Fuzzy Optimization in Vehicle Routing Problems

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Abstract*—This paper focuses on analyzing practical solution approaches for Vehicle Routing Problems (VRP) with uncertain information. Several variants of the basic problem and fuzzy optimization problem formulations are described. The fuzzy VRP is obtained when some of the elements in the formulation are fuzzy. The main versions of Fuzzy VRP that have appeared in the literature are reviewed and the standard approaches for solving the corresponding models are analyzed.*

Keywords—Vehicle Routing Problem, Fuzzy Optimization, Fuzzy Vehicle Routing Problem, VRP with Time Windows.

1 Introduction

Route planning problems are among the activities that have the highest impact in logistical planning, transport and distribution because of their effects on efficiency in resource management, service levels, and client satisfaction. Route distribution planning problems, also known as Vehicle Routing Problems (VRP), have been thoroughly studied in a variety of areas, such as Operations Research, Artificial Intelligence, etc. The standard VRP was originally introduced by Dantzig and Ramser (1959), and is NP-hard, which is a complex combinatorial optimization problem [5]. Several variants of the basic problem have been put forward and strong formulations have been proposed. Most of these problems can be modeled as linear programming problems. The most common solution techniques are exact methods that guarantee finding an optimal solution if it exists. These approaches have also been applied together with numerous heuristics solution techniques developed with enough flexibility in optimization systems and can be adapted to various practical contexts.

Given that the complex, flexible, and dynamic nature of real logistical planning produces a high degree of uncertainty related to the decision making process, is not always possible to have all of the necessary information available at the onset of the problem. Consequently the most common scenario provides incomplete or imprecise information of the parameters and variables. The use of fuzzy sets to approach these situations is very appropriate to build computing systems for the solution of solving decision and optimization problems. The modeling of these problems is complicated at various levels: not only are they difficult to define accurately and need to manage uncertainty, but there is also imprecision in the available information and stated preferences, restrictions and objectives by decision makers.

A frequent occurrence in real decision-making problems, such as those found in VRP, is the lack of precision or uncertainty in the information available in them. This characteristic complicates the process concerning the definition of their objectives and parameters. Although this type of uncertainty found in the nature of the data and their settings has traditionally been handled by means of probability theory, in most of the cases they cannot be considered random phenomena and therefore probability theory cannot be applied successfully.

The aim of this paper is to analyze how the relevance and the variety of applications found in VRP can be considered under the fuzzy context point of view. The organization of the paper follows. Section 2 introduces the broad class of optimization problems known as VRP. We also survey the concept of fuzzy optimization where the objective function and constraints have fuzzy parameters, and then we apply these concepts to the simple and classical VRP. Finally, we review the corresponding fuzzy versions, which are called Fuzzy VRP.

2 Vehicle Routing Problems

The VRP are one of the most widely studied class of problems in combinatorial optimization, and the literature provides several exact and heuristic solution techniques of general applicability. VRP are problems where several vehicles that must serve points of demands and satisfy a finite set of constraints while minimizing costs, distances or times [20].

The standard VRP (usually called capacitated VRP; CVRP) calls for the determination of a set of m routes whose total travel length is minimized such that: (1) each customer is visited exactly once by one route, (2) each route starts and ends at a single depot, (3) the total demand of the customers served by a route does not exceed a given vehicle capacity Q, and (4) the length of each route does not exceed a preset limit L. A constant speed is typically assumed so that to minimize distances, travel times and travel costs are considered equivalent. If each vehicle i is assigned to a route R_i, a feasible solution for the VRP is made up of a partition from V into m routes $R_1, R_2, ..., R_m$ and the corresponding permutations σ_i of $R_i \cup 0$ that specify client order along the routes.

