Visualising Rough Time Intervals in a Two-Dimensional Space

Yi Qiang[1], Katrin Asmussen[1], Matthias Delafontaine[1], Birger Stichelbaut[2], Guy De Tré[3], Philippe De Maeyer[1], Nico Van de Weghe[1]

1 CartoGIS Cluster, Dept. of Geography, Ghent University, Belgium
2 Dept. of Archaeology and Ancient History of Europe, Ghent University, Belgium
3 Dept. of Telecommunications and Information Processing, Ghent University, Belgium

E-mail: {Yi.Qiang, Katrin.Asmussen, Matthias.Delafontaine, Birger.Stichelbaut, Guy.DeTre, Philippe.DeMaeyer, Nico.VandeWeghe}@UGent.be

Abstract *— A lot of disciplines (e.g. archaeology) have to process imprecise temporal information. There are different possibilities to handle this kind of information, amongst them e.g. fuzzy set theory and rough set theory. In this paper, due to its capability in the context of many data acquisition applications, the focus has been set on rough set theory. To illustrate temporal information, an interval is often visualised by means of a one-dimensional segment in a one-dimensional space. An alternative representation of time intervals is called the Triangular Model (TM) by which a time interval is represented by a point in a two-dimensional space. In this paper, rough set theory is applied into TM, which gets extended to the Rough Triangular Model (RTM). In RTM, Rough Time Intervals (RTI) and their mutual relations can be visualised diagrammatically, which offers opportunities to visualise and analyse imprecise temporal information. Aerial photos, taken during World War I, containing imprecise temporal information with archaeological background, are used to illustrate the potentials of the model in processing RTI.*

Keywords — Imprecise temporal information, rough set theory, temporal reasoning, temporal relation, time interval, triangular model.

1 Introduction

For time intervals, the most widely adopted representation is the linear model where time intervals are modelled as finite linear segments in a one-dimensional space. Much research has been carried out on representing and reasoning about time intervals, most of which is based on this linear concept and simple temporal relations [1] [3] [6] [7] [10]. Though Rough Time Intervals (RTI), i.e. intervals starting and/or ending at uncertain time stamp are also frequently used in many disciplines, there is still a shortage of methods and tools to visualise them. An alternative temporal model, the Triangular Model (TM), has been proposed in [11] [12] [2]. This model is based on the W-diagram introduced in [4] [5]. Up till now, TM has remained mainly a theoretical concept. However, it seems to offer a promising design for several applications.

A lot of disciplines (e.g. archaeology, geography, psychology, and philosophy) are faced by the problem of having imprecise temporal information. To handle this information it can be reverted to different approaches, for instance to fuzzy set theory [13]. This theory aims to formalise inherently fuzzy concepts by permitting the gradual assessment of the membership of an element in relation to a set. Another way to deal with imprecise temporal information can be found in rough set theory [8] [9]. This theory introduces a concept of lower and upper approximation and a boundary region, describing a set where elements can or can not be decisively classified into a set X. In contrast to fuzzy set theory, rough set theory is particularly useful in the context of many data acquisition applications. Therefore, in this paper the rough set theory has been chosen to be applied into the TM.

This paper extends TM to RTM in order to visualise and analyse RTIs. First, the basics of TM are introduced in section two. The remainder of this section describes RTIs and their visualisation in the RTM. In section three, we illustrate how to visualise rough time relations of RTIs by means of the RTM. This section is followed by a description of an application of the RTM to incomplete temporal data which is deriving from an archaeological background. Finally, conclusions are drawn and future work is pointed out.

2 The Triangular Model

2.1 Representing time intervals with TM

Time is usually conceptualised as a linear, one-dimensional time line (Fig. 1). In this classical concept, a temporal interval I is visualised by a segment that is bounded by a begin point I^- and end point I^+. In the linear model, the vertical dimension is only used to differentiate multiple overlapping intervals, if used at all. An interval without duration is visualised as a point (zero dimension). The basic concept of TM is the construction of two lines through the extremes of a linear time interval (Fig. 2). For each time interval I, two straight lines (L_1 and L_2) are constructed, with L_1 going through I^-; L_2 going through I^+, and where the angle $\alpha_1 = \alpha_2$. The intersection of L_1 and L_2 is called the interval point I. The position of I in the two-dimensional space completely determines both, the beginning and end point of the interval (Fig. 3).

