# Non Additive Measures for Group Multi Attribute Decision Models

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***Abstract***— *This paper extends the Choquet integral, widely used in multi-attribute decision problems, to the non monotone case in the context of Group Decision Theory. Even if not so often, preference structures which violate the monotonicity axiom can be observed in real applications. Our aim is twofold. First, we propose the Choquet integral with non monotone non additive measure. Then, we apply the Choquet integral in the context of multi person decision problem, a typical framework of many real world applications, for which the Choquet integral was rarely proposed. Thus in our model this aggregation function is applied twice, both in the cases with possible negative interactions. For this reason, our proposal can be defined as two-step signed Choquet integral.*

***Keywords***— Non additive measures, Choquet integral, Group Decision Theory.

## 1 Introduction

In Multi Attribute decision models, non additive measures and the Choquet integral have been intensively applied in many real world decision problems. Despite simpler approaches, like simple additive weighting (SAW), non additive measures can model *interactions* between the criteria. This method is a very general aggregation tool, including as special cases many aggregation algorithms, as SAW, min and max operators, k-statistics, OWA, and many others. Nevertheless, monotonicity is usually considered a strict requirement. Only few papers explicitly considered the non monotone case. Even if rarer than for *preferential independence* axiom, some violations of the monotonicity axiom can be observed in the Decision Maker preference structure. Moreover, even if many real world applications exist so far, little results were obtained in the context of Group Decision Theory. This family of problem deals with complex decision about some alternatives that need to be scored by a tool of Experts, see [4] and the references therein. To simplify, suppose that an Experts Committee is demanded to evaluate a finite set of development strategies on the basis of some criteria. Non additive measures can help to solve this type of *multi person - multi attribute* decision problems.

Negative(non monotone) interactions among the criteria can be modeled by means of *signed* measures.

The aim of the paper is twofold; first we consider the possibility of *negative* interactions among the criteria, and then we apply a non additive aggregation algorithm in the context of Group Decision Theory. In this sense, we apply twice the Choquet integral, to aggregate the Expert's opinion for each alternative, and subsequently to aggregate the Expert alternative scores into a global numerical evaluation (*Group alternative score*). The latter is usually performed by a Decision Maker, which, on the basis of his confidence about the Committee members experience and/or capacity, averages their judgements not necessarily in a linear way. Thus we can define our approach as a two-step Choquet integral with *signed* measures, i.e. *two-step signed Choquet integral*. The paper is organized as follows. The next Section briefly resumes some definitions and properties of non additive measures and of the Choquet integral. Section 3 describes our proposed model, which is deeply analyzed in the following Section 4. Section 5 reports the application of the two-step signed Choquet integral to multi person - multi attribute decision problems, and finally in the last section a numerical example is reported.

## 2 The discrete Choquet integral

Aggregation has for purpose the simultaneous use of different pieces of information provided by several sources, in order to come to a conclusion or a decision. So aggregation functions transform a finite number of inputs, called arguments, into a single output. They are applied in many different domains and, in particular, aggregation functions play an important role in different approaches to decision making, where values to be aggregated are typically preferences or satisfaction degrees. Many functions of different type have been considered in connection with different situations and various properties of these functionals can be imposed by the nature of the considered aggregation problem. A class of aggregation operators can also be introduced axiomatically by means of a set of properties.

We denote by $E$ a non empty real interval. If the integer $n$ represents the number of values to be aggregated an aggregation operator is a function $A : E^n \to E$. To motivate the use of the Choquet integral as an aggregation operator, we present some basic mathematical properties of the aggregation functions.

- **Monotonicity** For all $\mathbf{x}, \mathbf{y} \in E^n$ if $x_i \leq y_i\,(i = 1, \ldots, n)$ then $A(\mathbf{x}) \leq A(\mathbf{y})$
- **Positive Homogeneity** If $\mathbf{x} \in E^n$ and $a \in \mathbb{R}, a > 0$ then $A(a\mathbf{x}) = aA\mathbf{x}$

Moreover we define $x_{-i}$ the element of $\mathbb{R}^{n-1}$ that is obtained from $x$ by eliminating component $i$, and we denote $(x_{-i}, y_i)$ as obtained from $x$ by replacing $x_i$ with $y_i$.

Now we present the concept of comonotonicity.