VRP allow us to consider several types of constraints, depending on the specific characteristics of the problem and decision-making process [4]. These possibilities lead to a variety of problems, beginning with the standard or basic VRP. Their descriptions follow:

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MFVRP- *Mix Fleet VRP*. This is a VRP in which the vehicles have different capacities, in other words, fleet of vehicles with a heterogeneous capacity. Each resource must consider these capacities for each route.

MDVRP- *Multi-depot VRP*. A company can have several depots or warehouses for use when supplying client demand. If clients are grouped around the depots then the problem can be viewed as a set of independent VRP problems. However, if the clients and depots are spread apart, then this scenario is known as a MDVRP.

PVRP - *Period VRP*. In the classical VRP the planning period of the route is one day. The PVRP expands the planning period to *M* days. During this period of *M* days, each client should be visited a given number of days.

SDVRP - *Split-up Delivery VRP*. This problem is a VRP in which a client is allowed to be serviced by several vehicles if the total cost decreases. Order size is important since the total size of the client orders must exceed the capacity of one vehicle.

PDVRP - *Pickup and Delivery VRP*. This variation is a VRP since it allows clients the possibility of returning certain goods that have to be delivered to other clients. Therefore, each vehicle that is fitted must consider client pickup and delivery amounts as well as a planned order.

VRPB – *VRP with Backhauls.* This problem is VRP where clients can order or return articles. Therefore the formulation requires that returned goods from clients fit in the vehicle. In addition, all deliveries must be completed before pickups begin because vehicles are back-loaded and pickups in the vehicles are considered uneconomical or unfeasible. Orders and pickups must be known ahead of time.

VRPTW- *VRP with Time Windows*. This variation is a VRP with the additional constraint that associates a time window to each client which identifies the only interval of time when the client is willing to receive goods or services. If a vehicle arrives to the client early the vehicle has to wait. If it arrives during the interval of the time window, the vehicle makes the delivery at that moment. Finally, if it arrives late the client is not serviced.

OVRP - *Open VRP*. This problem is a special class of VRP within all open delivery routes, since vehicles are not required to return to the depot.

Finally, other versions of VRP do exist. Some versions are mixes or hybrids of the previously mentioned typologies, such as the MDVRPTW or OPDVRP. Other problem types introduce specific characteristics which require the corresponding modifications in the model. For example, one version of a particularly interesting (and increasingly complex) problem is a dynamic version of the route problem with or without time windows (DVRP, Dynamic VRP). In this version the problem characteristics vary as time passes.

3 Fuzzy optimization problems

An *optimization problem* can be described as the search for the value of specific *decision variables* so that identified *objective functions* attain their optimum values. The value of the variables is subject to stated *constraints*. In these problems the objective functions are defined on a set of solutions that we will denote by *X*. The objective function is not subject to any condition or property nor is the definition of the set *X*. Typically the number of elements of *X* is very high, essentially eliminating the possibility of a complete evaluation of all its solutions while determining the optimal solution.

Optimization problems in their most general form involve finding an optimal solution according to stated criteria. In practice, however, many situations lack the exact information that is needed in the problem, including its constraints, or in other cases, where it is unreasonable to access such specific constraints or clearly defined objective functions. In these situations it is advantageous to model and solve the problem using soft computing and fuzzy techniques.

Among all the optimization problems, the models that have received the most attention and have offered the most useful applications in different areas are Linear Programming (LP) models, which is the single objective linear case with linear constraints. The classic problem of LP is to find the maximum or minimum values of a linear function subject to constraints that are represented by linear equations or inequalities. The most general formulation of the LP problem is:

$$\begin{aligned} \max \quad & z = cx \\ \text{subject to} \quad & Ax \le b \\ & x \ge 0 \end{aligned} \tag{1}$$

The vector $x = (x_1, x_2, ..., x_n) \in \mathbb{R}^n$ represents the decision variables. The objective function is denoted by *z*, the numbers c_j are coefficients and the vector $c = (c_1, c_2, ..., c_n) \in \mathbb{R}^n$ is known as the cost vector. The matrix $A = [a_{ij}] \in \mathbb{R}^{n \times m}$ is called the constraint or technological matrix and the vector $b = (b_1, b_2, ..., b_m) \in \mathbb{R}^m$ represents the independent terms or right-hand-side of the constraints.

