Probabilistic Decision Making with the OWA Operator and its Application in Investment Management

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Abstract*—We develop a new model for decision making under risk environment and under uncertainty. We introduce a new aggregation operator that unifies the probabilities and the ordered weighted averaging (OWA) operator in the same formulation. We call it the probabilistic ordered weighted averaging (POWA) operator. This aggregation operator provides a more complete representation of the decision problem because it is able to consider probabilistic information and the attitudinal character of the decision maker. We study different properties and families of the POWA operator. We also develop an illustrative example of the new approach in a decision making problem about selection of investments.*

Keywords— Decision making; OWA operator; Probabilities; Investment selection.

1 Introduction

Decision making problems [2-8,14,16,18] are very common in the literature. There are different ways and methods for solving the decision process. Usually, the method used for solving the decision problem depends on the available information. For example, when the decision maker has probabilistic information he will solve the problem calculating the expected value for each alternative. Thus, he will be in a problem of decision making under risk environment. However, in other problems the decision maker may not have probabilistic information. Therefore, he must use another approach for solving the problem such as the use of the ordered weighted averaging (OWA) operator [13] that reflects the attitudinal character (degree of optimism) of the decision maker. In this case, we are in a situation of decision making under uncertainty.

The OWA operator is a very useful technique for aggregating the information. It provides a parameterized family of aggregation operators between the maximum and the minimum. In decision making it is very useful for representing the attitudinal character of the decision maker. Since its appearance, the OWA operator has been studied and applied in a wide range of problems [1-4,6-7,9-18].

In [4], they presented a new model that tried to unify the concept of probability with the OWA operator. They introduced the immediate probabilities. This method formulates an aggregation operator that uses probabilities and OWA operators at the same time. Therefore, this approach permits to consider decision making problems under risk environment and under uncertainty at the same time. The concept of immediate probabilities has been further studied in [6-7,16]. It is very useful in some particular situations. However, further analysis (as those shown in the paper) show that it can only unify probabilities and OWAs giving a neutral degree of importance to each case. But the real situation is that sometimes the decision maker believes more on the probabilistic information or on the OWAs. Therefore, the real unification must consider the degree of importance of these to concepts in the aggregation process. The aim of this paper is to present a new formulation that unifies the probabilities and the OWA operators considering the degree of importance of each case in the aggregation process. We call this new approach as the probabilistic ordered weighted averaging (POWA) operator. The main advantage of this approach is that it provides more complete information of the decision process because we can unify the probabilistic information and the attitudinal character according to the available information in the decision process. We study some of its main properties and we see that the OWA operator and the usual probabilistic decision making are particular cases of this approach. We study other particular cases of the POWA operator such as the Hurwicz-POWA, the olympic-POWA and the S-POWA.

We study the applicability of the POWA operator and we see that it is incredibly broad because it can be used in most of the problems where it appears the probability. In this paper, we focus on a decision making problem about selection of investments. We see the usefulness of the POWA operator because we can use situations of risk and uncertainty at the same time.

This paper is organized as follows. In Section 2 we briefly review the OWA operator and the immediate probability. Section 3 presents the POWA operator and Section 4 different particular cases. In Section 5 we present a decision making problem with the POWA operator in investment selection and in Section 6 we give the main conclusions.

2 Preliminaries

2.1 The OWA operator

The OWA operator [13] is an aggregation operator that provides a parameterized family of aggregation operators between the minimum and the maximum. It can be defined as follows.

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Definition 1. An OWA operator of dimension n is a mapping OWA: $R^n \rightarrow R$ that has an associated weighting vector W of dimension n such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then:

$$\text{OWA}(a_1, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j \tag{1}$$

where b_j is the jth largest of the a_i.

Note that different properties can be studied such as the distinction between descending and ascending orders, different measures for characterizing the weighting vector and different families of OWA operators. Note that it is commutative, monotonic, bounded and idempotent. For further reading, refer, e.g., to [1-4,6-7,9-18].

