbf{L} = \langle L, \wedge, \vee, \otimes, \to, 0, 1\rangle$ where*

(a) *$\langle L, \wedge, \vee, 0, 1\rangle$ is a complete lattice with the least element 0 and the greatest element 1,*

(b) *$\langle L, \otimes, 1\rangle$ is a commutative monoid,*

(c) *$\otimes$ and $\to$ satisfy adjointness, i.e.*

$$a \otimes b \le c \iff a \le b \to c$$

for each $a, b, c \in L$ ($\le$ is the lattice ordering).

Definition 6 (Bělohlávek) *A triple $\langle B, A, r\rangle$ is an L-fuzzy context where $r : B \times A \to L$ and L is a complete residuated lattice. Define mappings $\uparrow$: ${}^BL \to {}^AL$ and $\downarrow$: ${}^AL \to {}^BL$ such that for every $f \in {}^BL$ and $g \in {}^AL$*

$$\uparrow(f)(a) = \bigwedge_{o\in B}\big(f(o) \to r(o, a)\big)$$

$$\downarrow(g)(o) = \bigwedge_{a\in A}\big(g(a) \to r(o, a)\big).$$

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Table 1: Example of L-fuzzy formal context.

	a_1	a_2	a_3	a_4
o_1	0,2	0,8	0,8	1
o_2	1	1	0,8	1
o_3	0,2	0,6	0,4	1
o_4	0,8	0,4	0,2	0
o_5	0,2	0,4	0,4	0,2

Table 2: 1-cut, 0,6-cut, 0,2-cut

	a_1	a_2	a_3	a_4
o_1				•
o_2	•	•		•
o_3				•
o_4				
o_5				

	a_1	a_2	a_3	a_4
o_1		•	•	•
o_2	•	•	•	•
o_3		•		•
o_4	•			
o_5				

	a_1	a_2	a_3	a_4
o_1	•	•	•	•
o_2	•	•	•	•
o_3	•	•	•	•
o_4	•	•	•	
o_5	•	•	•	•

The aim of this paper is to give a new equivalent definition of Belohlavek's mappings, but using so-called protto-fuzzy concepts as basic building units for constructing the fuzzy concepts. In the next section an l-cuts of L-fuzzy contexts will be defined for any truth degree $l \in L$, main properties of their concepts and a relationship of concepts of different cuts will be showed. Finally Proto-fuzzy concepts will be defined. Then an equality of new mappings with Bělohlávek ones will be showed.

2 Cuts of L-fuzzy context

Lets have an example of L-fuzzy context (Table 1). In our example is the lattice of truth degrees is
$\langle\{1;0{,}8;0{,}6;0{,}4;0{,}2;0,\};\leq\rangle$.

Definition 7 *Let $l \in L$ be an arbitrary truth degree. An l-cut of some L-fuzzy set $f \in {}^X L$ is a classical set denoted by*

$$f_l = \{x \in X : f(x) \geq l\}.$$

Definition 8 *An l-cut of the L-fuzzy formal context $\langle B, A, r\rangle$ for $l \in L$ is the classical context $\langle B, A, r_l\rangle$ where $r_l = \{(o, a) \in B \times A : r(o, a) \geq l\}$.*

Some of cuts of our example are in the tables.

Definition 9 *For every truth value $l \in L$ lets define mappings $\uparrow_l$: ${}^B2 \to {}^A2$ and $\downarrow_l$: ${}^A2 \to {}^B2$. For every object or attribute subset $X \subseteq B$ and $Y \subseteq A$ put*

$$\uparrow_l (X) = \{a \in A : (\forall o \in X) r(o, a) \geq l\}$$

$$\downarrow_l (Y) = \{o \in B : (\forall a \in Y) r(o, a) \geq l\}.$$

Lemma 1 *Let $K \subseteq L$ be an arbitrary subset of truth degrees. Then for every set of objects $X \subseteq B$ and attributes $Y \subseteq A$ holds that*

