

60 years “A Mathematical Theory of Communication” – Towards a “Fuzzy Information Theory”

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Abstract—When 60 years ago Shannon established “A Mathematical Theory of Communication” nobody could know the consequences for science and technology in the second half of the century. Shannon published his article in two parts in the July and October 1948 editions of the Bell System Technical Journal. However, it is very probable that this article wouldn’t have become famous without the help of Weaver, whose popular text “The Mathematics of communication” re-interpreted Shannon’s work for broader scientific audiences. Weaver’s “preface” and Shannon’s article were published together in the book *The Mathematical Theory of Communication* (1949) that represents the beginning of the then so-called “Information theory”. However, in his “introduction” Weaver went over and above Shannon’s mathematical theory mentioning not only the technical but also the semantic and influential problems of communication. This classification is very similar to the foundations of the Theory of Signs (1938) that was established by Morris. This paper deals with the connectivity between this Information theory and the Theory of Fuzzy sets and systems that appeared in the first half of the 1950s. Then the paper focuses to the non-technical but philosophical aspects of information theory and it advocates a fuzzy information theory that has to be appropriate to cover the concept of information – particularly with regard to the philosophical aspects.

Keywords— communication, fuzziness, information, philosophy, semiotics, signs, signals

1 Introduction

60 years ago, in July 1948, the mathematician and electrical engineer Claude Elwood Shannon (1916-2001) published “A Mathematical Theory of Communication” in the *Bell System Technical Journal* [1] (Fig. 1) and in July of the following year “The Mathematics of Communication” by the mathematician and physicist Warren Weaver (1894-1978) appeared in the *Scientific American* [2] (Fig. 2).

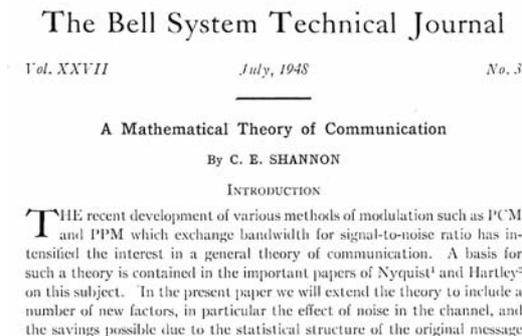


Figure 1: Title of Shannon’s publication in July 1948 [1].

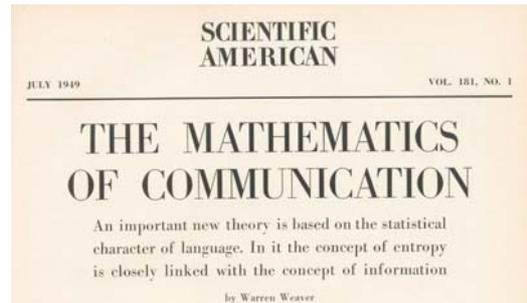


Figure 2: Title of Weaver’s publication in July 1949 [2].

However, other roots of the later so-called “Information Theory” can be found also in the *Cybernetics* of Norbert Wiener (1894-1964) [3], in the work of the Russian mathematician Andrei N. Kolmogorov (1903-1987) [4] and of the British statistician Ronald A. Fisher (1890-1962) [5].

Wiener wrote: “This idea occurred at about the same time to several writers, among them the statistician R. A. Fisher, Dr. Shannon of the Bell Telephone Laboratories, and the author. Fisher’s motive in studying this subject is to be found in classical statistical theory; that of Shannon in the problem of coding information; and that of the author in the problem of noise and message in electrical filters. Let it be remarked parenthetically that some of my speculations in this direction attach themselves to the earlier work of Kolmogorov in Russia, although a considerable part of my work was done before my attention was called to the work of the Russian school.” ([3], p. 18.)¹

Shannon’s “Mathematical Theory of Communication” and Wiener’s *Cybernetics* had appeared simultaneously, but in both cases the publication was delayed by the war. However, Wiener mentioned the fact that, since Shannon was a Bell employee, his research projects were geared toward realizing and marketing his findings as quickly as possible, whereas Wiener, as a college professor, was able to approach his research freely. He had “found the new realm of communication ideas a fertile source of new concepts not only in communication theory, but in the study of the living organism and in many related problems”.² In the manuscript

¹ Wiener cited Kolmogorov’s article [4].