2.2 Immediate probabilities

An immediate probability is a type of probability that includes the attitudinal character of the decision maker by using the OWA operator in the aggregation process. Therefore, it is an attemp to unify the probabilities with the OWA operator. It can be defined as follows.

Definition 2. An IPOWA operator of dimension n is a mapping IPOWA: $R^n \rightarrow R$ that has an associated weighting vector W of dimension n such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula:

$$\text{IPOWA}\ (a_1, \ldots, a_n) = \sum_{j=1}^{n} \hat{v}_j b_j \tag{2}$$

where b_j is the jth largest of the a_i, each argument has associated a probability v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0, 1]$, $\hat{v}_j = (w_j v_j / \sum_{j=1}^{n} w_j v_j)$ and v_j is the probability v_i ordered according to b_j, that is, according to the jth largest of the a_i.

Note that the IPOWA operator is a good approach for unifying probabilities and OWAs in some particular situations. But it is not always useful, especially in situations where we want to give more importance to the OWA operators or to the probabilities. One way to see why this unification does not seem to be a final model is considering other ways of representing $\hat{v}_j$. For example, we could also use $\hat{v}_j = [w_j + v_j / \sum_{j=1}^{n} (w_j + v_j)]$ or other similar approaches.

3 The probabilistic OWA operator

The probabilistic ordered weighted averaging (POWA) operator is an extension of the OWA operator for situations where we find probabilistic information. It can also be seen as a unification between decision making problems under uncertainty (with OWA operators) and under risk (with probabilities). This approach seems to be complete, at least as an initial real unification between OWA operators and probabilities.

However, note that some previous models already considered the possibility of using OWA operators and probabilities in the same formulation. The main model is the concept of immediate probability explained in Section 2 [4,6-7,16]. Although it seems to be a good approach it is not so complete than the POWA because it can unify OWAs and probabilities in the same model but it can not take in consideration the degree of importance of each case in the aggregation process. Other methods that could be considered are the hybrid averaging (HA) operator and the weighted OWA (WOWA) operator. These methods are focused on the weighted average (WA) but it is easy to extend them to probabilities because sometimes the WA is used as a subjective probability. As said before, these an other approaches are useful for some particular situations but they does not seem to be so complete than the POWA because they can unify OWAs with probabilities (or with WAs) but they can not unify them giving different degrees of importance to each case. Note that in future research we will also prove that these models can be seen as a special case of a general POWA operator (or its respective model with WAs) that uses quasi-arithmetic means. Obviously, it is possible to develop more complex models of the IP-OWA, the HA and the WOWA that takes into account the degree of importance of the OWAs and the probabilities (or WAs) in the model but they seem to be artificial and not a natural unification as it will be shown below.

In the following, we are going to analyze the POWA operator. It can be defined as follows.

Definition 3. A POWA operator of dimension n is a mapping POWA: $R^n \rightarrow R$ that has an associated weighting vector W of dimension n such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula:

$$\text{POWA}\ (a_1, \ldots, a_n) = \sum_{j=1}^{n} \hat{v}_j b_j \tag{3}$$

where b_j is the jth largest of the a_i, each argument a_i has an associated probability v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0, 1]$, $\hat{v}_j = \beta w_j + (1-\beta) v_j$ with $\beta \in [0, 1]$ and v_j is the probability v_i ordered according to b_j, that is, according to the jth largest of the a_i.

Note that it is also possible to formulate the POWA operator separating the part that strictly affects the OWA operator and the part that affects the probabilities. This representation is useful to see both models in the same formulation but it does not seem to be as a unique equation that unifies both models.

Definition 4. A POWA operator is a mapping POWA: $R^n \rightarrow R$ of dimension n, if it has an associated weighting vector W, with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$ and a probabilistic vector V, with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0, 1]$, such that:

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$$\text{POWA}(a_1, \ldots, a_n) = \beta \sum_{j=1}^{n} w_j b_j + (1-\beta) \sum_{i=1}^{n} v_i a_i \qquad (4)$$

where b_j is the jth largest of the arguments a_i and $\beta \in [0, 1]$.
In the following, we are going to give a simple example of how to aggregate with the POWA operator. We consider the aggregation with both definitions.