In many real situations not all the constraints and objective functions can be valued in a precise way. In these situations we are dealing with the general problem form of Fuzzy Linear Programming (FLP). FLP is characterized as follows: a_{ij}, b_j and c_i can be expressed as *fuzzy numbers*, x_i as variables whose states are fuzzy numbers, addition and multiplication operates with fuzzy numbers, and the inequalities are among fuzzy numbers.

Different FLP models can be considered according to the elements that contain imprecise information that is taken as a basis for the classification proposed in [22], [2]. Short descriptions of these models follow.

Models with feasible fuzzy set (fuzzy constraints)

This is the case where constraints can be satisfied, and consequently the feasible region can be defined as a fuzzy set; it should be defined by means of a membership function $\mu : \mathbb{R}^n \to [0,1]$. In such a situation, for each constraint, a desirable quantity *b* is considered, but the possibility that it is greater is accepted until a maximum *b*+*t* (*t* is referred to as a violation *tolerance level*). This model is represented by:

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$$\begin{array}{ll} \max & z = cx \\ \text{subject to} & Ax \leq_f b \\ & x \geq 0 \end{array} \qquad (2)$$

where the symbol $\leq_f$ indicates the imprecision of the constraints and where each fuzzy constraint $a_i x \leq_f b_i$ is specified by a membership function in the form:

$$\mu_i(a_i x) = \begin{cases} 1 & \text{if } a_i x \leq b_i \\ f_i(a_i x) & \text{if } b_i \leq a_i x \leq b_i + t_i \\ 0 & \text{if } b_i + t_i \leq a_i x \end{cases} \qquad (3)$$

which means that, for each constraint *i*, given the level of tolerance t_i, to each point (*n*-dimensional vector) *x is* associated a number $\mu_i(x) \in [0,1]$ known as the degree of fulfillment (or verification) of the constraint *i*. The functions f_i are assumed to be continuous and monotonous non-decreasing functions. In particular, Verdegay [21], using the representation theorem for fuzzy sets, proves that the solutions for the case of linear functions f_i can be obtained from the auxiliary model:

$$\begin{array}{ll} \max & z = cx \\ \text{subject to} & Ax \leq b + t(1-\alpha) \\ & x \geq 0, \alpha \in [0,1] \end{array} \qquad (4)$$

where $t = (t_1, t_2, \ldots, t_m)$.

Models with fuzzy goals

A optimization problem with fuzzy goals allows the objective function value to be slightly below the minimum goal for a maximization problem, and similarly for a minimization problem. The corresponding linear model is expressed in the following way:

$$\begin{array}{ll} \widetilde{\max} & z = cx \\ \text{subject to} & Ax \leq b \\ & x \geq 0 \end{array} \qquad (5)$$

If t_0 is the maximum quantity that the objective function should be inferior to the minimum goal c_0, then each vector *x* is associated to a number $\mu_0(x)$, which represents the degree that the decision maker considers to be an achieved goal. It is defined according to the following function:

$$\mu_0(x) = \begin{cases} 1 & \text{if } cx > c_0 \\ f_0(cx) & \text{if } c_0 - t_0 \leq cx \leq c_0 \\ 0 & \text{if } cx < c_0 - t_0 \end{cases} \qquad (6)$$

where f_0 is a continuous, monotonous non-decreasing function. An operative model that provides satisfactory solutions can be found in [27].