² Wiener in conversation with Bello (technology editor of the journal *Fortune*) on 13 October 1953, Box 4.179, MC22, Wiener Papers, MIT. Quoted from [6], p. 138.

for his book *Invention: The Care and Feeding of Ideas*, which he never finished, Wiener wrote in 1957:

“Once I had alerted myself and the public in general to the statistical element in communication theory, conformation began to flow in from all sides. At the Bell Telephone Laboratories there was, and is, a young mathematical physicist by the name of Claude Shannon. He had already applied mathematical logic to the design of switching systems, and throughout all his work he has shown a love for the discrete problems, the problems with a small number of variable quantities, which come up in switching theory. I am inclined to believe that from the very start, a large part of his ideas in communication theory and its statistical basis were independent of mine, but whether they were or not, each of us appreciated the significance of the work of the other. The whole subject of communication began to assume a new statistical form, both for his sorts of problems and for mine. This is not the point for me to give the genealogy of every single piece of apparatus which this new statistical communication theory has fostered, but I can say that the impact of the work has gone from one end of communication theory to the other, until now there is scarcely a recent communication invention which has not been touched by statistical considerations. Thus, this whole wide spreading branch of science represents a subtle working out of concepts implicit in Gibbs and in the Lebesgue-Borel team, but if I may say so, implicitly implicit, so that until some forty years had passed, no one could have seen the direction in which the earlier thought was bound to lead. This is, in my mind, a key example of a change in intellectual climate, and of the effect it has had both in discovery and in invention.” ([7], p. 22f)

Today Shannon’s name is associated almost unanimously with mathematical information and communication theory – these terms are often considered synonyms – although people had spoken at first of the “Wiener-Shannon Communication Theory”.



Figure 3: C. E. Shannon, W. Weaver and N. Wiener.

However, it was not least Warren Weaver’s article that popularized Shannon’s theory and therefore it became – with little changes and the new title “Recent Contributions to the Mathematical Theory of Communication” – as an “introduction” to Shannon’s text in their joint book-publication *The Mathematical Theory of Communication* [8]. Even more Weaver’s manuscript was not only a brief and understandable sketch of Shannon’s theory and their technical problems – in this text Weaver presented self-

contained and more philosophical considerations on the semantic and influential problems with the concept of information that is in the core of Shannon’s theory and that was called in the following years “Information theory”. We will deal with these philosophical subjects in section 3. Beforehand we will give a few remarks on the genesis of the theory of fuzzy sets and systems, especially against the background of the developments of information theory in the 1950s.

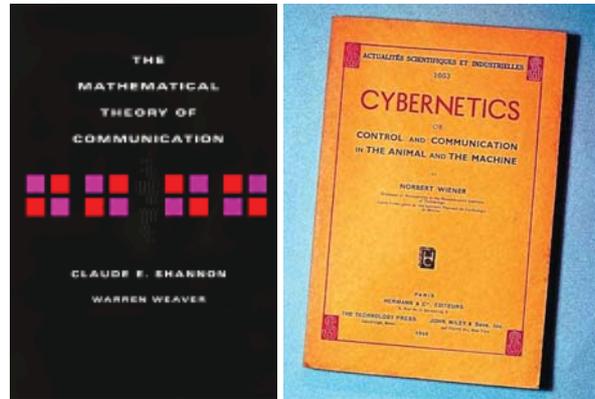


Figure 4: The books of Shannon and Weaver and Wiener.

2 Information theory and the theory of Fuzzy Sets and Systems

2.1 A Historical Sketch - in General

Inspired by Wiener’s *Cybernetics*, Shannon’s and Weaver’s *Mathematical Theory of Communication* and the digital computer era that started during the wartime with the *Electronic Numerical Integrator and Computer* (ENIAC) and the *Electronic Discrete Variable Computer* (EDVAC), both designed by John P. Eckert (1919-1995) and John W. Mauchly (1907-1980), the young emigrant Lotfi A. Zadeh continued his studies in electrical engineering at MIT in Boston after his emigration from Iran into the USA in the 1940s. When he started his doctoral studies at the Columbia University in 1946, he became acquainted with these new milestones in science and technology. Shannon and Wiener delivered lectures in New York about the new theories they had developed during the War. In 1950 Zadeh acted as a moderator at a debate on digital computers at Columbia University, held between Shannon, Edmund Callis Berkeley (1923-1988), the author of the book *Giant Brains or Machines That Think* published in 1949 [9], and Francis Joseph Murray (1911-1996), mathematician and consultant to IBM.