Example 1. Assume the following arguments in an aggregation process: (20, 40, 50, 30). Assume the following weighting vector $W = (0.2, 0.2, 0.2, 0.4)$ and the following probabilistic weighting vector $P = (0.4, 0.2, 0.3, 0.1)$. Note that the probabilistic information has a degree of importance of 60% while the weighting vector W a degree of 40%. If we want to aggregate this information by using the POWA operator, we will get the following. The aggregation can be solved either with (3) or (4). With (3) we calculate the new weighting vector as:

$$\hat{v}_1 = 0.4 \times 0.2 + 0.6 \times 0.3 = 0.26$$
$$\hat{v}_2 = 0.4 \times 0.2 + 0.6 \times 0.2 = 0.2$$
$$\hat{v}_3 = 0.4 \times 0.2 + 0.6 \times 0.1 = 0.14$$
$$\hat{v}_4 = 0.4 \times 0.4 + 0.6 \times 0.4 = 0.4$$

And then, we calculate the aggregation process as follows:

POWA = 0.26 × 50 + 0.2 × 40 + 0.14 × 30 + 0.4 × 20 = 33.2.

With (4), we aggregate as follows:

POWA = 0.4 × (0.2 × 50 + 0.2 × 40 + 0.2 × 30 + 0.4 × 20) + 0.6 × (0.4 × 20 + 0.2 × 40 + 0.3 × 50 + 0.1 × 30) = 33.2.

Obviously, we get the same results with both methods.
From a generalized perspective of the reordering step, it is possible to distinguish between the descending POWA (DPOWA) and the ascending POWA (APOWA) operator by using $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the DPOWA and w^*_{n-j+1} the jth weight of the APOWA operator.
If B is a vector corresponding to the ordered arguments b_j, we shall call this the ordered argument vector and W^T is the transpose of the weighting vector, then, the POWA operator can be expressed as:

$$\text{POWA}(a_1, \ldots, a_n) = W^T B \qquad (5)$$

Note that if the weighting vector is not normalized, i.e., $W = \sum_{j=1}^{n} w_j \neq 1$, then, the POWA operator can be expressed as:

$$\text{POWA}(a_1, \ldots, a_n) = \frac{1}{W} \sum_{j=1}^{n} \hat{v}_j b_j \qquad (6)$$

The POWA is monotonic, commutative, bounded and idempotent. It is monotonic because if $a_i \geq u_i$, for all a_i, then, POWA($a_1, a_2, \ldots, a_n$) ≥ POWA($u_1, u_2 \ldots, u_n$). It is commutative because any permutation of the arguments has the same evaluation. That is, POWA($a_1, a_2, \ldots, a_n$) = POWA($u_1, u_2, \ldots, u_n$), where ($u_1, u_2, \ldots, u_n$) is any permutation of the arguments ($a_1, a_2, \ldots, a_n$). It is bounded because the POWA aggregation is delimitated by the minimum and the maximum. That is, Min$\{a_i\}$ ≤ POWA($a_1, a_2, \ldots, a_n$) ≤ Max$\{a_i\}$. It is idempotent because if $a_i = a$, for all a_i, then, POWA($a_1, a_2, \ldots, a_n$) = a.
Another interesting issue to analyze are the measures for characterizing the weighting vector W. Following a similar methodology as it has been developed for the OWA operator [7,13] we can formulate the attitudinal character, the entropy of dispersion, the divergence of W and the balance operator. Note that these measures affect the weighting vector W but not the probabilities because they are given as some kind of objective information.

4 Families of POWA operators

First of all we are going to consider the two main cases of the POWA operator that are found by analyzing the coefficient β. Basically, if $\beta = 0$, then, we get the probabilistic approach and if $\beta = 1$, the OWA operator.
By choosing a different manifestation of the weighting vector in the POWA operator, we are able to obtain different types of aggregation operators. For example, we can obtain the probabilistic maximum, the probabilistic minimum, the probabilistic average and the probabilistic weighted average.