Models with fuzzy costs as objective function coefficients

These models are those whose costs are not fully known (with imprecision). Therefore, they are represented by an *m*-dimensional fuzzy vector $c^f = (c_1^f, c_2^f, \ldots, c_n^f)$, and the following model:

$$\begin{array}{ll} \max & z = c^f x \\ \text{subject to} & Ax \leq b \\ & x \geq 0 \end{array} \qquad (7)$$

Evidently, *z* is also a fuzzy number, but *x* can be a vector of fuzzy or non-fuzzy numbers, and each fuzzy cost is described by its corresponding membership function $\mu_i(x)$. Each coefficient c_i^f of the objective function is a plane fuzzy number of the L-R type with modal interval $[\underline{c}_j, \overline{c}_j]$ and membership functions g_j and h_j (which can be linear, parabolic, etc.). Delgado et al. [7] prove that the solution can be obtained with the multi-objective auxiliary model:

$$\begin{array}{ll} \max & z = [c^1 x, \ldots, c^{2^n} x] \\ \text{subject to} & Ax \leq b \\ & x \leq 0, \alpha \in [0,1], c_j^k \in \{g_j^{-1}(1-\alpha), h_j^{-1}(1-\alpha)\} \\ & k = 1, \ldots, 2^n, j = 1, \ldots, n. \end{array} \qquad (8)$$

Models with fuzzy coefficients in the technological matrix

Consider a problem of this type:

$$\begin{array}{ll} \max & z = cx \\ \text{subject to} & A^f x \leq_f b^f \\ & x \geq 0 \end{array} \qquad (9)$$

where the values of the technological matrix and the coefficients are fuzzy numbers. Fuzzy constraints can also be included. Delgado et al. [6] also include imprecision in the constraints. They propose considering fuzzy solutions that are solved with the application of an ordered function *g* for the constraints. This new formulation is expressed by the auxiliary problem:

$$\begin{array}{ll} \max & z = cx \\ \text{subject to} & a_i^f x \leq_g b_i^f + t_i^f(1-\alpha), \ i = 1, \ldots m \\ & x \geq 0, \alpha \in [0,1] \end{array} \qquad (10)$$

where the symbol $\leq_g$ stands for a comparison relation between fuzzy numbers.

4 Fuzzy optimization Vehicle Routing Problems

Although there are different stochastic approaches to modeling and solving the VRP in the literature, this is not the case with the proposed approaches from fuzzy set theory. Furthermore, the literature offers very little in terms of modeling VRP proposals, both from the standpoint of the solutions modeled as Fuzzy Mathematical Programming, and Fuzzy VRP (FVRP). However, as we will discuss later, several proposals have been introduced in recent years, such as the VRPTW and Dynamic VRP.

Specifically, if we look at the FVRP models in the literature, the majority only assume vagueness for some of the following elements that are described in the model: a) Fuzzy demands (to be collected): customer demand is a imprecise variable and 2) Fuzzy times: service time and travel time can be imprecise variables.

The first problem deals with the demand vector of each client's ordered goods. Planning the ordered quantity by the clients is difficult to establish with sufficient notice and precise form, therefore we do not have access to a specific quantity. In other words, the information about vehicle demand at some nodes is often not precise enough. Thus,

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there is often uncertainty regarding the amount of demand at some nodes. This FVRP with fuzzy customer demand was first instructed by Teodorovic and Kikuchi [18]. In this paper, they treated the travel time and the transportation costs between two nodes in a network as fuzzy numbers. They modified the Clarke and Wright algorithm where travel times in a network are treated as fuzzy numbers. Later, Teodorović and Pavković [19] solve a VRP when demand at the nodes is uncertain and is represented by a triangular fuzzy number. The model is based on the heuristic "sweeping" algorithm, which uses fuzzy approximate reasoning procedures to decide whether or not to include a node in the route. It first uses the approximate reasoning algorithm to calculate the preference index. Once the membership function of the preference index has been determined, defuzzification must take place. In recent years, these same authors have proposed solutions to this problem [15], [16], where actual demand value is known only after the visit to the node. Their solution combines Bee and Ant systems with rules based on fuzzy logic. A new paper [14] has recently appeared which considers the VRP with uncertain demand at node. It uses the approximate reasoning algorithm to determine the preference strength to send the vehicle to next node, and the improved sweeping algorithm with vehicle coordinated strategy to determine a set of vehicle routes that minimizes costs.

