Fuzzy analogical model of adaptation for case-based reasoning

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Abstract— *Case-based reasoning is a recognized paradigm and has been explored in both applied and methodological directions. In the several phases of CBR, the adaptation phase is certainly the most problematic whereas the most characteristic and interesting phase. We propose to view this task through a fuzzy analogical scheme. The adaptation is realized by focusing on the relation existing between the problem to be solved and the retrieved cases. Two approaches are proposed here: the relation can be captured by a fuzzy linguistic modifier or by a fuzzy interpolation.*

Keywords— Case-based reasoning, analogy, interpolation, adaptation, fuzzy sets.

1 Introduction

Case-based reasoning (CBR) has been explored in both applied and methodological directions. If the retrieval phase of the process has been widely investigated, less modelling has been proposed for the adaptation phase [13]. Derivational analogy [10] has been proposed to solve this problem when general domain knowledge is available, but it remains mainly a domain-dependent approach. More formal analysis of transformational analogy, assuming knowledge about differences in problems and their solutions, has been considered. In particular, a formal model of transformational adaptation has been proposed [2] on the basis of the consideration of a quality function evaluation.

2 Adaptation modeling in CBR

2.1 Presentation of the problem

Let us consider a problem space P and the solution space S and P' the subset of P of already solved problems. For a problem p in P, case-based reasoning can be regarded as the research of a solution s in S which can be associated with p. Let us denote by IAS the application "is a solution of" p and its solution s: $IAS : P \to S$. A commonly used method is to look for the most resembling elements p' to p in P' already associated with a solution s' in S. Either the best fitting problem is only taken into account and its associated solution is considered, or a group of similar problems is retrieved and various methods can be used to take them into account, by means of an aggregation method or a prototype-based approach for instance.

After the retrieval step, it is generally necessary to incorporate the dissimilarity between p and p' to produce a transformed view s' of the solution s. The transformation is for instance based on a degree of uncertainty attached to the solution s, on the proximity between elements of S, on a modulation of the linguistic expression of s, or on the use of constraints.

Several types of situations occur. In the first type of situation, S is a finite set of crisp solutions and the element s which will be associated with a given p is one of them, associated with the most similar p' in P'.

For *example*, we consider the assignment of a student grant to attend a conference, P is a set of students characterized by several attributes, S the solution space. In this case, S will be reduced to a two-element space "grant, no grant". The student p will be assigned a grant if the most similar element in P' has been assigned a grant.

The only possible adaptation in this case is the assignment of a degree of uncertainty to s'. Obviously, if a decision is to be made, a level of acceptable uncertainty will be chosen and no decision will be possible if this level is not reached.

In the second type of situation, S is infinite. For a given element p in P, the element s which will be associated with p is constructed from the solutions associated with the most similar problems p' in P'. A particular case corresponds to a continuous universe S and a finite number of linguistic values represented by fuzzy sets of S.

For instance, continuing the previous example, we can consider that a grant assignment can be defined as {refused, small, medium, high}. For a given student description p, the grant assignment may be expressed as "rather small" or "very high" if the value of the grant is supposed to depend on his/her merits or characteristics (his/her country is very far away, his/her grades are very good…).

We propose in this paper several adaptation techniques derived from fuzzy set based analogical reasoning and modelling. Our purpose is to propose a general approach based on fuzzy analogy to define adaptation methods in CBR.

2.2 Fuzzy analogy as a model for adaptation in CBR

Problems and solutions are, in many cases, described by means of linguistic terms or approximate values derived from expert knowledge, for instance "if the quality of the paper is very high then the grant assignment is highly recommended". A convenient knowledge representation is thus fuzzy set based. Let us denote by $[0,1]^{\Omega}$ the set of fuzzy sets of any universe Ω.

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A number of works have already presented various utilizations of fuzzy logic based representations in CBR [1][11][12][14] or analogy [20]. Very little attention has been drawn to the adaptation problem and we focus on this aspect in the present paper.