Remark 1. The probabilistic maximum is found when $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$. The probabilistic minimum is formed when $w_n = 1$ and $w_j = 0$ for all $j \neq n$.

Remark 2. More generally, the step-POWA is formed when $w_k = 1$ and $w_j = 0$ for all $j \neq k$. Note that if $k = 1$, the step-POWA is transformed to the probabilistic maximum, and if $k = n$, the step-POWA becomes the probabilistic minimum operator.

Remark 3. The probabilistic average is obtained when $w_j = 1/n$ for all j, and the probabilistic weighted average is obtained when the ordered position of i is the same as the ordered position of j.

Remark 4. For the median-POWA, if n is odd we assign $w_{(n+1)/2} = 1$ and $w_{j*} = 0$ for all others. If n is even we assign for example, $w_{n/2} = w_{(n/2)+1} = 0.5$ and $w_{j*} = 0$ for all others.

Remark 5. For the weighted median-POWA, we select the argument b_k that has the kth largest argument such that the sum of the weights from 1 to k is equal or higher than 0.5 and the sum of the weights from 1 to $k - 1$ is less than 0.5.

Remark 6. The olympic-POWA is generated when $w_1 = w_n = 0$, and for all others $w_{j*} = 1/(n - 2)$. Note that it is possible

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to develop a general form of the olympic-POWA by considering that $w_j = 0$ for $j = 1, 2, \ldots, k, n, n-1, \ldots, n-k+1$, and for all others $w_{j*} = 1/(n-2k)$, where $k < n/2$. Note that if $k = 1$, then this general form becomes the usual olympic-POWA. If $k = (n-1)/2$, then this general form becomes the median-POWA aggregation. That is, if n is odd, we assign $w_{(n+1)/2} = 1$, and $w_{j*} = 0$ for all other values. If n is even, we assign, for example, $w_{n/2} = w_{(n/2)+1} = 0.5$ and $w_{j*} = 0$ for all other values.

Remark 7. Note that it is also possible to develop the contrary case, that is, the general olympic-POWA operator. In this case, $w_j = (1/2k)$ for $j = 1, 2, \ldots, k, n, n-1, \ldots, n-k+1$, and $w_j = 0$, for all other values, where $k < n/2$. Note that if $k = 1$, then we obtain the contrary case for the median-POWA.

Remark 8. Another interesting family is the S-POWA operator. It can be subdivided into three classes: the "or-like," the "and-like" and the generalized S-POWA operators. The generalized S-POWA operator is obtained if $w_1 = (1/n)(1-(\alpha+\beta)) + \alpha$, $w_n = (1/n)(1-(\alpha+\beta)) + \beta$, and $w_j = (1/n)(1-(\alpha+\beta))$ for $j = 2$ to $n-1$, where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. Note that if $\alpha = 0$, the generalized S-POWA operator becomes the "and-like" S-POWA operator, and if $\beta = 0$, it becomes the "or-like" S-POWA operator.

Remark 9. Another family of aggregation operator that could be used is the centered-POWA operator. We can define a POWA operator as a centered aggregation operator if it is symmetric, strongly decaying and inclusive. Note that these properties have to be accomplished for the weighting vector W of the OWA operator but not necessarily for the weighting vector P of the probabilities. It is symmetric if $w_j = w_{j+n-1}$. It is strongly decaying when $i < j \leq (n+1)/2$ then $w_i < w_j$ and when $i > j \geq (n+1)/2$ then $w_i < w_j$. It is inclusive if $w_j > 0$. Note that it is possible to consider a softening of the second condition by using $w_i \leq w_j$ instead of $w_i < w_j$. We shall refer to this as softly decaying centered-POWA operator. Another particular situation of the centered-POWA operator appears if we remove the third condition. We shall refer to it as a non-inclusive centered-POWA operator.