**Definition 2.1** *If $x, y$ are elements of $\mathbb{R}^n$ then $x, y$ are said* comonotonic *if $x_i < x_j$ implies that $y_i \leq y_j$.*

Two vectors $x, y$ are comonotonic if they have the same ranking of their components or there exists a permutation $\sigma : \{1, \ldots, n\} \to \{1, \ldots, n\}$ such that $x_{\sigma(i)}$ and the corresponding $y_{\sigma(i)}$ are arranged in nondecreasing order.

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Our representation result depends on the following axioms related to the concept of comonotonicity.

- **Comonotonic Monotonicity** If $\mathbf{x}, \mathbf{y} \in E^n$ are comonotonic and $x_i \leq y_i$ $(i = 1, \dots, n)$ then $A(\mathbf{x}) \leq A(\mathbf{y})$
- **Comonotonic Separability** If $\mathbf{x}, \mathbf{y} \in E^n$ are comonotonic then for every $i$, $A(x_{-i}, x_i) \geq A(y_{-i}, x_i)$ iff $A(x_{-i}, y_i) \geq A(y_{-i}, y_i)$

The comonotonic separability axiom is obviously a variation of an additive separability axiom and it has been applied successfully in decision making under risk and uncertainty, see [5]and the references therein. It states that preferences between alternatives depend only on the components that differ between the vectors under cons ideration, as long as these alternatives maintain the attributes' ordering.
In order to introduce a non-additive approach to aggregation operators we propose a non-additive integral operator and so we consider the integral as a particular averaging operator. The use of variants of the Choquet integral allows some flexibility in the way criteria are combined. Non additive measures are in the current literature a commonly used method to aggregate numerical information. This is particularly due to the fact that in cooperation with integral aggregation functions, they are a well-founded framework able to capture interactions among the involved variables. Usually the monotonicity property is required in most of practical applications. Monotone measures can model both *synergic* and *redundance* interactions among the criteria. Conversely, they are unable to represent the *neutralization* effect. Such a situation appears, for instance, where increasing one criterion by alone has a positive effect, but the contemporary increase of two criteria has a negative effect. Of course, such a situation cannot be represented by monotone measures.
In particular in this paper we consider a non-monotone Choquet integral as in [5], [8] and [12] and we define a non-monotone Choquet measure and a Choquet integral for a n-dimensional vector.
If $N$ is a finite index set $N = \{1, \dots, n\}$, a real valued set function $v : 2^N \to \mathbb{R}$, with $v(\emptyset) = 0$, is called a *non additive signed measure*. If $A \subseteq B \subseteq N$ implies that $v(A) \leq v(B)$, then the function is said to be *monotone* and $v$ is called a *non-additive measure*. If $v$ is a non-additive measure then $v(A) \geq 0$ for all $A \in 2^N$. We say that a measure is *additive* if $v(A \cup B) = v(A) + v(B)$.
We note that if $S \subseteq N$, $v(S)$ can be viewed as the *importance* of the set of elements $S$. We introduce now the discrete Choquet integral on $N$ viewed as an aggregation function that generalizes the weighted arithmetic mean.

**Definition 2.2** *The* Choquet integral $C_v$ *of a vector* $\boldsymbol{x} \in \mathbb{R}^n$ *with respect to a non-additive signed measure* $v$ *is*

$$C_v(\boldsymbol{x}) := \int_0^{+\infty} v(\boldsymbol{x} \geq \alpha)\, d\alpha + \int_{-\infty}^{0} (v(\boldsymbol{x} \geq \alpha) - v(N))\, d\alpha$$

*where* $v(\boldsymbol{x} \geq \alpha) = v(\{i \in N : x(i) \geq \alpha\})$.

This formula can be interpreted as an expectation operator with respect to a generalized measure. Then, the Choquet integral of a vector $\mathbf{x} \in \mathbb{R}^n$ with respect to a non-additive measure $v$ can be represented as the following weighted sum:

$$C_v(\mathbf{x}) = \sum_{1=1}^{n} (x_{(i)} - x_{(i-1)}) v(A_{(i)}) \tag{1}$$

and $\sigma : \{1, \dots, n\} \to \{1, \dots, n\}$ is such that $x_{\sigma(i)}$ are arranged in nondecreasing order and $A_{(i)} = \{i, \dots n\}$ and $A_{(n+1)} = \emptyset$. The Choquet integral can be also computed using the Möbius representation of $v$, see [8].