15 years later Zadeh, who was then a professor of electrical engineering at Berkeley, established the new mathematical theory of Fuzzy Sets and Systems [10-12]. Already in 1962 he described the basic necessity of a new scientific tool to handle very large and complex systems in the real world: “we need a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities which are not describable in terms of probability distributions. Indeed, the

need for such mathematics is becoming increasingly apparent even in the realm of inanimate systems, for in most practical cases the a priori data as well as the criteria by which the performance of a man-made system are judged are far from being precisely specified or having accurately-known probability distributions” [13].

The potential of the new techniques of the theory of Fuzzy sets and Fuzzy systems urged Ebrahim H. Mamdani, a professor of electrical engineering at Queen Mary College in London, to attempt the implementation of a fuzzy system under laboratory conditions. He expressed the intention to his doctor student Sedrak Assilian, who designed a fuzzy algorithm to control a small steam engine within a few days. The concepts of so-called linguistic variables and Zadeh’s max-min composition were suitable to establish fuzzy control rules because *input*, *output* and *state* of the steam engine system range over fuzzy sets. Thus, Assilian and Mamdani designed the first real fuzzy application when they controlled the system by a fuzzy rule base system [14]. In 1974, Assilian completed his Ph. D. thesis on this first fuzzy control system [15].

The steam engine heralded the Fuzzy boom that started in the 1980s in Japan and later pervaded the Western hemisphere. Many fuzzy applications, such as domestic appliances, cameras and other devices appeared in the last two decades of the 20th century. Of greater significance, however, was the development of fuzzy process controllers and fuzzy expert systems that served as trailblazers for scientific and technological advancements of fuzzy sets and systems.³

Very little is known about the historical connectivity between the Information theory and the theory of Fuzzy Sets and Systems. In the following paragraph we review some links across the boundaries of these fields.

2.2 In detail: Signals, Noise and Uncertainty

Shannon intended to establish a general theory of communication and a basis for such a theory was already given in two former papers of other Bell-engineers, Harry Nyquist and Ralph V. L. Hartley [17, 18].⁴ 20 years later, Shannon extended this theoretical basis. He included “new factors, in particular the effect of noise in the channel, and the savings possible due to the statistical structure of the original message and due to the nature of the final destination of the information.” ([1], p. 379) To illustrate this mathematical theory, Shannon had drawn a diagram of a general communication model (Fig. 5):

- An information source produces a series of messages that are to be delivered to the receiver side. Transmission can occur via a telegraph or teletypewriter system, in which case it is a series of letters. Transmission can also occur via a telephone or radio system, in which case it is a function of time $f(t)$, or else

it is a function $f(x, y, t)$, such as in a black and white television system, or it consists of complicated functions.

- The transmitter transforms the message in some way so that it can produce signals that it can transmit via the channel. In telegraphy, these are dotdash codes; in telephony, acoustic pressure is converted into an electric current.
- The channel is the medium used for transmission. Channels can be wires, light beams and other options.
- The receiver must perform the opposite operation to that of the transmitter and in this way reconstructs the original message from the transmitted signal.
- The destination is the person or entity that should receive the message.

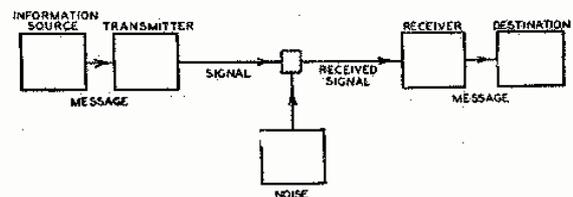


Figure 5: Shannon’s diagram ([1], p. 381).

According to this, Shannon conceived of communication purely as the transmission of messages – completely detached from the meaning of the symbols!

In the second paragraph of his paper Shannon states: “The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point.” ([1], p. 379) Eventually, in chapter 12 that is entitled “Equivocation and Channel Capacity”, he applies his theoretical framework to this problem: “If the channel is noisy it is not in general possible to reconstruct the original message or the transmitted signal with *certainty* by any operation on the received signal E . There are, however, ways of transmitting the information which are optimal in combating noise.” His solution of the problem is as follows: “We consider a communication system and an observer (or auxiliary device) who can see both what is sent and what is recovered (with errors due to noise). This observer notes the errors in the recovered message and transmits data to the receiving point over a “correction channel” to enable the receiver to correct the errors.” ([1], p. 408) Shannon also indicated this situation schematically (Fig. 6).