$$\uparrow_{(\bigvee K)} (X) = \bigcap_{l \in K} \uparrow_l (X)$$

$$\downarrow_{(\bigvee K)} (Y) = \bigcap_{l \in K} \downarrow_l (Y).$$

Proof:
$\boxed{\subseteq}$ Cuts of the L-fuzzy context were defined such that for every $l_1, l_2 \in L$ if $l_1 \leq l_2$ then $r_{l_1} \supseteq r_{l_2}$. Hence for every subset of objects X or subset of attributes Y, $\uparrow_{l_1} (X) \supseteq \uparrow_{l_2} (X)$ and $\downarrow_{l_1} (Y) \supseteq \downarrow_{l_2} (Y)$, which for every $l \in K$ implies $\uparrow_{\bigvee K} (X) \subseteq \uparrow_l (X)$ and $\downarrow_{\bigvee K} (Y) \subseteq \downarrow_l (Y)$. And from above we have $\uparrow_{(\bigvee K)} (X) \subseteq \bigcap_{l \in K} \uparrow_l (X)$ and $\downarrow_{(\bigvee K)} (Y) \subseteq \bigcap_{l \in K} \downarrow_l (Y)$.
$\boxed{\supseteq}$ Let a be an arbitrary attribute from $\bigcap_{l \in K} \uparrow_l (X)$. For all $l \in K$ and for every object $o \in X$, $r(o, a) \geq l$. From the properties of supremum is $r(o, a) \geq \bigvee K$ for all objects $o \in X$. It means that $a \in \uparrow_{(\bigvee K)} (X)$. Hence $\bigcap_{l \in K} \uparrow_l (X) \subseteq \uparrow_{(\bigvee K)} (X)$.
The second part can be prooved dually. □

Lemma 2 *For all $l \in L$ the pair $(\uparrow_l, \downarrow_l)$ forms a Galois connection between the power-set lattices ${}^B2$ and ${}^A2$.*

Now lets define the concept on the l-cut for some truth degree $l \in L$.

Definition 10 *Let $\langle B, A, r\rangle$ be the L-fuzzy context. A pair $\langle X, Y\rangle$ is called an l-concept iff*

$$\uparrow_l (X) = Y, \text{ and } \downarrow_l (Y) = X,$$

hence the pair is a concept in a classical context $\langle B, A, r_l\rangle$. The set of all l-concepts will be assigned $\mathrm{C}_l(B, A, r)$, shortly C_l.

2.1 *Relationship of concepts in different cuts*

Lemma 3 *Let $l_1, l_2 \in L$ be an arbitrary truth values, such that $l_2 \leq l_1$. Let $\langle X, Y\rangle \in \mathrm{C}_{l_1}$. Then there exists an interval $\mathcal{I}$ in concept lattice C_{l_2}, such that for every l_2-concept $\langle Z, W\rangle \in \mathcal{I}$ holds that $X \subseteq Z$ and $Y \subseteq W$.*

Proof: From $\langle X, Y\rangle \in \mathrm{C}_{l_1}$ we know that $\uparrow_{l_1} (X) = Y$ and $\downarrow_{l_1} (Y) = X$. So as the greatest element of wanted interval we can use the $\langle \downarrow_{l_2}\uparrow_{l_2} (X), \uparrow_{l_2} (X)\rangle$ and the least one $\langle \downarrow_{l_2} (Y), \uparrow_{l_2}\downarrow_{l_2} (Y)\rangle$. From the fact $r_{l_1} \subseteq r_{l_2}$ we have inclusions

$$Y = \uparrow_{l_1} (X) \subseteq \uparrow_{l_2} (X)$$

$$X = \downarrow_{l_1} (Y) \subseteq \downarrow_{l_2} (Y).$$

From closure property of conclusion of mappings we have

$$\downarrow_{l_2}\uparrow_{l_2} (X) \supseteq X \text{ and } \uparrow_{l_2}\downarrow_{l_2} (Y) \supseteq Y.$$