The second problem, (fuzzy times in both service and travel) is characterized by other pieces of information that are increasingly imprecise, given the daily circumstances found in routing networks and traffic. In these cases service time, time windows and travel time are expressed as fuzzy numbers.

The traditional deterministic VRP is expanded to the situation so that the VRP has fuzzy travel time features. In [11] a simple description of the VRP with fuzzy traveling time, that is, a mathematical model for the problem, is built. It puts forward the concept of level effect function which can quantify the location of fuzzy number intensively and globally, and sets up the uncertain degree of measurement. In this paper the solution to the problem is based on a genetic algorithm.

In [17] the travel time based on the fuzzy mathematical model of the VRP takes the time window as a fuzzy variable. Information entropy and the path chosen by the use of random disturbance control strategy to the Ant algorithm is used to the vehicle routing problem with fuzzy travel time.

Reference [9] focuses on modeling and solution of the dynamic VRP with time-dependent and fuzzy travel time. A model of this problem is set up based on fuzzy service times of the customer, its demand and its time windows, which is regarded and ordering as a triangular fuzzy number. A hybrid genetic algorithm, which is seasoned with the model and combined with the ant colony algorithm, is presented.

In [10] the authors present a model of the real world vehicle routing and dispatching problem. The time-dependent and fuzzy travel speeds are introduced into the model. A dispatching period is divided into some time slices and each time slice is designated a triangular fuzzy speed. The method of comparing two triangular fuzzy numbers is applied to check whether or not customers' time windows are satisfied. A hybrid intelligent approach combining a genetic algorithm and an ant colony algorithm is proposed for solving the dispatching model.

Results in both types of models with uncertainty in demand or in time have been published based on fuzzy variables, fuzzy random variables, stochastic programming and Chance Constrained Programming (CCP). These concepts were introduced by Charnes and Cooper and later by Liu, and generally are applied with heuristics to find solutions to the VRP [26], [24].

In [13] the author considers the VRP with time window while assuming that the travel times cannot be precisely known, but can be regarded as fuzzy variables. Since the travel times are fuzzy variables, every customer will be visited at a fuzzy time. Credibility is introduced as a measure of confidence in the constraints so that it ensures that all customers are visited within their time windows with a confidence level, then following Chance Constraint Programming and also a hybrid intelligent algorithm by integrating fuzzy simulation and GA to solve the VRP.

To the best of our knowledge, there is little evidence in the literature on the properties of mathematical programming with random fuzzy coefficients and VRP with random fuzzy demands. The only related study appears to be a paper [8] which proposed a method to solve a class of model with random fuzzy coefficients in both the objective functions and constraint functions and applied it in the CVRP with random fuzzy demands. Based on the concept of random fuzzy variables introduced in [3], the objective is to provide workable formulations and exact algorithms for a class of uncertainty.

Reference [12] deals with a variation of uncertain VRP where the customers' demands are random fuzzy variables and the travel times between customers are random variables. The travel times between customers follow given probability distributions. The authors develop a programming model with random fuzzy and random variables for VRP which consider capacity and arrival time constraints and present a stochastic programming formulation which includes probabilistic constraints and apply a pure genetic algorithm (GA).

In [23] and [25] the authors considered a fuzzy multi-objective modeling approach for capacitated VRP with fuzzy random parameters. The first paper is based on the mixed integer linear programming model, and suggested an interactive heuristics approach using triangular fuzzy number and crisp equivalent formulae to address the demand solution. The authors in the second paper use travel time and customer demand and treat them as fuzzy random variables and chance-constrained programming is presented and converted to a crisp equivalent model under some assumptions. The authors present a hybrid multi-objective particle swarm optimization that incorporates specific heuristics to solve such a problem.