**Proposition 2.1** *The Choquet integral* $C_v : \mathbb{R}^n \to \mathbb{R}$ *can be written as*

$$C_v(\boldsymbol{x}) = \sum_{T \subseteq N} a(T) \bigwedge_{i \in T} x_i, \qquad \boldsymbol{x} \in \mathbb{R}^n$$

*where* $a$ *is the Möbius representation of* $v$.

The Choquet integral has important properties for aggregation.

**Proposition 2.2** *Let* $A$ *be an aggregation function defined on* $E^n$.

*i) If* $A$ *is a non-monotone Choquet integral then it is positive homogeneous, comonotonic monotone and comonotonic additive.*

*ii)* $A$ *is a non-monotone Choquet integral if it satisfies positive homogeneity and comonotonic separability.*

*iii)* $A$ *is a non-monotone Choquet integral if it is continuous and comonotonic additive.*

*Proof* The proof of part i) is immediate by the definition of Choquet integral.
Now it is important to note that a comonotonic separable aggregation function $A$ is comonotonic additive that is if $\mathbf{x}, \mathbf{y} \in E^n$ are comonotonic then $A(\mathbf{x} + \mathbf{y}) = A(\mathbf{x}) + A(\mathbf{y})$. In fact if $\mathbf{x}, \mathbf{y}, \mathbf{z} \in E^n$ are comonotonic and $A(\mathbf{x}) \geq A(\mathbf{y})$ then by comonotonic separability $A(\mathbf{x} + \mathbf{z}) \geq A(\mathbf{y} + \mathbf{z})$. Then if we consider two comonotonic vectors $\mathbf{x}, \mathbf{y} \in E^n$ there exist $c, d \in E$ such that $A(\mathbf{x}) = \mathbf{c} = (c, \dots, c)$ and $A(\mathbf{y}) = \mathbf{d} = (d, \dots, d)$. By the fact that each comonotonic vector is comonotonic with each other vector with equal component we can prove $A(\mathbf{x} + \mathbf{y}) = A(\mathbf{c} + \mathbf{y}) = A(\mathbf{c} + \mathbf{d}) = A(\mathbf{c}) + A(\mathbf{d}) = A(\mathbf{x}) + A(\mathbf{y})$. Now we can conclude since a homogeneous function that satisfies comonotonic additivity is a non-monotone Choquet integral by theorem 1 of [8] and a continuous function that satisfies comonotonic additivity is a non-monotone Choquet integral by corollary 2.2 of [14].

It is elementary verified that if $A$ satisfies the hypothesis of proposition 1.1 and it is also monotone then it is a monotone Choquet integral.

## 3 The model

We consider a decision problem where $K = \{1, \dots, k\}$ Experts have to score $M$ alternatives on the basis of some criteria, each of them normalized in the common scale [0,1]. The preference structure of the $k$-th Expert, $(1 \leq k \leq K)$, is represented by a set of non additive measures $m_k$, defined on the space of criteria. Despite the majority of Multi Criteria Group decision models, we admit that each Expert is characterized

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by his own criteria set, even if the sets can partially overlap or coincide as a particular case. Moreover, the preference structure of each Expert will be represented by a *signed* measure, including the possibility of *neutralization* effects among the criteria themselves. Finally, at higher level, an other measure $u$ is used to aggregate the individual opinions into a global score. Assuming that a Decision Maker (DM) has to score the alternatives on the basis of the judgements expressed by the Experts, the measure $u$ can be interpreted as the degree of confidence that the DM feels towards the Experts themselves, or towards each coalition of Experts. Since the Choquet integral is twice applied, we deal with a *two-step* Choquet integral [11][1]. For a better problem formulation, let us consider the following definitions.
i) The criteria set for the $k$-th Expert $E_k$ is the set $N_i = \{c_{i_1}, \ldots, c_{i_n}\}$, with $n_i = |N_i|$ is the cardinality of $N_i$. It follows that $\mathbf{x}^i = (x_{i_1}, \ldots, x_{i_n})$ is the *profile* of criteria associated with an alternative by the Expert $E_k$ and an alternative is characterized by a vector $\mathbf{x}$ in $\mathbb{R}^n$ where $n = \sum_{i=1}^{K} n_i$.

ii) The preference structure of the $k$-th Expert is represented by a non additive signed measure $m_k$, defined on $2^{N_k}$.

iii) There are $M$ alternatives, each of them characterized by the vector $\mathbf{x}_j$ in $\mathbb{R}^n$, $j = 1, .., M$.