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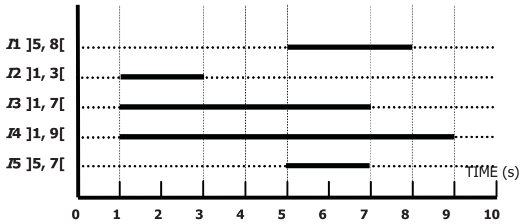

Figure 1: Linear representation of time intervals.

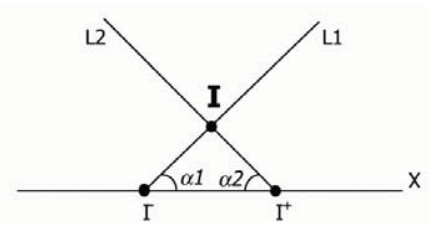

Figure 2: Construction of a simple interval point.

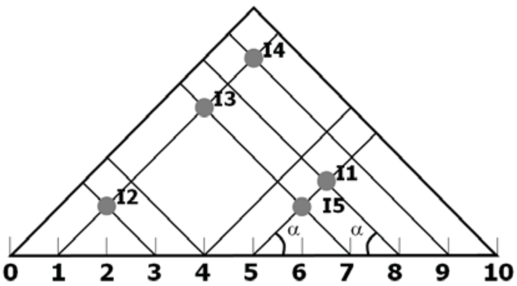

Figure 3: Time intervals in TM.

2.2 *Representing RTIs with TM*

It is often difficult for scientists to obtain precise temporal information about events or processes. Geological periods, for example, are always associated with uncertain starting and ending times. In history and archaeology as well, many events are lacking precise temporal demarcation. For instance, consider the impreciseness coming along with radiocarbon dating.

Scientifically underpinned treatments for imprecise information handling are already existing. For instance, fuzzy set theory [13] aims to formalize inherently fuzzy concepts by permitting the gradual assessment of the membership of an element in relation to a set. It extends conventional (crisp) set theory and handles the concept of partial truth, i.e. truth values between 0 (complete false) and 1 (complete true).

Another way to deal with imprecise information can be found in rough set theory [8] [9]. The main purpose of rough set theory is the induction of approximations of concepts, which are represented by the upper approximation $\overline{B}$ and the lower approximation $\underline{B}$. Within $\underline{B}$, elements can be decisively classified into a set X; outside $\overline{B}$, elements are not members of X. The difference between $\overline{B}$ and $\underline{B}$ forms a boundary region. If the boundary region is nonempty, a set is said to be rough; otherwise the set is crisp. In the boundary region, elements can not be decisively classified as members or not members of X.

Rough set theory is particularly useful in the context of temporal information, due to the nature of data acquisition in many scientific applications. Remote sensing, for instance, is a world-wide applied tool that relies on images taken at discrete time stamps. One can determine the state of a feature on these snapshots, whereas this state is uncertain in between two time stamps. However, we rely on the very natural assumption that this state does not change in between two snapshots which show similar states, i.e. the uncertain parts only remain in between two snapshots showing different states. Hence, we might consider a period of snapshots showing similar states (closed time interval) as a lower approximation for this state and its neighbouring periods of uncertainty (two open time intervals) as boundary region; both then cumulate into the state's upper approximation (open time interval) (Fig. 4). Thus, a feature's state can be considered as a rough set or more specifically a RTI. Next to remote sensing, this idea applies to numerous other fields, such as, soil core sampling, socio-economical census and surveys, and opinion polls.

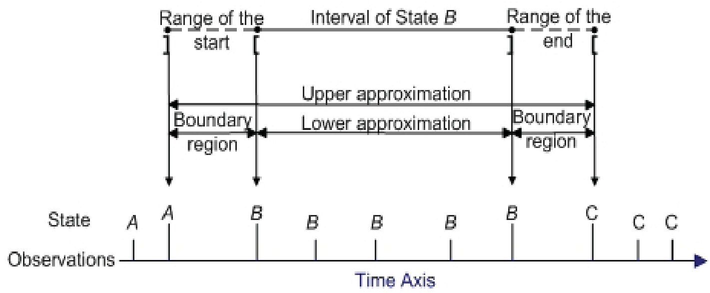

Figure 4: Rough set interval in remote sensing.

In the classical linear model, RTIs are represented as linear segments with an uncertain beginning range and an uncertain end range (Fig. 5). If there is a huge amount of rough intervals, it is quite difficult for humans to abstract information from this representation. Therefore the analyse capacities of the linear model are quite limited. For an advanced temporal analysis based on visualisation an alternative is needed.