All of the above VRP can be formalized as problems of combinatorial optimization. These problems in their most general form, have integer LP formulations, based on the proposal given by Bodin *et al*. [1], and can be described, assumed that the depot is the node 0, N is the number of

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customers to be served by K vehicles and the decision variables are $x_{ij}^k \in \{0,1\}$, i,j = 1, 2, …, N, k = 1, 2, …, K, where $x_{ij}^k = 1$ if vehicle k travels from customer i to j and 0 otherwise.

In particular, objectives and constraints may be formulated as follows.

Objectives

a. If c_{ij}^k is the *cost of travelling* from customer i to customer j by vehicle k. The total travel cost is an objective function to be minimized and is expressed by:

$$\min \sum_{k=1}^{K}\sum_{i=0}^{N}\sum_{j=0}^{N} c_{ij}^k x_{ij}^k \tag{11}$$

In general, this cost can be expressed as total time or distance traveled by the vehicles.

b. If t_{ij}^k denotes the time needed to go from customer i to customer j and δ_i^k is the required time by vehicle k to unload the demand to the customer i. The objective function is to minimize the total travel time.

$$\min\left(\sum_{k=1}^{K}\sum_{i=0}^{N}\sum_{j=0}^{N} t_{ij}^k x_{ij}^k + \sum_{k=1}^{K}\sum_{i=0}^{N}\sum_{j=0}^{N} \delta_i^k x_{ij}^k\right) \tag{12}$$

c. Other possible objective function is to maximize the number of customers served by the vehicles.

$$\max \sum_{k=1}^{K}\sum_{i=0}^{N}\sum_{j=0}^{N} x_{ij}^k \tag{13}$$

Constraints

Constraints (14) and (15) ensure that each customer is served exactly once.

$$\sum_{k=1}^{K}\sum_{i=0}^{N} x_{ij}^k = 1, \quad j = 1,2,\dots,N \tag{14}$$

$$\sum_{k=1}^{K}\sum_{j=0}^{N} x_{ij}^k = 1, \quad i = 1,2,\dots,N \tag{15}$$

Constraint (16) ensures route continuity.

$$\sum_{i=0}^{N} x_{it}^k - \sum_{j=0}^{N} x_{tj}^k = 0, \quad t = 1,2,\dots,N;\ k = 1,2,\dots,K \tag{16}$$

Constraint (17) shows that the total length of each route k has a limit D_k, with d_{ij}^k the distance from i to j.

$$\sum_{k=1}^{K}\sum_{i=0}^{N} d_{ij}^k x_{ij}^k \le D_k, \quad k = 1,2,\dots,K \tag{17}$$

Constraint (18) ensures that the total demand of any route must not exceed the capacity Q_k of the vehicle k, with the demand of customer i, q_i.

$$\sum_{j=0}^{N} q_j\left(\sum_{i=0}^{N} x_{ij}^k\right) \le Q_k, \quad k = 1,2,\dots,K \tag{18}$$

Constraints (19) and (20) ensure that each vehicle is used no more than once.

$$\sum_{j=0}^{N} x_{0j}^k \le 1, \quad k = 1,2,\dots,K \tag{19}$$

$$\sum_{i=0}^{N} x_{i0}^k \le 1, \quad k = 1,2,\dots,K \tag{20}$$

This formalization assumes that the decision-maker has access to specific information on the components that define the problem; that is, on objective functions and constraints. However in real world problems it is more common that the information one has is actually imprecise or incomplete.

Real world situations reveal the difficulties of analyzing different types of models and problems that are affected by incomplete, imprecise or vague information or constraints. These problems, however, can be modeled in terms of fuzzy sets, leading to the field of fuzzy optimization. The solutions to these problems are fuzzy solutions. Discussions concerning solutions do not focus on their feasibility, nor if they are optimal solutions or not. We, in turn, have chosen to discuss the degree of feasibility and optimality of the solution.