We consider the universe D of descriptions of problems p present in P and the application $\alpha : P \to [0,1]^D$ such that a problem p is described by a fuzzy set $\alpha(p)$ of D. We consider also the universe E of expressions of solutions in S. Each solution in S is associated by means of a one-to-one mapping γ to a fuzzy set $\gamma(s)$ of E.

Our purpose is to provide a model of transformational adaptation in case-base reasoning, starting from the opportunity to use analogy [18] to describe the assignment of a solution to a problem.

A scheme of analogy has been defined in a fuzzy framework [5] in order to mimic the most classic human means of deriving a solution to a new problem on the basis of past experiences in an automatic approach.

Starting from this scheme, and considering the specific environment of case-based reasoning, we propose the following definition of an analogical scheme in CBR. We consider two similarity relations R_1 on $P \times P$ and R_2 on $S \times S$, such that $R_1(p,p') = 1$ if and only if p and p' are similar, $R_2(s,s') = 1$ if and only if s and s' are similar.

Definition 1. For a given application $IAS : P \to S$ and two similarity relations R_1 on $P \times P$ and R_2 on $S \times S$, an *analogical scheme* on (P,S) is a function (see figure 1):

$$\Re_{IAS,R_1R_2} : P \times S \times P \to S \qquad (1)$$

satisfying $\forall p \in P$ and $\forall s \in S$ such that $s = IAS(p)$

(i) $s = \Re_{IAS,R_1R_2}(p,s,p)$

(ii) $\forall p' \in P$ such that $R_1(p,p') = 1$, $s' = \Re_{IAS,R_1R_2}(p,s,p')$ if and only if $s' = IAS(p')$ and $R_2(s,s') = 1$.

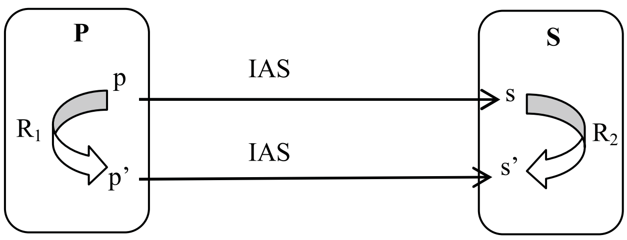

Figure 1. Global analogical scheme for case-based reasoning x

Definition 1 means that if we want to solve a problem p' and we find a problem p similar to p' with a solution $s = IAS(p)$ in S, then the solution s' of p' will be similar to s. At this point, several problems p can be similar to p' and s' will be similar to all their solutions.

It is now necessary to define an operational way of evaluating the similarity R_1 of problems on the basis of their descriptions and the similarity R_2 of solutions on the basis of their expressions.

2.3 *Measures of similitude*

In many case-based reasoning applications, gradual evaluations of similarities are used, for instance based on distances between descriptions in D and between solutions in S or based on the compatibility with prototypes [19]. Let S_1 and S_2 be measures of similitude respectively defined on D and E to compare fuzzy sets. Several forms are available, depending of the properties we require from measures of similitude [8].

Following Tversky's seminal work on features of similarity [21], we define a measure of similitude on any universe Ω as follows, for a given *fuzzy set measure* $M : [0,1]^\Omega \to R^+$ such that $M(A) = 0 \Leftrightarrow A = \varnothing$ and M is monotonous with respect to the inclusion $\subseteq$ of fuzzy sets. We suppose also given an operation – of *difference* of fuzzy sets such that $A - A'$ is monotonous with respect to A and $A \subseteq A'$ implies $A - A' = \varnothing$.

With the convention that we use the same symbol for a fuzzy set and its membership function, the most used difference is defined for any y in Ω by:

$$A - A'(y) = \max(0, A'(y) - A(y)) \qquad (2)$$

Definition 2. A *measure of similitude* on Ω is a mapping $S_\Omega : [0,1]^\Omega \times [0,1]^\Omega \to [0,1]$ defined as:

$$S_\Omega(A,A') = F(M(A \cap A'), M(A' - A), M(A - A')) \qquad (3)$$

for a given mapping $F : R^{+3} \to [0,1]$ such that $F(u,v,w)$ is non-decreasing in u, non-increasing in v and w.