From $Y = \uparrow_{l_1} (X) \subseteq \uparrow_{l_2} (X)$ we have

$$\downarrow_{l_2} (Y) \supseteq \downarrow_{l_2}\uparrow_{l_2} (X)$$

and from properties of concepts from [3] we know that it is equivalent to

$$\uparrow_{l_2}\downarrow_{l_2} (Y) \subseteq \uparrow_{l_2} (X).$$

□

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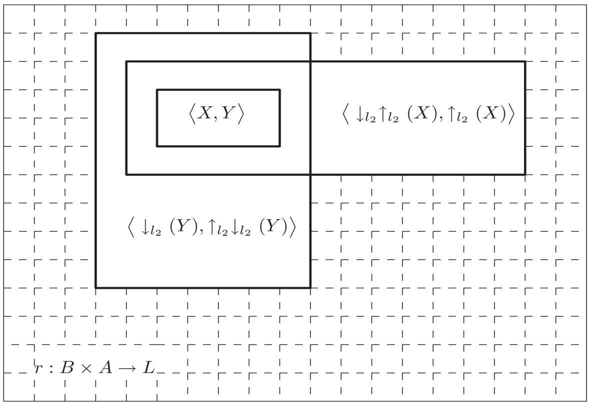

Figure 1: l_1-concept and its l_2-superconcepts

Definition 11 *Define an ordering "$\preceq$" on the set of all l-concepts of all cuts of L-fuzzy context. Lets have two concepts of two different cuts $\langle X, Y \rangle \in \mathrm{C}_{l_1}$ and $\langle Z, W \rangle \in \mathrm{C}_{l_2}$ (because of these pairs are concepts in two different cuts so we can assign them $\langle X, Y \rangle_{l_1}$ and $\langle Z, W \rangle_{l_2}$) then $\langle X, Y \rangle_{l_1} \preceq \langle Z, W \rangle_{l_2}$ iff $X \subseteq Z$ and $Y \subseteq W$ and $l_2 \leq l_1$.*

The lattice in the picture 3 isn't the concept lattice. It is the lattice of all l-concepts for all $l \in L$ and lines is assigning new ordering from definition.

Lemma 4 *Let $l_1, l_2 \in L$ are arbitrary truth values such that $l_2 \leq l_1$. Let $\langle X_1, Y_1 \rangle, \langle X_2, Y_2 \rangle \in \mathrm{C}_{l_1}$, such that $\langle X_1, Y_1 \rangle \preceq \langle X_2, Y_2 \rangle$. Than the geatest and least elements of corresponding intervals of l_2-concepts are ordered same as corresponding l_1-concepts.*

Proof: If $X_1 \subseteq X_2$ from properties of closure operator we have $\downarrow_{l_2}\uparrow_{l_2}(X_1) \subseteq \downarrow_{l_2}\uparrow_{l_2}(X_2)$ And from $Y_2 \subseteq Y_1$ we have $\downarrow_{l_2}(Y_1) \subseteq \downarrow_{l_2}(Y_2)$. □

3 Proto-fuzzy concepts

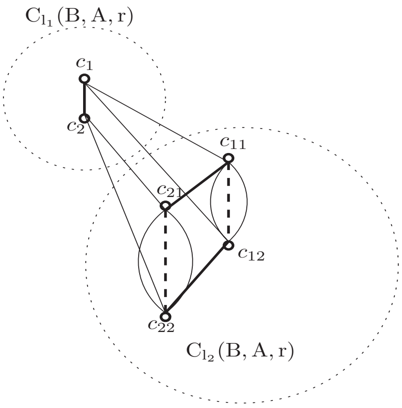

Figure 2: The relationship of concepts of different cuts

As we can see in the figure 3, some of l-concepts are equal, but in different cuts. If we fix some l-concept and look on the set of truth degrees in which cuts the conceot exists, we can see two important properties described in next theorems.

Theorem 2 is sayng that for every concept of some cut, a set of all truth degrees in which cut the concept exists is closed under its supremum.

Theorem 2 *Let $K \subseteq L$ be an arbitrary set of truth degrees, $\langle X, Y \rangle \in \mathrm{C}_l$ for all $l \in K$. Then $\langle X, Y \rangle \in \mathrm{C}_{\bigvee K}$.*

Proof: The lemma 1 implies

$$\uparrow_{\bigvee K}(X) = \bigcap_{l \in K} \uparrow_l (X) = \bigcap_{l \in K} Y = Y,$$

$$\downarrow_{\bigvee K}(Y) = \bigcap_{l \in K} \downarrow_l (Y) = \bigcap_{l \in K} X = X.$$

Hence $\langle X, Y \rangle \in \mathrm{C}_{\bigvee K}$. □

Next theorem 3 is saying that if some concept exist in two different cuts, then exists in every cut between them.