In TM, a RTI is represented by a polygon. Four lines are constructed respectively from the earliest/latest beginning and the earliest/ latest end, forming a diamond (Fig. 6). This polygon indicates a zone within which the uncertain interval can be found. We call this representation the Rough Triangular Model (RTM).

In some cases, the beginning and end range have intersections. Take I_5 in Fig. 6 for example, we only know that it starts between 3 and 5, and ends between 3 and 5. In this case, the interval is represented as a triangle incumbent on the x-axis in RTM. In some other cases, intervals are partially rough: they have a precise beginning/end, but an imprecise end/beginning. These semi-rough intervals can be represented by lines (e.g. I_2 in Fig. 6). There are still other possibilities such as semi-open intervals, two-sided open intervals and totally indefinable intervals. In this paper, we are focussing on the

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first type of RTIs which are having a twofold, non-overlapping boundary region.

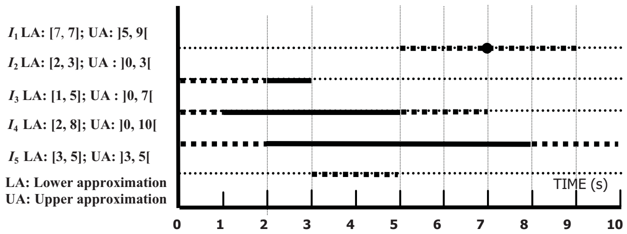

Figure 5: Linear representation of RTIs[1].

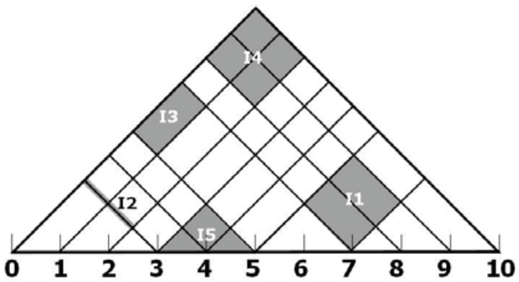

Figure 6: Visualising RTIs in RTM.

3 Rough temporal relations

When time intervals are rough, the relations between them also become rough. RTM provides the possibility of visualising and analysing temporal relations in a two-dimensional space

3.1 Representing fine temporal relations in TM

Based on the basic work of Allen [1], an interval I is represented as a pair (I^- ; I^+) with real numbers I^- and I^+, denoting respectively the beginning and end points of the interval. This means that Allen only deals with simple intervals having a specified duration. Let the beginning I^- and end point I^+ of two simple intervals have the following three possible relations: smaller than (<), equal (=) and larger than (>). Then, thirteen possible fine relationships between two intervals can be defined (see Tab. 1).

Table 1: Thirteen Allen relations [1].

Allen's temporal relations			
I_1 *equal* I_2	if $I_1^- = I_2^-$	^ $I_1^+ = I_2^+$	
I_1 *starts* I_2	if $I_1^- = I_2^-$	^ $I_1^+ < I_2^+$	
I_1 *started-by* I_2	if $I_1^- = I_2^-$	^ $I_2^+ < I_1^+$	
I_1 *finishes* I_2	if $I_1^+ = I_2^+$	^ $I_1^- > I_2^-$	
I_1 *finished-by* I_2	if $I_1^+ = I_2^+$	^ $I_2^- > I_1^-$	
I_1 *meets* I_2	if $I_1^+ = I_2^-$		
I_1 *met-by* I_2	if $I_2^+ = I_1^-$		
I_1 *overlaps* I_2	if $I_2^- > I_1^-$	^ $I_1^+ < I_2^+$	^ $I_1^+ > I_2^-$
I_1 *overlapped-by* I_2	if $I_1^- > I_2^-$	^ $I_1^- < I_2^+$	^ $I_2^+ < I_1^+$
I_1 *during* I_2	if $I_1^- > I_2^-$	^ $I_1^+ < I_2^+$	
I_1 *contains* I_2	if $I_2^- > I_1^-$	^ $I_2^+ < I_1^+$	
I_1 *before* I_2	if $I_1^+ < I_2^-$		
I_1 *after* I_2	if $I_2^+ < I_1^-$		

[1] In this paper, we use ISO standard notation to distinguish open intervals and close intervals. In this notation,]a, b[denotes open interval, [a, b] denotes close interval and]a, b] denotes left-open but right-close intervals. Since Allen's intervals are open, we use]a,b[here to denotes Allen's intervals.