A measure of similitude can be regarded as a fuzzy relation on $[0,1]^\Omega \times [0,1]^\Omega$ and a fuzzy version of a similarity relation. Given a function F, we use measures of similitude on D and E to determine the similarities on P and S.

We assume that

$$R_1(p, p') = 1 \Leftrightarrow S_D(\alpha(p), \alpha(p')) \geq \varepsilon \qquad (4)$$

and $$R_2(s, s') = 1 \Leftrightarrow S_E(\gamma(s), \gamma(s')) \geq \varepsilon, \qquad (5)$$

for a chosen threshold ε.

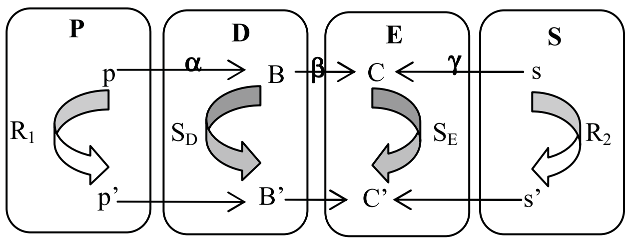

Figure 2. Operational analogical scheme for CBR

We obtain the operational analogical scheme described in Figure 2. The mapping β from D to E is defined in such a way that, for any B in $[0,1]^D$ and C in $[0,1]^E$, $C = \beta(B)$ if

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and only if there exist p in P and s in S satisfying $B=\alpha(p)$, $C=\gamma(s)$ and $s=IAS(p)$. We have $IAS=\gamma^{-1}\circ\beta\circ\alpha$.
Let us note that, crisp values being particular cases of fuzzy sets, the general scheme remains valid of values of attributes are precise. The measures of similitude must of course be chosen in an appropriate way [15].

2.4 Operational analogical scheme for CBR

It is now necessary to define an operational way of deriving s' from p' on the basis of the two measures of similitude S_D and S_E, and the analogical scheme $\Re_{IAS,R_1,R_2}$. We use a fuzzy version of analogical scheme as follows.

Definition 3. For a given application $\beta:[0,1]^D\rightarrow[0,1]^E$ and two measures of similitude S_D on $[0,1]^D\times[0,1]^D$ and S_E on $[0,1]^E\times[0,1]^E$, an $\varepsilon-$*analogical scheme* on (D,E) is a function defined for a given threshold $\varepsilon\in[0,1]$ as:

$$\Re^{\varepsilon}_{\beta,S_DS_E}:[0,1]^D\times[0,1]^E\times[0,1]^D\rightarrow[0,1]^E \qquad (6)$$

satisfying the following conditions $\forall B\in[0,1]^D$ and $\forall C\in[0,1]^E$ such that $C=\beta(B)$

(iii) $C=\Re^{\varepsilon}_{\beta,S_DS_E}(B,C,B)$

(iv) $\forall B'\in[0,1]^D$ such that $S_D(B,B')\geq\varepsilon$, then $C'=\Re^{\varepsilon}_{\beta S_DS_E}(B,C,B')$ if and only if $C'=\beta(B')$ and $S_E(C,C')\geq\varepsilon$.

This means that we need to determine a fuzzy $\varepsilon-$analogical scheme $\Re^{\varepsilon}_{\beta,S_DS_E}$ on $\left([0,1]^D,[0,1]^E\right)$ compatible with $\Re_{IAS,R_1R_2}$ (see figure 1). This compatibility is satisfied in the following way.