Theorem 3 (Convexity) *Let $l_1, l_2 \in L$ be an arbitrary truth degrees, and let $\langle X, Y \rangle \in \mathrm{C}_{l_1} \cap \mathrm{C}_{l_2}$. Then for all $l \in L$, such that $l_1 \leq l \leq l_2$, $\langle X, Y \rangle \in \mathrm{C}_l$.*

Proof: The lemma 1 implies, that for every set of object X and any two arbitrary truth degrees $k, m \in L$, such that $k \leq m$ holds

$$\uparrow_k (X) \subseteq \uparrow_k (X) \cap \uparrow_m (X) = \uparrow_{\bigvee\{k,m\}} (X) = \uparrow_m (X)$$

and

$$\downarrow_k (Y) \subseteq \downarrow_k (Y) \cap \downarrow_m (Y) = \downarrow_{\bigvee\{k,m\}} (Y) = \downarrow_m (Y).$$

So

$$Y = \uparrow_{l_1} (X) \supseteq \uparrow_l (X) \supseteq \uparrow_{l_2} (X) = Y,$$

$$X = \downarrow_{l_1} (Y) \supseteq \downarrow_l (Y) \supseteq \downarrow_{l_2} (Y) = X.$$

Hence $\uparrow_l (X) = Y$ and $\downarrow_l (Y) = X$, which implies $\langle X, Y \rangle \in \mathrm{C}_l$. □

Definition 12 *Let $\langle X, Y \rangle \in \bigcup_{k \in L} \mathrm{C}_k(B, A, r)$ be the concept of some cut of the fuzzy context $\langle B, A, r \rangle$. Triple $\langle X, Y, l \rangle$ such that $l = \bigvee \{k \in L : \langle X, Y \rangle \in \mathrm{C}_k(B, A, R)\}$ will be called a* proto-fuzzy concept. *The set of all proto-fuzzy concepts will be denoted by* $\mathrm{PFC}(B, A, r)$.

Definition 13 *Define a mapping* $\mathrm{pd} : {}^{B}2 \times {}^{A}2 \to L$ *such that for every set of objects $Z \subseteq B$ and set of attributes $W \subseteq A$ is*

$$\mathrm{pd}(Z, W) = \bigvee \{l \in L : (\exists X \subseteq B)(\exists Y \subseteq A)$$

$$\langle X, Y, l \rangle \in \mathrm{PFC}(B, A, r) Z \subseteq X W \subseteq Y\}.$$

This mapping assigns to every pair of sets of objects and attributes the truth degree of highest proto-fuzzy concept which owns them. Notation pd means proto-degree of input sets.

4 Alternative definition of Bělohlávek's mappings

Lets go back to introduction.

Definition 14 *A triple $\langle B, A, r \rangle$ is L-fuzzy context where $r : B \times A \to L$ and L is the complete residuated lattice. Define mappings $\uparrow : {}^{B}L \to {}^{A}L$ and $\downarrow : {}^{A}L \to {}^{B}L$ such that for every $f \in {}^{B}L$ and $g \in {}^{A}L$*