Remark 10. Another type of aggregation that could be used is the E-Z POWA weights. In this case, we should distinguish between two classes. In the first class, we assign $w_{j*} = (1/q)$ for $j* = 1$ to q and $w_{j*} = 0$ for $j* > q$, and in the second class, we assign $w_{j*} = 0$ for $j* = 1$ to $n-q$ and $w_{j*} = (1/q)$ for $j* = n-q+1$ to n.

Remark 11. A further interesting type is the non-monotonic-POWA operator. It is obtained when at least one of the weights w_j is lower than 0 and $\sum_{j=1}^{n} w_j = 1$. Note that a key aspect of this operator is that it does not always achieve monotonicity. Therefore, strictly speaking, this particular case is not a POWA operator. However, we can see it as a particular family of operators that is not monotonic but nevertheless resembles a POWA operator.

Remark 12. Note that other families of POWA operators could be used following the recent literature about different methods for obtaining OWA weights. Some of these methods are explained in [1,7,9,15,18].

5 Decision making with the POWA operator in investment selection

The POWA operator is applicable in a wide range of situations where it is possible to use probabilistic information and OWA operators. Therefore, we see that the applicability is incredibly broad because all the previous models, theories, etc., that uses the probability can be extended by using the POWA operator. The reason is that all the problems with probabilities deal with uncertainty. Usually, in most of the problems it is assumed a neutral attitudinal character against the probabilistic information but we are still under uncertainty. Thus, sometimes we may prefer to be more or less optimistic against the probabilistic information. Note that this problem can be proved by looking to utility theory. In this theory, people prefer more safety results independently that the expected results (obtained with probabilities) give better results to a more risky alternative. For example, people prefer 100.000 euros with probability 1 instead of 90% probability of obtaining 1.000.000 euros and 10% probability of obtaining 0 euros. If we calculate the expected result (with probabilities) we get that the first alternative gives 100.000 euros and the second one 900.000 euros. However, most of the people will prefer the first alternative because there is no risk involved. This problem has been solved by the economists by using utility theory but as we can see, the use of the POWA operator is also useful for solving this type of problems.

Summarizing some of the main fields where it is possible to apply the POWA operator, we can mention:

- Statistics (especially in probability theory).
- Mathematics
- Economics
- Decision theory
- Engineering
- Physics
- Etc.

In this paper, we focus on an application in decision making about selection of investments. The main reason for using the POWA operator is that we are able to assess the decision making problem considering probabilities and the attitudinal character of the decision maker. Thus, we get a more complete representation of the decision problem.

In the following, we present a numerical example of the new approach in investment selection. We analyze a company that operates in Europe and North America that wants to invest some money in a new market. They consider five alternatives.

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- A_1 = Invest in the Asian market.
- A_2 = Invest in the South American market.
- A_3 = Invest in the African market.
- A_4 = Invest in all three markets.
- A_5 = Do not invest money in any market.

In order to evaluate these investments, the investor has brought together a group of experts. This group considers that the key factor is the economic situation of the world economy for the next period. They consider 5 possible states of nature that could happen in the future: S_1 = Very bad economic situation, S_2 = Bad economic situation, S_3 = Regular economic situation, S_4 = Good economic situation, S_5 = Very good economic situation.

The results of the available investments, depending on the state of nature S_i and the alternative A_k that the decision maker chooses, are shown in Table 1.

Table 1: Available investment alternatives

	S_1	S_2	S_3	S_4	S_5
A_1	20	40	60	70	80
A_2	50	70	90	40	20
A_3	30	40	50	80	60
A_4	40	40	50	60	70
A_5	80	60	50	40	30

In this problem, the experts assume the following weighting vector: W = (0.3, 0.2, 0.2, 0.2, 0.1). They assume that the probability that each state of nature will happen is: P = (0.1, 0.3, 0.3, 0.2, 0.1). Note that the OWA operator has an importance of 40% and the probabilistic information an importance of 60%. With this information, we can calculate how the attitudinal character of the decision maker may affect the probabilistic information. For doing so, we will use (3) to calculate the "attitudinal probabilities". The results are shown in Table 2.