Thus, if we suppose that any of the parameters: cost c_{ij}^k, distance d_{ij}^k, times t_{ij}^k and δ_i^k, and demand q_i can be fuzzy, the traditional model becomes a Fuzzy VRP. Intuitively, when any of these quantities are fuzzy numbers, the objective functions become fuzzy as well. If these parameters are approximately known, they can be represented by the fuzzy numbers $\tilde{c}_{ij}^k$, $\tilde{d}_{ij}^k$, $\tilde{t}_{ij}^k$, $\tilde{\delta}_i^k$ and $\tilde{q}_i$ respectively, with their corresponding membership functions. Then, for instance, the objective function can be expressed as:

$$\widetilde{\min} \sum_{k=1}^{K}\sum_{i=0}^{N}\sum_{j=0}^{N} \tilde{c}_{ij}^k x_{ij}^k \tag{21}$$

Similar changes occur with other objective functions. In the same way, constraints (17) would be expressed by:

$$\sum_{k=1}^{K}\sum_{i=0}^{N} \tilde{d}_{ij}^k x_{ij}^k \preceq \tilde{D}_k, \quad k = 1,2,\dots,K \tag{22}$$

with similar changes to the other constraints. However, constraints (14) to (16) are the same as those found in the crisp model.

Note that quantities Q_k and D_k also must be considered as fuzzy parameters. In addition, the summation symbol Σ in the objective functions and constraints refers to an addition of fuzzy numbers and $\preceq$ a fuzzy relation between fuzzy numbers. The meaning of min operator $\widetilde{\min}$ is also ambiguous, because it depends on the fuzzy numbers ranking index to be used. Hence there is a need to seek appropriate procedures for its solution.

Following the pattern in general Fuzzy Optimization described in a previous section, four different types of problems can be considered. Two of these problems include imprecision/uncertainty in the objective function(s), such as the case with fuzzy goals and the case with fuzzy costs. The remaining two problems consider fuzzy comparison in the constraints and in the coefficients of the technological matrix. In addition a fifth problem, the general fuzzy problem could be studied in which all of the parameters will be subject to fuzzy considerations.

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In practice, the search for optimal solutions to FVRP can be done with the following approaches. The simplest approach applies procedures for the fuzzification and defuzzification of variables. It transforms the imprecise information in fuzzy parameters and uses procedures that integrate fuzzy arithmetic to obtain fuzzy solutions. The fuzzy solution is then transformed into a crisp one using some known formulation. This approach may also be used for the introduction of sophisticated fuzzy rules in the decision-making processes to improve their quality. Linguistic variables could be used to facilitate the incorporation of "intelligent" procedures as automatic reasoning, adaptive control or automatic learning.

Another common approach consists in applying the Theory of Possibility and Chance Constraint Programming. It treats the fuzzy models, becoming the crisp equivalent, and by adding Chance Constraints changes the model. It assumes that the constraints will hold with at least a possibility α, and the chance is represented by the possibility that the constraints are satisfied.

The previous approaches combined with some metaheuristics provide robust optimization methods to obtain efficient solutions. Although in each planning problem we can establish a set of constraints, many of which can define different planning problems, this set which is defined with parameters and imprecise variables can also be considered the same if the constraints are not necessarily strictly or exactly established. These parameters or imprecise variables could include the times that cannot exceed a specified bound, or the time windows required to visit clients. In other words, it is possible to establish problems with constraints where not all of the constraints need to be satisfied and with the same degree of precision. In this formulization constraints can be called hard, which represent those constraints that must be satisfied exactly, or soft, which are not subject to the same demanding criteria.

5. Conclusions

Fuzzy Vehicle Routing Problems have been analyzed. Strong formulations are available but most algorithmic development focuses on a limited number of prototype problems. We analyze the use of sufficiently flexible and comprehensive fuzzy approaches to address the various imprecision required in practice.

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