$$\uparrow (f)(a) = \bigwedge_{o \in B} \big(f(o) \to r(o, a)\big)$$

$$\downarrow (g)(o) = \bigwedge_{a \in A} \big(g(a) \to r(o, a)\big).$$

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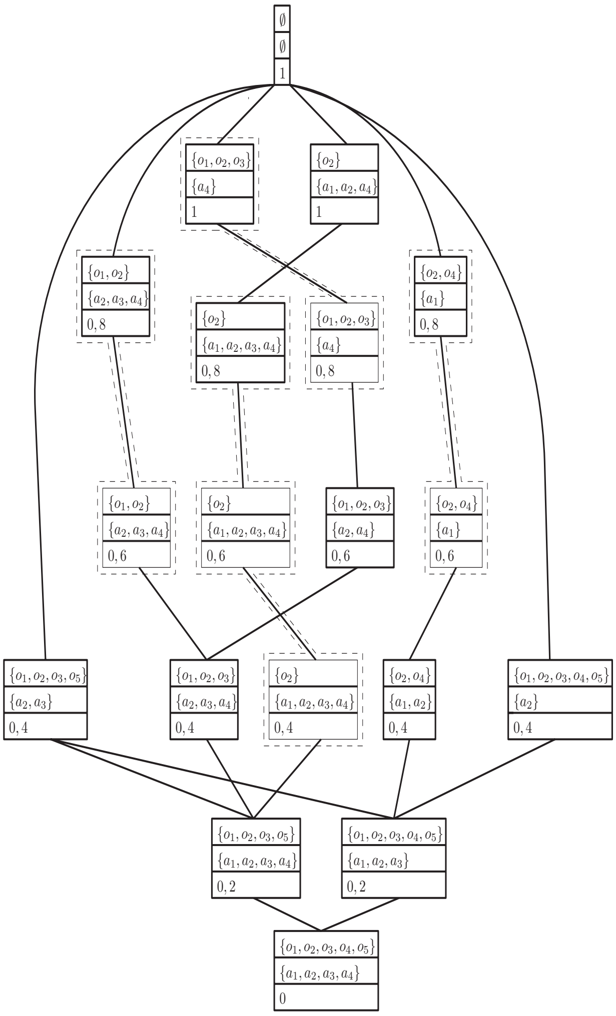

Figure 3: Lattice of all l-concepts of example for all $l \in L$, $\left\langle \cup_{l\in L} \mathrm{C}_l(B, A, r), \preceq \right\rangle$

Lets define new mappings and show that they are equivalent to the mappings above.

Definition 15 *A triple $\langle B, A, r\rangle$ is L-fuzzy context where $r : B \times A \to L$ and L is the complete residuated lattice. Define mappings $\Uparrow : {}^B L \to {}^A L$ and $\Downarrow : {}^A L \to {}^B L$ such that for every $f \in {}^B L$ and $g \in {}^A L$*

$$\Uparrow (f)(a) = \bigwedge_{l\in rng(f)} \big(l \to \mathrm{pd}(f_l, \{a\})\big)$$

$$\Downarrow (g)(o) = \bigwedge_{l\in rng(g)} \big(l \to \mathrm{pd}(\{o\}, g_l)\big).$$

Theorem 4 *For every set Z of objects, an every set W of attributes,*

$$\bigwedge_{(o,a)\in Z\times W} r(o, a) = \mathrm{pd}(Z, W).$$

Proof:

$\boxed{\leq}$ Let $l = \bigwedge_{(o,a)\in Z\times W} r(o, a)$. If $a \in W$ then $(\forall o \in Z) r(o, a) \geq l$, i.e. $a \in \uparrow_l (Z)$, so $W \subseteq \uparrow_l (Z)$. Take $Y = \uparrow_l (Z)$ and $X = \downarrow_l (Y)$ clearly $\langle X, Y\rangle \in \mathrm{C}_l(B, A, r)$. Hence $\langle X, Y, m\rangle \in \mathrm{PFC}(B, A, r)$ where $m = \bigvee\{k \in L : \langle X, Y\rangle \in \mathrm{C}_k(B, A, r)\}$.

Because $W \subseteq \uparrow_l (Y)$ and $Z \subseteq \downarrow_l \uparrow_l (Z) = \downarrow_l (Y) = X$, we have