Table 2: Attitudinal probabilities

	$\hat{v}_1$	$\hat{v}_2$	$\hat{v}_3$	$\hat{v}_4$	$\hat{v}_5$
A_1	0.18	0.2	0.26	0.26	0.1
A_2	0.3	0.26	0.14	0.2	0.1
A_3	0.24	0.14	0.26	0.26	0.1
A_4	0.18	0.2	0.26	0.26	0.1
A_5	0.18	0.26	0.26	0.2	0.1

With this information, we can aggregate the expected results for each state of nature in order to make a decision. In Table 3, we present different results obtained by using different types of IOWAAC operators.

Table 3: Aggregated results

	Prob.	OWA	PAM	POWA
A_1	54	60	54	56.4
A_2	63	61	59.4	62.2
A_3	52	57	52	54
A_4	50	55	50.8	52
A_5	52	57	52	54

Note that we can also obtain these results by using (4). Then, we will calculate separately the OWA and the probabilistic approach as shown in Table 4.

Table 4: First aggregation process

	Prob.	AM	OWA
A_1	54	54	60
A_2	63	54	61
A_3	52	52	57
A_4	50	52	55
A_5	52	52	57

After that, we will aggregate both models in the same process considering that the OWA model has a degree of importance of 40% and the probabilistic information 60% as shown in Table 5.

Table 5: Final aggregated results

	Prob.	OWA	PAM	POWA
A_1	54	60	54	56.4
A_2	63	61	59.4	62.2
A_3	52	57	52	54
A_4	50	55	50.8	52
A_5	52	57	52	54

Obviously, we get the same results with both methods. If we establish an ordering of the alternatives, a typical situation if we want to consider more than one alternative, then, we get the results shown in Table 6. Note that the first alternative in each ordering is the optimal choice.

Table 6: Ordering of the investments

	Ordering
Probabilistic	$A_2 \succ A_1 \succ A_3 = A_5 \succ A_4$
OWA	$A_2 \succ A_1 \succ A_3 = A_5 \succ A_4$
PAM	$A_2 \succ A_1 \succ A_3 = A_5 \succ A_4$
POWA	$A_2 \succ A_1 \succ A_3 = A_5 \succ A_4$

As we can see, depending on the aggregation operator used, the ordering of the investments may be different. Therefore, the decision about which investment select may be also different. Note that in this example, we get the same results for all the cases but in other problems or other types of aggregation operators we may find different results and decisions.

6 Conclusions

We have presented an approach for decision making with probabilities and OWA operators. We have developed a new aggregation operator that unifies the probabilities with the OWA operators. We have called it the POWA operator. The main advantage is that it gives a more complete representation of the decision problem because we are taking into account the probabilistic information and the attitudinal character of the decision maker. We have studied this new formulation and we have seen that it includes decision making problems under risk environment and under uncertainty as particular cases. We have studied some of its main properties and different families of POWA operators.

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We have also studied the applicability of the POWA operator and we have seen that there are a lot of potential applications that can be developed because in almost all the studies where it appears the probabilities, it is possible to extend the analysis by using the POWA operator. The reason is that the probabilistic information is always affected by the uncertainty. Therefore, it is always possible to consider the attitudinal character of the decision maker by using the OWA operator. We have focused on a decision making problem about selection of investments. We have seen the usefulness of using the POWA operator because we are able to consider probabilities and OWAs at the same time.

In future research, we expect to develop further extensions to this approach by adding new characteristics in the problem such as the use of order inducing variables, uncertain information (interval numbers, fuzzy numbers, linguistic variables, etc.), generalized and quasi-arithmetic means and distance measures. We will also extend this approach to situations where we use the WA instead of probabilities and further developments that have been initially developed in [7]. We will also consider different applications giving special attention to business decision making problems such as strategic and product management.

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