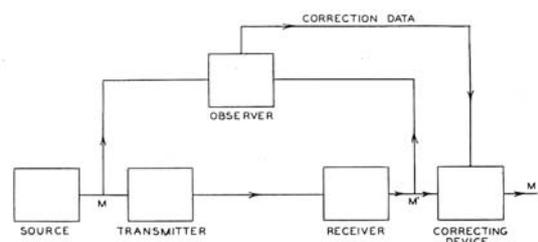


Figure 6: Shannon’s diagram of a communication system with correction device ([1], p. 409).

³ A more detailed presentation of the history of the theory of Fuzzy sets and systems can be found in [16].

⁴ A more detailed presentation of the aspect of these papers is given in my paper for the 2009 annual meeting of NAFIPS [19].

Shannon's introduction of the observer is of great theoretical interest but – of course – the system has to transmit the data on errors and the correction data via communication channels to the observer. When Warren Weaver explained the mathematics of Communication in the first version of his paper [2], he wrote: "If noise is introduced, then the received message contains certain distortions, certain errors. Certain extraneous material, that would certainly lead to increased uncertainty. [...] To get the useful information in the received signal we must subtract the spurious portion. This is accomplished, in the theory, by establishing a quantity known as the "equivocation", meaning the amount of ambiguity introduced by noise." ([2], p. 13) And some paragraphs later he emphasized: "However clever one is with the coding process, it will always be true that after the signal is received there remains some undesirable uncertainty about what the message was". ([2], p. 13)

Let's see how Lotfi Zadeh dealt with this fundamental problem of communication in the 1950s!

In 1949, the year when the book of Shannon and Weaver appeared, Zadeh wrote the Ph. D. thesis on *Frequency Analysis of Variable Networks*, under supervision of Professor John Ralph Ragazzini, and in 1950 he was appointed to an assistant professor. He was enhanced by Wiener's cybernetics and Shannon's information theory. In February 1952, he presented *Some Basic Problems in Communication of Information* at the meeting of the Section of Mathematics and Engineering of the New York Academy of Sciences and in the following, one of these problems will be sketched, that deals with the *recovery process* of transmitted signals. In the proceedings of this meeting, Zadeh wrote: "Let $X=\{x(t)\}$ be a set of signals. An arbitrarily selected member of this set, say $x(t)$, is transmitted through a noisy channel Γ and is received as $y(t)$. As a result of the noise and distortion introduced by Γ , the received signal $y(t)$ is, in general, quite different from $x(t)$. Nevertheless, under certain conditions it is possible to recover $x(t)$ – or rather a time-delayed replica of it – from the received signal $y(t)$." ([20], p. 201)

In this paper, he didn't examine the case where $\{x(t)\}$ is an ensemble; he restricted his view to the problem to recover $x(t)$ from $y(t)$ "irrespective of the statistical character of $\{x(t)\}$ " [5, p. 201]. Corresponding to the relation $y = \Gamma x$ between $x(t)$ and $y(t)$ he represented the recovery process of $y(t)$ from $x(t)$ by $x = \Gamma^{-1}y$, where Γ^{-1} is the inverse of Γ , if it exists, over $\{y(t)\}$.

Zadeh represented signals as ordered pairs of points in a signal space Σ , which is imbedded in a function space with a delta-function basis, and to measure the disparity between $x(t)$ and $y(t)$ he attached a distance function $d(x, y)$ with the usual properties of a metric. Then he considered the special case in which it is possible to achieve a perfect recovery of the transmitted signal $x(t)$ from the received signal $y(t)$. He supposed that " $X = \{x(t)\}$ consist of a finite number of discrete signals $x_1(t), x_2(t), \dots, x_n(t)$, which play the roles of symbols or sequences of symbols. The replicas of all these

signals are assumed to be available at the receiving end of the system. Suppose that a transmitted signal x_k is received as y . To recover the transmitted signal from y , the receiver evaluates the 'distance' between y and all possible transmitted signals x_1, x_2, \dots, x_n , by the use of a suitable distance function $d(x, y)$, and then selects that signal which is 'nearest' to y in terms of this distance function (Fig. 7). In other words, the transmitted signal is taken to be the one that results in the smallest value of $d(x, y)$. This in brief, is the basis of the reception process." ([20], p. 201) By this process the received signal x_k is always 'nearer' – in terms of the distance functional $d(x, y)$ – to the transmitted signal $y(t)$ than to any other possible signal x_i , i.e. $d(x_k, y) < d(x_i, y)$, $i \neq k$, for all k and i .