$$\mathrm{pd}(Z, W) \geq m \geq l = \bigwedge_{(o,a)\in Z\times W} r(o, a).$$

$\boxed{\geq}$ Let $\langle X, Y, l\rangle \in \mathrm{PFC}(B, A, r)$, $Z \subseteq X$ and $W \subseteq Y$. Then $l = \bigvee\{k \in L : \langle X, Y\rangle \in \mathrm{C}_k(B, A, r)\}$ and it follows from Theorem 2 that $\langle X, Y\rangle \in \mathrm{C}_l(B, A, r)$. It means that, for all $o \in X$ and $a \in Y$ is $r(o, a) \geq l$
Hence $\bigwedge_{(o,a)\in Z\times W} r(o, a) \geq \bigwedge_{(o,a)\in X\times Y} \geq l$. It follows that $\bigwedge_{(o,a)\in Z\times W} r(o, a) \geq \bigvee\{l \in L : (\exists X \subseteq B)(\exists Y \subseteq A)\langle X, Y, l\rangle \in \mathrm{PFC}(B, A, r), Z \subseteq X, W \subseteq Y\} = \mathrm{pd}(Z, W)$.
□

Theorem 5 *For above defined mappings holds*

$$\uparrow = \Uparrow \textit{ and } \downarrow = \Downarrow.$$

Proof: Note these three facts:

- Because $\to$ is antitone in the first argument, $f(o) \geq l$ implies
$$f(o) \to r(o, a) \leq l \to r(o, a)$$
- Because $\{o \in B : f(o) \geq l\} \supseteq \{o \in B : f(o) = l\}$, we have
$$\bigwedge_{o\in B:f(o)\geq l} (l \to r(o, a)) \leq \bigwedge_{o\in B:f(o)=l} (l \to r(o, a))$$
- $$\bigcup_{l\in \mathrm{rng}(f)} \{o \in B : f(o) \geq l\} =$$
$$= \bigcup_{l\in \mathrm{rng}(f)} \bigcup_{m\in \mathrm{rng}(f):m\geq l} \{o \in B : f(o) = m\} =$$
$$= \bigcup_{l\in \mathrm{rng}(f)} \{o \in B : f(o) = l\}$$

Using the previous facts we obtain

$$\bigwedge_{l\in \mathrm{rng}(f)} \bigwedge_{o\in B:f(o)=l} (f(o) \to r(o, a)) =$$
$$= \bigwedge_{l\in \mathrm{rng}(f)} \bigwedge_{o\in B:f(o)\geq l} (f(o) \to r(o, a)) \leq$$
$$\leq \bigwedge_{l\in \mathrm{rng}(f)} \bigwedge_{o\in B:f(o)\geq l} (l \to r(o, a)) \leq$$
$$\leq \bigwedge_{l\in \mathrm{rng}(f)} \bigwedge_{o\in B:f(o)=l} (l \to r(o, a)) =$$
$$= \bigwedge_{l\in \mathrm{rng}(f)} \bigwedge_{o\in B:f(o)=l} (f(o) \to r(o, a))$$

It follows that both inequalities are in fact equalities, hence (using Theorem 4)

$$\uparrow (f)(a) = \bigwedge_{o\in B} (f(o) \to r(o, a)) =$$

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$$= \bigwedge_{l\in \mathrm{rng}(f)} \bigwedge_{o\in B: f(o)=l} (f(o) \to r(o,a)) =$$

$$= \bigwedge_{l\in \mathrm{rng}(f)} \bigwedge_{o\in B: f(o)\geq l} (l \to r(o,a)) =$$

$$= \bigwedge_{l\in \mathrm{rng}(f)} (l \to \bigwedge_{o\in B: f(o)\geq l} r(o,a)) =$$

$$= \bigwedge_{l\in \mathrm{rng}(f)} (l \to \bigwedge_{o\in f_l} r(o,a)) =$$

$$= \bigwedge_{l\in \mathrm{rng}(f)} (l \to \mathrm{pd}(f_l, \{a\})) = \Uparrow (f)(a)$$

The second part can be proved dually. □

4.1 *One-sided fuzzy concepts*

In [8] we defined so-called one-sided fuzzy concepts, the pairs consisting of one classical set of objects and fuzzy set of attributes and defined mappings which are creating them.