Property 1: Let us suppose that $IAS=\gamma^{-1}\circ\beta\circ\alpha$. For any p and p' in P, and s in S, the following holds:

$$s'=\Re_{IAS,R_1R_2}(p,s,p')\Leftrightarrow\gamma(s')=\Re^{\varepsilon}_{\beta,S_DS_E}(\alpha(p),\gamma(s),\alpha(p')).$$

Proof. If $p'=p$, then $s'=IAS(p')=IAS(p)=s$ by (i), also $\alpha(p')=\alpha(p)$ and $\gamma(s')=\gamma(s)$. Then (iii) is ensured.
Now, if $s'=\Re_{IAS,R_1R_2}(p,s,p')$, then $R_1(p,p')=1$, $R_2(s,s')=1$, and consequently the following inequalities hold:

$S_D(\alpha(p),\alpha(p'))\geq\varepsilon$, $S_E(\gamma(s),\gamma(s'))\geq\varepsilon$,

which entails $\gamma(s')=\Re^{\varepsilon}_{\beta,S_DS_E}(\alpha(p),\gamma(s),\alpha(p'))$ by (iv).

Conversely, if $\alpha(p')=\alpha(p)$, then $\gamma(s)=\gamma(s')$, and $s=s'$. Then (i) is ensured.

Now, if $\gamma(s')=\Re^{\varepsilon}_{\beta,S_DS_E}(\alpha(p),\gamma(s),\alpha(p'))$, then for $S_D(\alpha(p),\alpha(p'))\geq\varepsilon$, we get $\gamma(s')=\beta(\alpha(p'))$ and $S_E(\gamma(s),\gamma(s'))\geq\varepsilon$. For p and p' such that $S_1(p,p')=1$, $s'=IAS(p')$ since γ is a one-to-one mapping, and $R_2(s,s')=1$ by (5). Consequently, (ii) holds.

Continuing the previous *example*, we can consider a population P of students p, described by various attributes, such as the quality of their submitted paper, their ages, their countries… Let us consider universes of paper quality Q, of ages A, of countries C, we note D=Q×A×C and we consider fuzzy descriptions of students p, i.e. fuzzy sets of D, for instance (rather good quality, young, far country). Each student p will be assigned an amount of financial support in S which may be expressed by a fuzzy set of the universe of natural numbers E=IN, such as a “small grant”.

3 Transformational adaptation in CBR

Since we have established a link between a comparison of problems in P or solutions in S and a comparison of their descriptions in D or expressions in E, we can work on (D,E) instead of (P,S) to determine methods of transformational adaptation in CBR.

3.1 Retrieval of a similar problem

We don’t focus in this paper on the retrieval step of CBR. Let us only mention that there exist many ways to use the similarity relations R_1 to retrieve problems in P' which will be used to determine a solution in S to a given problem p.
For instance, the comparison can be restricted to typical problems in P'. The definition of prototypes can easily be solved through methods based on particular measures of similitude and dissimilarity in D [16]. Let P'' be the set of prototypes. They can be particular problems of P' or they can be abstract problems with descriptions in D summarizing the descriptions of a class of problems in P'.
The comparison can also be restricted to clusters of P' determined in agreement with solutions in S.
Let us remark that the threshold ε can be freely chosen by the user. If he/she chooses $\varepsilon=0$, he/she takes into account all possible problems in P, and the determination of a unique p' associated with a given p lies in this case on the retrieval method. For instance, the retrieval method can be based on an optimization technique and p' will be the most similar to p, corresponding to the greatest value of $S_D(\alpha(p),\alpha(p'))$, under given additional constraints in some cases. Otherwise, the threshold ε can be considered as a level of decidability: if there exists no p' such that $S_D(\alpha(p),\alpha(p'))\geq\varepsilon$, then there is no already solved problem sufficiently similar to p and no solution can be proposed, unless more information is obtained.
If we now suppose that there exists a subset P_0 of such problems p', a method must be picked to determine a unique p'_0 in P_0 which will be used to find a solution to p by adapting $s'_0=IAS(p'_0)$. Either we use an aggregation

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method to combine all p'_0 in P_0, or we look for the maximum of $\{S_D(\alpha(p), \alpha(p'))/ p' \in P_0\}$, under additional constraints if necessary. Another solution is to look for the k-best values among $\{S_D(\alpha(p), \alpha(p'))/ p' \in P_0\}$, with k chosen by the user, for instance $k = 2$. A refinement of the method considers a restriction of the search to elements of P_0 significantly different, with an additional threshold of decidability ε' such that, if the two best values in $\{S_D(\alpha(p), \alpha(p'))/ p' \in P_0\}$ correspond to fuzzy subsets b and b' of D such that $|S_D(\alpha(p), b) - S_D(\alpha(p), b')| \le \varepsilon'$ and $\beta(b) \neq \beta(b')$, then no solution can be proposed to solve p, unless additional information is obtained.