But at the end of his reflection of this problem Zadeh conceded that "in many practical situations it is inconvenient, or even impossible, to define a quantitative measure, such as a distance function, of the disparity between two signals. In such cases we may use instead the concept of neighborhood, which is basic to the theory of topological spaces" ([20], p. 202). Spaces such as these, Zadeh surmised, could be very interesting with respect to applications in communication engineering. Therefore, this problem of the recovery process of transmitted signals which is a special case of Shannon's *fundamental problem of communication*, the problem "of reproducing at one point either exactly or approximately a message selected at another point", was one of the problems that initiated Zadeh's thoughts about not precisely specified quantitative measures, i. e. or cloudy or fuzzy quantities. – About 15 years later he proposed his new 'concept of neighborhood' which is now basic to the theory of fuzzy systems!

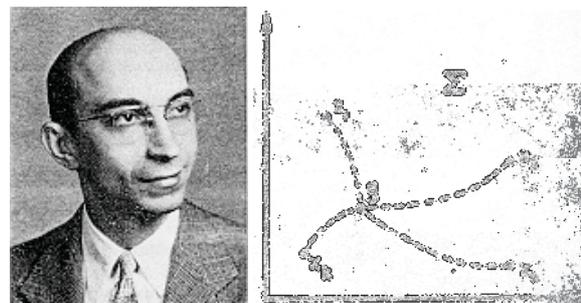


Figure 7: Lotfi A. Zadeh (born 1921) and his illustration: "Recovery of the input signal by means of the comparison between the distances between the received signal y and all possible transmitted signals" [20].

3 Fuzziness and Information Theory

In this section we concentrate our attention from the problem of communication to some philosophical considerations in the first half of the 20th century by Charles William Morris (1901-1976) and Warren Weaver. However, Morris was an engineer by training and he did his Ph. D. in philosophy under George Herbert Mead (1863-1931), the founder of social psychology, and Weaver was a mathematician and physicist. Nevertheless, both scientists wrote philosophical

works on the subjects of signs and their transmission. Moreover, there is a distinct similarity between the two theoretical works that has to be underlined.

3.1 Theory of Signs, semiotics, and Information theory

The study of sign processes, or signification and communication is called “semiotics” since the 1930s. Great work to formalize this field was done by members of the Vienna Circle but also by the philosophers and linguists Charles Sanders Peirce (1839–1914), Ferdinand de Saussure (1857–1913) and Louis Hjelmslev (1899–1965), but here we will limit our considerations to the fundamental work of semiotics by Morris.

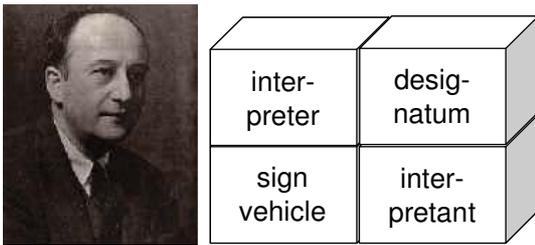


Figure 8: Ch. W. Morris; an illustration of the four components of the semiosis process.

Morris was in touch with some members of the Vienna Circle and he was a member of the Unity of Science movement. When he intended in his 1938 published *Foundations of the Theory of Signs* [21] a science of signs “on a biological basis and specifically with the framework of the science of behavior”, he defined *semiotics* as a *universal* theory of signs and an *interdisciplinary* undertaking. In his view, the mission of semiotics as a science of signs is analyzing language as a social system of signs. Language is a system of signs which produces dispositions to social behavior, and in order to understand the uses and effects of signs we have to understand that and how signs influence social behavior. The process by which a sign-vehicle may function as a sign is called *semiosis*.⁵ In Morris’ foundation there are four components of the semiosis (Fig. 8):

- 1) the *sign vehicle* – this is the object or event which functions as a sign,
- 2) the *designatum* – this is the kind of object or class of objects which the sign designates,
- 3) the *interpretant* – this is the disposition of an interpreter to initiate a response-sequence as a result of perceiving the sign, and
- 4) the *interpreter* – this is the person for whom the sign-vehicle functions as a sign.

He also divided *semiotics* into three interrelated disciplines:

- 1) syntactics – the study of the methods by which signs may be combined to form compound signs,

- 2) semantics – the study of the signification of signs,
- 3) pragmatics – the study of the origins, uses, and effects of signs.