Definition 16 *For L-context $\langle B, A, r\rangle$ define mappings $\upuparrows$: ${}^{B}2 \to {}^{A}L$ and $\downdownarrows$: ${}^{A}L \to {}^{B}2$. For an arbitrary set of objects $X \in {}^{B}2$ and an L-fuzzy set of attributes $g \in {}^{A}L$ put*

$$\upuparrows (X)(a) = \bigwedge_{o\in X} r(o,a)$$

$$\downdownarrows (g) = \{o \in B : (\forall a \in A) g(a) \leq r(o,a)\}.$$

By the theorem 4 we can write $\upuparrows (X)(a) = \mathrm{pd}(X, \{a\})$. Bělohlávek's theorem ([2]) say:

Theorem 6 *For $X \in {}^{B}2$ and $g \in {}^{A}L$ and mappings $\uparrow, \downarrow, \upuparrows, \downdownarrows$ we have*

$$\upuparrows (X) = \uparrow (X') \text{ and } \downdownarrows (g) = (\downarrow (g))_1$$

and X' means L-fuzzy set of objects corresponding to X, and $(\downarrow (g))_1$ means 1-cut of $\downarrow (g)$.

With the theorem 5 we can in previous theorem change the mappings $\uparrow, \downarrow$ by $\Uparrow, \Downarrow$.

5 Sketch of algorithm for generating all fuzzy concepts

```
Set<Object> B;
Set<Attribute> A;
Set<TrueDegree> L;
LFuzzy binary relation r;
Set<PFConcept> PC(B,A,r)
Set<FConcept> fcs;

public Set<FuzzyConcept> generateAllFC(
        Set<Proto-fuzzy concept> PC(B,A,r)
 ){

// generating all basic L-fuzzy concepts

 for ( PFConcept pc : PC(B,A,r) ){
  Set<Object> objs = pc.getObjects();
  Set<Attribute> atrs = pc.getAttributes();
  True Degree deg = pc.getTruthDegree();
   for ( TruthDegree m : L )
    for ( TruthDegree k : L )
     if ( m== k --> deg ){
      //new L-fuzzy set of Objects
      LFSObjs f = new LFSObjs(
        <obj,m> if obj : objs,
        <obj,0> if obj : B-objs
      );
      //new L-fuzzy set of Attributes
      LFSAtrbs g = new LFSAtrbs(
        <atr,k> if atr : atrs,
        <atr,0> if atr : A-atrs
      );
//every fuzzy concept will remeber of which
//of proto fuzzy concepts was created
      fcs.add(
       new FConcept(
                f ,
                g ,
                new Set<PFConcept>{ pfc } )
             );
     }
  }

// creating connected L-fuzzy concepts

 for ( LFuzzyConcept fc1 : fcs )
  for ( LFConcept fc2 : fcs ){
   Set<PFConcept> pfcs1 = fc1.getPFConcepts();
   Set<PFConcept> pfcs2 = fc2.getPFConcepts();
   boolean ordered = true;
   for ( PFConcept pfc1 : pfcs1 )
    for ( PFConcept pfc2 : pfcs2 )
     if ( !pfc1 <= pfc2 AND !pfc2 <= pfc2 )
      ordered = false;
   if ( ordered )
    fcs.add( createNewFC( fc1 , fc2 ) );
  }

  return fcs;
}

public FConcept createNewFConcept(
   FConcept fc1 ,
   FConcept fc2 )
{
//new L-fuzzy set of Objects
LFSObjs f = new LFSObjs(
 <obj,fc1.getDeg(obj)>
   if fc1.getDeg(obj)>=fc2.getDeg(obj),
 <obj,fc2.getDeg(obj)>
   if fc2.getDeg(obj)>=fc1.getDeg(obj)
);
//new L-fuzzy set of Attributes
LFSAtrbs g = new LFSAtrbs(
 <atr,fc1.getDeg(atr)>
   if fc1.getDeg(atr)>=fc2.getDeg(atr),
 <atr,fc2.getDeg(atr)>
   if fc2.getDeg(atr)>=fc1.getDeg(atr)
);
//Union of sets of proto-fuzzy concepts
Set<PFConcept> pfc =
  unite(
     fc1.getPFConcepts,
```

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```
    fc2.getPFConcepts
      )

return new FConcept( f , g , pfc );
}
```

6 Future work

Our future work will be to finish the sketched algorithm, to proove his good working and apply it.

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