Let $v_k(\mathbf{x}) = \int \mathbf{x}^k \, d\, m_k$ for $1 \leq k \leq K$ and $v(\mathbf{x}) = (v_1(\mathbf{x}), \ldots, v_K(\mathbf{x}))$.
Then we define the two-step Choquet integral $C(\mathbf{x})$,

$$C(\mathbf{x}) = \int v(\mathbf{x}) \, d\, u.$$

## 4 Multi-step non monotone Choquet integral

In this section we give some basic definitions and we present some results on multi-step non monotone Choquet integral. The two-step (monotone) Choquet integral has been investigated mainly in [13] and [11]. Let us now give a formal definition of a multi-step non-monotone Choquet integral.

**Definition 4.1** *Let $\Gamma \subseteq \mathbb{R}^n$. For any $i \in N$, the projection $\mathbf{x} \mapsto x_i$ is a* 0-step non-monotone Choquet integral*. Let us consider $F_i : \Gamma \to \mathbb{R}$, $i \in M := \{1, \ldots, m\}$, being $k_i$-step non-monotone Choquet integrals, and a non-additive signed measure $\upsilon$ on $M$. Then*

$$F(\mathbf{x}) := C_\upsilon(F_1(\mathbf{x}), \ldots, F_m(\mathbf{x}))$$

*is a $k$-*step non-monotone Choquet integral*, with $k := \max\{k_1, \ldots, k_m\} + 1$. A* multi-step non-monotone Choquet integral *is a $k$-step non-monotone Choquet integral for some integer $k > 1$.*

Let us first recall the concept of piecewise linear function.

**Definition 4.2** *A real-valued function $F$ on a convex closed subset $\Gamma \subseteq \mathbb{R}^n$ is* piecewise linear *if $\Gamma$ can be written as a union of closed subspaces $\Gamma_1, \ldots, \Gamma_q$ of the same dimension as $\Gamma$, such that $F|_{\Gamma_i}$ is linear, $i = 1, \ldots, q$. A linear function $G$ on $\mathbb{R}^n$ which coincides with $F$ on some $\Gamma_i$ is a* component *of $F$.*

[1] Anywise, in the quoted reference the two-step Choquet integral was limited to the monotone case only.

The following proposition gives the basic properties of non-monotone multi-step Choquet integrals.

**Proposition 4.1** *The non-monotone multi-step Choquet integral is a continuous, positively homogeneous and piecewise linear function.*

*Proof* The non monotone Choquet integral is a piecewise linear and positively homogeneous function, then also the multi-step non monotone Choquet integral. Clearly, any piecewise linear function is continuous.

Recall that the multi-step Choquet integral is not comonotonic additive, in general, and hence it cannot be described by a 1-step Choquet integral.

**Proposition 4.2** *If $F$ is a 2-step non-monotone Choquet integral and the measure of the second level is additive then $F$ coincides with a 1-step non-monotone Choquet integral.*

*Proof* If we consider two comonotonic vectors $\mathbf{x}, \mathbf{y}$ then for every $i = 1, \ldots, m$ $F_i(\mathbf{x} + \mathbf{y}) = F_i(\mathbf{x}) + F_i(\mathbf{y})$ since $F_i$ is comonotonic additive. Then it is easy to prove that $F(\mathbf{x}+\mathbf{y}) = F(\mathbf{x}) + F(\mathbf{y})$ where $F(\mathbf{x}) = C_\upsilon(F_1(\mathbf{x}), \ldots, F_m(\mathbf{x}))$ and $C_\upsilon$ is a linear functional. Then $F$ is a positively homogeneous and comonotonic additive functional. Now the positively homogeneous function $F$ that satisfies comonotonic additivity is a non-monotone Choquet integral by theorem 1 of [8].

## 5 A Multi Criteria Group Decision problem

In the proposed model, the two-step Choquet integral will be used to compute the score of any alternative, proceeding in two sequential phases. Firstly, the individual measures $m_k$ defined on $2^{N_k}$ are used to compute the individual score. Subsequently all the individual scores are aggregated with the measure $u$ on the space $2^K$. Both in the cases signed measure can be admitted, modeling negative interactions.
The measure $u$ represents the preference structure of the DM about the Expert's coalitions. For instance, consider a rule like the following one

*If the first and the second Experts score equally an alternative, then score this alternative with their common value, despite any other opinion of the remaining Experts of the group*

This rule can be implemented assigning one to the coalition formed by the Experts n. $1, 2$, that is $m\{X\} = 1$ if $A \cap X \neq \emptyset$ with $A = \{1, 2\}$.
The steps of the aggregation algorithms are then the following ones

i) $\forall k$, $1 \leq i \leq k$, aggregate each of the individual measure using the Choquet integral: $v_k(\mathbf{x}) = \int \mathbf{x}^k \, d\, m_k$, obtaining the individual score for the alternative ($\mathbf{x}$)

ii) at the top level (the DM level), aggregate the individual ranking, obtained at the previous step, using the measure $m$ on the space $2^K$ applying again the Choquet integral: $C(\mathbf{x}) = \int v(\mathbf{x}) \, d\, u$.