Using TM [12], these relations can be visualised. Each relation thereby corresponds to a specific Fine Relation Zone (FRZ) within TM. Given a study period beginning at 0 and ending at 100, all examined intervals are located within the isosceles triangular of I]0, 100[. To obtain the best visualisation, the reference interval I_2]33,66[is chosen to be located in the centre of TM. As shown in Fig. 7b several intervals (I_{1a}, I_{1b}, I_{1c}, I_2) may exist *before* interval I_2]33,66[. All possible intervals for which $I_1^+ < I_2^-$ applies are generalised, with respect to interval I_2, into the FRZ *before,* displayed by the black triangle in Fig. 7c. Note that as Allen worked with open intervals, also the interval zone corresponding to I]0,33[is open. The right boundary of FRZ *before* represents all intervals for which applies $I_1^+ = I_2^-$. Therefore, the intervals have their end point at 33, resulting in the *meets* relationship.

Comparing the visualisations of TM with the linear model, both visualise the same fine interval relations, as displayed in Fig. 7a and Fig. 7c. The benefit of TM in visualising time relations gets spontaneously definite. The time intervals in Fig. 7c is faster to capture than the one in Fig. 7a. An overview of positions and names of the thirteen FRZs is given in Fig. 8.

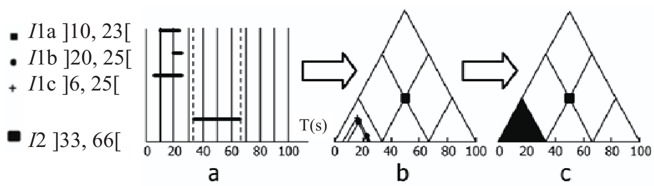

Figure 7: Visualisation of fine intervals by means of the linear model a) and TM b). FRZ *before* within TM c).

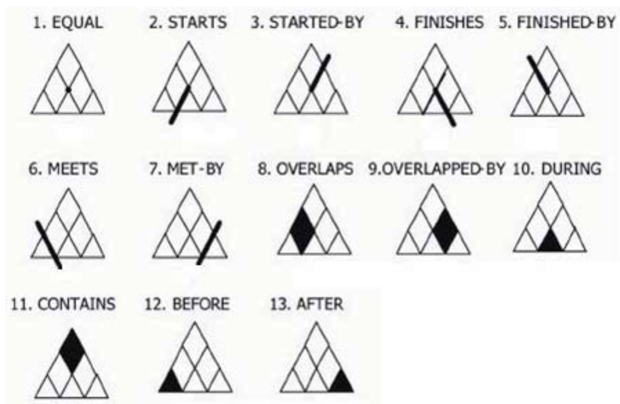

Figure 8: Thirteen FRZs in TM.

3.2 Relation zones in RTM

While TM represents fine intervals and fine interval relations, RTM represents rough intervals and its relation zones are Rough Relation Zones (RRZs).

To transform FRZs into RRZs, all point and line zones of TM (*equal, starts, started-by, finish, finished-by, meets,* and *met-*

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by) are expanded according to the duration of the boundary regions of the reference interval I_2. Hence, two polygon zones arise from the intersections of the expanded line zones *starts* and *meets* as well as from the intersections of the expanded line zones *finishes* and *met-by* (Fig. 9). Note that the absolute position of these zones may change according to the used data, while the relative position always remains.

Transforming the TM to RTM, fifteen relation zones are generated; nine of these are expanded zones (Fig. 9 and Tab. 2). The names of the expanded zones are based on the names of FRZs, but preceded by 'maybe'. Therefore these expanded relation zones are named *maybe* zones.

In TM, only one specific fine relation is possible within a FRZ (Tab. 2). Different from that, in RTM *maybe* zones represent zones where several fine relations are possible (Tab. 2). This is caused by the fact that a RRZ consists of parts of corresponding neighbouring FRZs in TM. The dashed lines in Fig. 9 represent the borders of original FRZs. In RTM, the positions of these dashed lines are uncertain and can be anywhere within the corresponding *maybe* zone. This is different from fuzzy set interval where the dashed lines have gradually-changing probability of appearance. Thus, we only use flat colour (white) to mark the interior of *maybe* zones. Note that each maybe zone contains the relations of its neighbouring zones in RTM (Fig. 9).