We now focus on transformational adaptation techniques to determine the solution s of problem p.

3.2 Transformational adaptation by means of modifiers

We first consider that the retrieval method provides a unique p', such that $R_1(p, p') = 1$, best fitting the considered problem p. The value $S_D(\alpha(p), \alpha(p'))$ of the measure of similitude gives an information on the level of resemblance between p and p'. This value is used to adapt the solution $s = IAS(p)$ in order to define $s' = IAS(p')$. The $\varepsilon-$analogical scheme introduced in Definition 3 indicates that s will be determined in such a way that $S_E(\gamma(s), \gamma(s')) \ge \varepsilon$. It is clear that, $C' = \gamma(s')$ being a fuzzy set of D, there exist many fuzzy sets $C = \gamma(s)$ such that this condition is satisfied.

A convenient way to define C from C' is to use a modifier [4], such that s' will be a linguistically modified form of s. For example, if s is "small", then s' may be "rather small", or "more or less small", depending on the context.
A modifier defined for a universe Ω (Ω being either D or E) is a mapping $m : [0,1]^\Omega \to [0,1]^\Omega$.
We restrict ourselves to $\varepsilon-$*modifiers* m on Ω such that

$$M(m(X) - X) = 1 - \varepsilon \quad (7)$$

The modifier is expansive if $X \subseteq m(X)$ for any X in $[0,1]^\Omega$. Examples of expansive modifiers are "more or less" or "approximately". Modifiers m are defined by parameterized mathematical transformations such as homotheties [3].
The transformational adaptation process is then the following: given the measure of similitude S_Ω, we define the solution s to problem p in such a way that $S_E(\gamma(s), \gamma(s')) \ge \varepsilon$. The form of m depends on the chosen measure of similitude and the parameters defining m are deduced from ε.

We have proved in [6] that the choice of a particular measure of similitude and the choice of the modifier are linked. Working with expansive modifiers, for instance, leads to use a particular measure of similitude called a measure of *inclusion*, defined as a reflexive measure of similitude (r.m.s.) such that $F(0, v, w) = 0$ for any v and w and $F(u, v, w)$ is independent of v. Classic examples of measures of inclusion are the following:

$$- \; S_\Omega(X, X') = \frac{M(X \cap X')}{M(X)}, \quad (8)$$

M standing for the fuzzy cardinality of X,

$$M(X) = \int_{y \in \Omega} X(y) \quad (9)$$

$$- \; S_\Omega(X, X') = 1 - M(X - X'), \quad (10)$$

M standing for the height of X:

$$M(X) = \sup_{y \in \Omega} X(y). \quad (11)$$

Property 2. Let us suppose that the difference of fuzzy sets is defined by (2), the fuzzy set measure by (11) and the similarity by (10). For any given problem p associated with a description B, an $\varepsilon-$modifier m provides the expression of a solution s of p compatible with the $\varepsilon-$analogical scheme i.e. such that $m(C') = \Re^{\varepsilon}_{\beta S_D S_E}(B', C', B)$ for any B, B', C'.

Proof. If m is an $\varepsilon-$modifier, then $M(m(C') - C') = 1 - \varepsilon$, which means that $\sup_{x \in E}(m(C') - C') = 1 - \varepsilon$. In the case where $B = B'$, then $S_D(B, B) = 1$, $\varepsilon = 1$. Then $M(m(C') - C') = 0$ by (7). Then $m(C') = C = C'$ and consequently $C' = \Re^{\varepsilon}_{\beta S_D S_E}(B', C', B')$, satisfying condition (iii). In the case where $\varepsilon \neq 1$, we have $m(C')$ is such that $m(C') = \gamma(s)$ by construction and $S_E(C', m(C')) = \varepsilon$ by (7). Consequently $m(C') = \Re^{\varepsilon}_{\beta S_D S_E}(B', C', B)$.