3.2 Weaver’s three levels in Information theory

It seems that Warren Weaver was familiar with Morris’ 10 years old classification of the semiosis when he wrote his paper on Shannon’s Mathematical theory of communication [2]. Already in the third paragraph of his paper he wrote “In communication there seem to be problems at three levels: 1) technical, 2) semantic, and 3) influential. The technical problems are concerned with the accuracy of transference of information from sender to receiver. They are inherent in all forms of communication, whether by sets of discrete symbols (written speech), or by a varying two-dimensional pattern (television). The semantic problems are concerned with the interpretation of meaning by the receiver, as compared with the intended meaning of the sender. This is a very deep and involved situation, even when one deals only with the relatively simple problems of communicating through speech. [...] The problems of influence or effectiveness are concerned with the success with which the meaning conveyed to the receiver leads to the desired conduct on his part. It may seem at the first glance undesirable narrow to imply that the purpose of all communication is to influence the conduct of the receiver. But with any reasonably broad definition of conduct, it is clear that communication either affects conduct or is without any discernible and provable effect at all.” ([2], p. 11)

In the revised version of the paper that was published in [8], Weaver explained the trichotomy of the communication problem in extenso and he divided it into three levels:

- **Level A** contains the purely technical problem involving the exactness with which the symbols can be transmitted,
- **Level B** contains the semantic problem that inquires as to the precision with which the transmitted signal transports the desired meaning,
- **Level C** contains the pragmatic problem pertaining to the effect of the symbol on the destination side: What influence does it exert?

He underscored very clearly the fact that Shannon’s theory did not even touch upon any of the problems contained in levels *B* and *C*, that the concept of information therefore must not be identified with the “meaning” of the symbols: “In fact, two messages, one of which is heavily loaded with meaning and the other of which is pure nonsense, can be exactly equivalent, from the present viewpoint, as regards information.” [8] However, there is plenty of room for fuzziness in the levels *B* and *C*. The interpretation of meaning of signs, e. g. linguistic signs, names, words, is obviously a fuzzy process, and influence or effectiveness the exerted to the receiver’s side is a fuzzy process, too. We will have this fuzziness at the back of our mind following Weaver’s continuing considerations.

⁵ The term *semiosis* was introduced by Ch. S. Peirce to describe “a process that interprets signs as referring to their objects” [22].

3.3 Fuzziness in the diagram of a communication system

It is quite plain: Weaver went over and above Shannon’s theory: “The theory goes further. Though ostensibly applicable only to problems at the technical level, it is helpful and suggestive at the levels of semantics and effectiveness as well.” Weaver stated, that Shannon’s formal diagram of a communication system (Fig. 5) “can, in all likelihood, be extended to include the central issues of meaning and effectiveness. [...] One can imagine, as an addition to the diagram, another box labeled “Semantic Receiver” interposed between the engineering receiver (which changes signals to messages) and the destination. This semantic receiver subjects the message to a second decoding the demand on this one being that it must match the statistical semantic characteristics of the message to the statistical semantic capacities of the totality of receivers, or of that subset of receivers which constitutes the audience one wishes to affect.

Similarly one can imagine another box in the diagram which inserted between the information source and the transmitter, would be labeled “Semantic Noise” (not to be confused with “engineering noise”). This would represent distortions of meaning introduced by the information source, such as a speaker, which are not intentional but nevertheless affect the destination, or listener. And the problem of semantic decoding must take this semantic noise into account. It is also possible to think of a treatment or adjustment of the original message that would make the sum of message meaning plus semantic noise equal to the desired total message meaning at the destination.” ([2], p. 13)

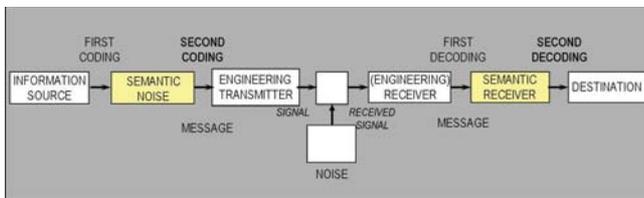


Figure 9: Shannon’s diagram with Weaver’s addition of the two boxes “Semantic Receiver” and “Semantic Noise”.

Fig. 9 shows Weaver’s two additional “fuzzy” boxes in Shannon’s diagram of a communication system. We will interpret the “first coding” between the information source and the “Semantic Noise” as a *fuzzification* and the “second decoding” between the “Semantic Receiver” and the destination as a *defuzzification*.

4 Outlook

In future work we will proceed with this generalization of Weaver’s philosophical considerations on Shannon’s Mathematical theory of communication to a “Fuzzy information theory”.

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