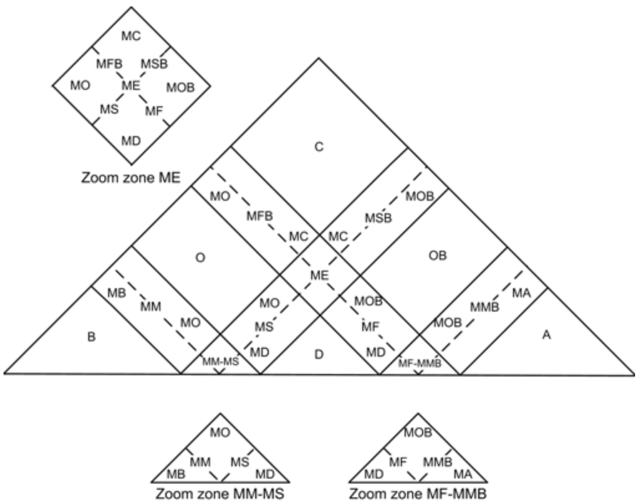

Figure 9: RRZs in RTM.

Table 2: Thirteen FRZs in TM and fifteen RRZs in RTM

FRZ		RRZ		
Name	**Abbr.**	**Name**	**possible relations**	**Abbr.**
equal	E	*maybe equal*	*contains, started-by, overlapped-by, finishes, during, starts, overlaps, finished-by, equal*	ME
starts	S	*maybe starts*	*starts, overlaps, during*	MS
started-by	SB	*maybe started-by*	*started-by, overlapped-by, contains*	MSB
finishes	F	*maybe finishes*	*finishes, during, overlapped-by*	MF
finished-by	FB	*maybe finished-by*	*overlapps, finished-by, contains*	MFB
meets	M	*maybe meets*	*meets, before, overlaps*	MM
met-by	MB	*maybe met-by*	*met-by, overlapped-by, after*	MMB
overlaps	O	*overlaps*	*overlaps*	O
overlapped-by	OB	*overlapped-by*	*overlapped-by*	OB
during	D	*during*	*during*	D
contains	C	*contains*	*contains*	C
before	B	*before*	*before*	B
after	A	*after*	*after*	A
/	/	*maybe meets or maybe starts*	*before, meet, overlaps, during, starts*	MM-MS
/	/	*maybe finishes or maybe met-by*	*during, finishes, overlapped-by, met-by, after*	MF-MMB

Unlike with FRZs, the border lines of RRZs do not make up separate relation zones but belong to one of their neighbouring RRZs. Due to their definition, the boundary regions in the beginning and the end of RTIs are open intervals (Fig. 4). The corresponding assignments are illustrated in Fig. 10, where the arrows are pointing into the zone to which the respective border line is assigned.

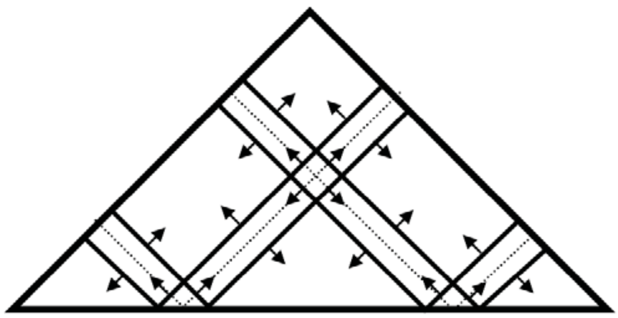

Figure 10: Assignment of the borders of RRZs.

3.3 *Applying RTM*

In order to evaluate whether RTM is helpful for the analysis of imprecise temporal information; we tested RTM with data deriving from an archaeological context. During World War I, aerial photos covering the Belgian-German front line in West-Flanders (Belgium) have been taken. From these aerial photos, we can observe whether a feature such as a fire trench, a gun position or a barrack, was not yet present, was present or was destroyed. For each feature, several photos with according date of acquisition exist. Hence, if we can find (1)

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the last photo where the feature is not yet found, (2) the first photo where the feature is found, (3) the last photo where the feature is found, and (4) the first photo where the feature is destroyed, we can derive a RTI representing the feature's uncertain lifetime (Fig. 4).