The properties of a measure of inclusion show that $S_\Omega(X, X')$ is independent of $X' - X$. This means that the choice of a measure of inclusion on E provides $S_E(m(C'), C')$ independent of $C' - m(C')$. This leads to use an expansive modifier m, since it corresponds to $C' - m(C') = \varnothing$ and $C' - m(C')$ has no influence on the value of a measure of inclusion.
An example of such an expansive $\varepsilon-$modifier is the following:

$$C(x) = m(C')(x) = \min\left(1, \frac{C'(x)}{\varepsilon}\right) \text{ for any } x \in E,$$

if we use (10) for M, s expressed as "approximately s'".

Considering again the previous *example*, if a "good" paper's author is assigned a "high grant", a similar paper's author is assigned an "approximately high" grant. The utilization of an expansive modifier reveals a cautious approach.

3.3 Transformational adaptation by means of modifiers in a gradual environment

In a different context, we consider the specific case where there is a gradual link between a variation on D and a

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variation on E. An example is expressed by "the better the quality of a student's paper, the higher the assigned grant".
Several methods are then possible to obtain a solution s to a problem p. We assume that D and E are included in the universe **R** of real numbers.

We consider again that the retrieval method provides a unique p', such that $R_1(p, p') = 1$, best fitting the considered problem p. An appropriate form of modifier corresponds to translations of X to the left or to the right [9], with an amplitude of translation λ (positive or negative), to obtain the modified version $m(X)$. All components of the comparison between X and X' are taken into account, namely $X \cap X'$, $X - X'$ and $X' - X$.
The classic definition of a translatory modifier is the following, if the universe E is $[0,1]$, to simplify:

$m(C')(x) = C'(x + \lambda)$ if $x + \lambda \in [0,1]$

$m(C')(x) = C'(0)$ if $x + \lambda \le 0$

$m(C')(x) = C'(1)$ if $x + \lambda \ge 1$.

This means that $m(C')$ is generally the translation of C', except at both ends of E. It is easy to check that $M(m(C') - C') = M(C' - m(C')) = |\lambda|$, except at both ends, where $M(m(C') - C') \le |\lambda|$ or $M(C' - m(C')) \le |\lambda|$, if we consider the fuzzy set measure (9).
Such a modifier is associated with any form of r.m.s., defined by $F(u, v, w)$, and depending on u, v, w. We can consider the measure of similitude (10) associated with the fuzzy set measure (9).
The amplitude must be defined in our case with respect to ε. Choosing $|\lambda| = 1 - \varepsilon$ ensures that $S_E(m(C'), C') \ge \varepsilon$. We then obtain the following result :

Property 3. Let us suppose that the universe is $[0,1]$. If the difference of fuzzy sets is defined by (2), the fuzzy set measure by (9) and the similarity by (10), then a translatory modifier m defined by an amplitude ε such that $|\lambda| = 1 - \varepsilon$ satisfies $m(C') = \Re^{\varepsilon}_{\beta S_D S_E}(B', C', B)$ for any B, B', C', according to definition 3.

The solutions we deduce in S from the use of such modifiers in E is more diverse. Depending on the sign and the value of the amplitude λ, depending also on the context, the solution s can take into account a reinforcement or a weakening of $\gamma^{-1}(C')$. In more concrete terms, if a student p' in P' is described by "good quality paper" in D and the solution $s' = IAS(p')$ expressed by "high assigned grant" in E, then a student p similar to p' will correspond to an assignment of "very high grant" (reinforcement) or "rather high grant" (weakening). To determine which of these two possibilities is the right one, more information must be taken into account about the relative position of B and B'.