In RTM, fifteen RRZs can be created for one feature (Fig. 9). Lots of similar intervals will yield overlapped RRZs. If different grey shades are added to denote the number of overlaps in the concerning area, there will appear some patterns which reflect characteristics of the underlying data.

Fig. 11 and Fig. 12 visualise the overlaps of *maybe equal* zones of gun positions and breastworks respectively. More overlapped areas are marked with dark grey, and vice versa. Comparing these two visualisations, we can have a direct overview about how the two types of features are temporally distributed. As in the model most polygons of gun positions are distributed right of the breastworks zones, comparing Fig. 11 with Fig. 12, we may observe that gun positions exist relatively later than breastworks. It also can be observed that breastworks generally exist longer than gun positions because most of the breastworks zones are higher on the *y*-axis than the zones of the gun positions.

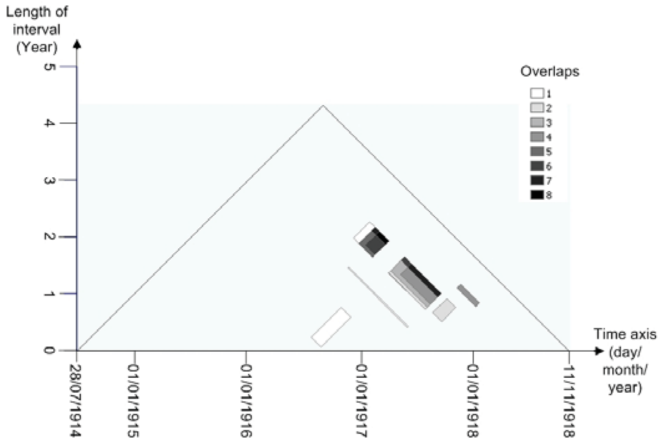

Figure 11: Overlaps of maybe equal zones of gun positions.

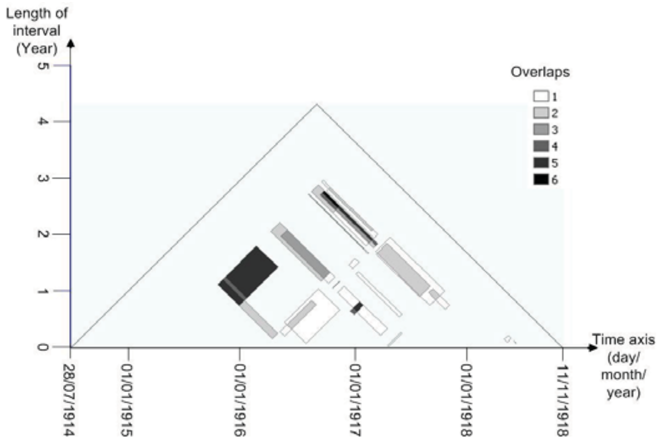

Figure 12: Overlaps of maybe equal zones of breastworks.

If we select *contain* interval zones of gun positions (Fig. 13), the display of the model can be divided into several zones according to the colour grade. If an interval point is in the dark zone, it contains intervals of most features. In other words, in the dark zone, all intervals contain most features lifetime intervals. In natural language, we could say: if an interval is in a dark zone, it witnesses the lifetime of most features. This approach can be applied to other RRZs and even to combinations of relation zones.

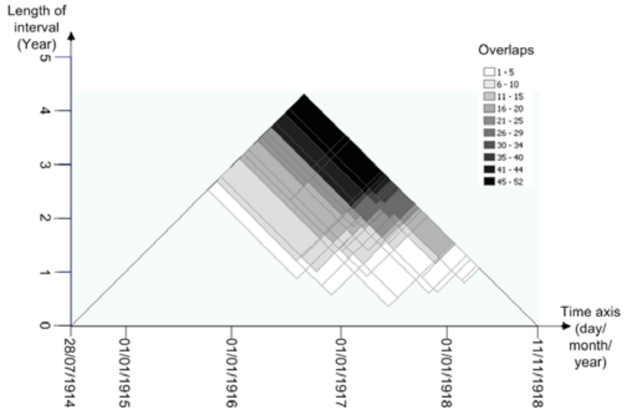

Figure 13: The overlaps of the *contain* zone of gun positions.

4 Conclusions and future work

Since a lot of disciplines (e.g. archaeology, geography, psychology, and philosophy) are faced by the problem of having imprecise temporal information we extended TM into RTM in order to visualise RTIs and temporal relations between them. We do not intend to create a new or extend an existing temporal calculus or temporal logic. Our approach takes special care for the intuitive and visualization aspects. In RTM, RTIs are represented by simple geometries (e.g. lines and polygons) in a two-dimensional space. Compared to the classical linear model, which has limited analytical capacities, RTM gives people a direct overview of the temporal distribution of RTIs. When handling a huge amount of rough intervals, RTM provides a compact visualisation pattern, which helps in further exploratory analysis. Furthermore, the temporal relations between RTIs can be visualised as zones, i.e. RRZs, in the two-dimensional space, which gives potentials in visual queries of rough temporal relations.

In this paper, we described the basics of RTM and RRZs. However, further research needs to be done.

First, as explained in 2.2; also other types of RTIs should be considered, e.g. partially rough intervals, one-sided open intervals and intervals with overlapped begin and end boundary regions. This would imply a wide modification of the RRZs. This major change in the division of the RRZ consequently has to be followed by an adaptation of visualisation and interpretation. Whether these visualisation and analyses are delivering feasible results has to be tested in further research.

Second, the idea of conceptual neighbourhood has proved its importance in qualitative reasoning about time and space. When temporal relations become rough, there are more possibilities that one relation continuously changes to another relation without passing through other relations. Thus, the conceptual neighbourhood diagram of rough temporal relations will change accordingly. Also, the practical use of conceptual neighbourhood of temporal relations of rough intervals has to be studied in detail.

Third, so far we dedicated the borders of the RRZs of the RTM to one of the RRZs. But following the example of the TM, these lines could be relation zones by themselves. That would mean that additionally to the fifteen RRZ we would have nineteen line zones and probably six point zones. In total RTM would be build up out of forty zones. The additional line zones would include, for example, line zones of TM like *starts*, *started-by*, *finishes*, *finished-by, meets* and *met-by*. But also new line zones would be created like the line zones which are located between the zones *maybe starts or maybe meets* and *maybe meets.* Within this line zone a wide range of relations are possible. Whether this great number of RRZ provides useful visualisations and if the interpretation could be aided by a computer still needs to be determined. Also whether this interpretation would deliver helpful information still needs to be investigated.

Finally, this model can be more useful if it is implemented as an interactive tool. More flexible and interactive visualisation tools can help to better analyse complex temporal data.

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References

[1] J.F. Allen. Maintaining Knowledge about Temporal Intervals. *Communications of the ACM,* 26:832-843, 1983.

[2] G. De Tré, et al. Towards a flexible Visualisation Tool for dealing with Temporal Data. *In proc. of "FQAS" LN AI*, 4027: 109-120, 2006.

[3] C. Freksa. Temporal Reasoning based on Semi-Intervals. *Artificial Intelligence*, 54:199-227, 1992.

[4] Z. Kulpa. Diagrammatic Representation for a Space of Intervals. *Machine Graphics & Vision* 6: 5-24, 1997.

[5] Z. Kulpa. Diagrammatic Representation of Interval Space in Proving Theorems about Interval Relations. *Reliable Computing* 3:209-217, 1997.

[6] V. Kumar, R. et al. Allen. Metadata Visualization for Digital Libraries: Interactive Timeline Editing and Review. *In proc. of "ACM Digital Libraries"*, Pittsburgh, USA, 1998.

[7] J.D. Mackinlay. The Perspective Wall: Detail and Context Smoothly Integrated. *In proc. "ACM CHI'91"*, New York, 1991.

[8] Z. Pawlak. Rough Sets. *Int. J. of Information and Computer Science*, 11:341-356, 1982.

[9] Z. Pawlak. Rough Sets: Theoretical Aspects of Reasoning About Data. *Kluwer Academic Publishers,* Dordrecht, The Netherlands, 1991.

[10] S. Schockaert, M.D. Cock, and E.E. Kerre. Fuzzifying Allen's Temporal Interval Relations. *IEEE Transactions on Fuzzy Systems*, 16: 517-533, 2008.

[11] N. Van de Weghe, et al. The Triangular Model as an Instrument for Visualising and Analysing Residuality. *J. of Archaeological Science* 34: 649-655, 2007.

[12] N. Van de Weghe. Development of a Conceptual Data Model for digital spatio temporal Geographical Information. *In proc. "ER"*, *LNCS*, Tampere, Finland, 2002.

[13] L.A. Zadeh. Fuzzy Sets. *Information and Control* 8: 338-353, 1965.

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