3.4 Transformational adaptation by means of interpolation

We still consider that there is a gradual link between a variation on D and a variation on E. We now suppose that we take into account two problems in P' similar to p in order to find a solution $s = IAS(p)$ in S in a more comprehensive way.
The problems in P' are associated with descriptions represented by a family $D' = \{\alpha(p_1), \alpha(p_2)...\}$ of fuzzy sets of D and we further suppose that there exists an order $\prec$ on D' such that:

$\alpha(p_1) \prec \alpha(p_2) \prec ... \prec \alpha(p_i) \prec \alpha(p_{i+1}) \prec ... \prec \alpha(p_n)$

Let $E' = \{\gamma(s_1), \gamma(s_2)...\}$ be the family of associated expressions of solutions represented by fuzzy sets in E, with $s_i = IAS(p_i)$ for any $i = 1, 2...$, supposed to be equipped with the same order $\prec$.
Without any loss of generality, we can then suppose that: $\gamma(s_1) \prec \gamma(s_2) \prec ... \prec \gamma(s_i) \prec \gamma(s_{i+1}) \prec ... \prec \gamma(s_n)$.

For a given p in P, we propose to look for the most similar p', namely p_i, and consider the pair (p_i, p_{i+1}) if $\alpha(p_i) \prec \alpha(p)$ or the pair (p_{i-1}, p_i) if $\alpha(p) \prec \alpha(p_i)$. For the sake of simplicity, we consider only the first case, such that $\alpha(p_i) \prec \alpha(p) \prec \alpha(p_{i+1})$.
Because of the assumption of graduality, we will have $\gamma(s_i) \prec \gamma(s) \prec \gamma(s_{i+1})$. The methods to determine s are again various. We propose to use a method based on interpolation [7][18]. The general spirit of the method is based on the basic analogical process "s is to s_i and s_{i+1} as p is to p_i and p_{i+1}".
The method to determine $C = \gamma(s)$ is based on the following steps: first, we compare $\alpha(p)$ to $(\alpha(p_i), \alpha(p_{i+1}))$. Secondly, we construct a family of possible fuzzy sets of E similar to the pair $(\gamma(s_i), \gamma(s_{i+1}))$ in a way analogous to the way $\alpha(p)$ is similar to the pair $(\alpha(p_i), \alpha(p_{i+1}))$. Finally, we deduce one element C from this family.

The detailed process we have proposed takes into account both location in the universe E and shape (understood as their membership function) of fuzzy sets and we proceed as follows.
Step1. We determine the location $loc(C)$ of C, considering that it is to the locations of $\gamma(s_i)$ and $\gamma(s_{i+1})$ as the location of $\alpha(p)$ is to those of $\alpha(p_i)$ and $\alpha(p_{i+1})$.
Step 2. We translate $\alpha(p_i)$ and $\alpha(p_{i+1})$ towards $\alpha(p)$ with respect to locations, to obtain B'_i and B''_i.
Step 3. We compare the shapes of B'_i and B''_i to the shape of $\alpha(p)$.
Step 4. We translate $\gamma(s_i)$ and $\gamma(s_{i+1})$ to $loc(C)$ to obtain respectively C'_i and C''_i.
Step 5. We construct two fuzzy sets C' and C'' of E with location $loc(C)$ such that the shape of C' (resp. C'') can be

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compared to the shape of C'_i (resp. C''_i) in the same way as the shape of $\alpha(p)$ is compared with the shape of B'_i (resp. B''_i).

Step 6. We aggregate C' and C'' to construct C.

The interest of such a method appears when the family D' is sparse, which means that it is possible to find problems p falling into a "gap" between already solved problems, their similarity being relatively low. This method can also be used in ordinary cases, with a concern of quality of the solution, since taking into account two elements provides a more gradual, and then more robust, treatment of the problem.

4 Conclusion

We have pointed out several methods enabling the user to perform the transformational adaptation of the solution to a similar problem. These methods are diverse and their utilization depends on the context. Their interest is to ensure a gradual passage between cases and the global utilization of the set of already solved problems. Experiments will be available in the future to compare methods.

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