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Let us recall that the *relative importance* for monotone measure of a single criterion is usually measured by a suitable index like the Shapley index, [6]. The Shapley index for the $i$-th criterion and for the $k$-th Expert is given by

$$p_k(i) = \sum_{T \subseteq N \setminus \{i\}} \frac{(n_i - t - 1)!t!}{n_i!}[m_k(T \cup i) - m_k(T)] \quad (2)$$

with $t = card(T)$. The Shapley index measures the *average relative importance* of a criterion and varies between zero and one. In the case of non monotone measure, it can happen that the Shapley index is close to zero even if the criterion is *important*. This is due to possible conflicting interactions that compensate positive marginal gains with negative ones. To this purpose, in [1] the *extended* Shapley index was introduced:

$$q_k(i) = \sum_{T \subseteq N \setminus \{i\}} \frac{(n_i - t - 1)!t!}{n_i!}|m_k(T \cup i) - m_k(T)| \quad (3)$$

for the $i$-th criterion and for the $k$-th Expert. Both the *extended* Shapley indices measures the *relative* importance of the criterion. If the measure is monotone, the two indices coincide. Otherwise, it can happen that the Shapley index is close to zero, while the *extended* index is high. This means that the criterion is *important*, but, on average, it is *neutral*, it is neither a benefit, nor a cost (for some coalitions it is a cost, while for others it is a benefit). Then, in the non monotone case, both the two indices should be considered.
For a better comprehension of the proposed methodology, consider the following multi person - multi attribute decision problem. Suppose that an Expert Committee is required to evaluate among some different investment projects that are the finite set of the available alternatives. Each project is characterized by a finite set of criteria. Every Expert scores independently any coalition of the criteria. Anywise, the Experts are not forced to consider *all* the criteria. Conversely, each Expert can consider an his *own* subset of criteria. This preference is formalized by a non additive signed measure, which can model *strict conflict* between the criteria. For instance, an Expert can consider three criteria valid for the judgements of the alternatives, and even if all of them, considered by alone, can be seen as *benefit*, the subset of the first and the second one induce a conflict, so that such a coalition receive an inferior score than the minimum between the first and the second criterion scores, which is usually implied in the monotone case.

## 6 A numerical example

Consider a decision problem where two alternatives have to be scored, on the basis of the four criteria values (alternative *profile*) reported in Table 1.
Suppose that three Experts are involved in the decision process. Then the DM gives a final evaluation of the alternatives on the basis of the Expert's individual scores. As above described, the decision process splits into two subsequent phases: the Expert's scoring, and the aggregation of each Expert's score into an aggregated score. If signed non additive measures are used for the aggregation in both the activities, the problem can be approached with a two-step non monotone Choquet integral.

Table 1: Alternative profile

| $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|
| 0.4 | 0.2 | 0.8 | 0.6 |
| 0.1 | 0.7 | 0.4 | 0.5 |

Table 2: Expert n. 1

| $m^1(1)$ | $m^1(2)$ | $m^1(1,2)$ |
|---|---|---|
| 0.4 | 0.2 | 1 |

In the first phase, suppose that the first Expert takes only the first two criteria into account, the second discharges the first criterion, while the third Expert considers the first and the fourth criteria only. The values assigned by the three Experts directly assigned or implicitly obtained using a suitable questionnaire as proposed by [3] to every coalition are reported in Tables from 2 up to 4. The first Expert considers more important the first criterion with respect to the third one, but the coalition formed by both the two Experts receives a weight greater than the sum of the two criteria weights. Thus the first Expert exhibits a *synergic*, or *disjunctive*, effect. In MCDA literature, this is sometimes knew as *andness*-type effect, while the opposite redundant behavior is named *orness*-type effect, see [7]. The second Expert is characterized by a *linear* behavior, as it can be easily checked. Both the first and the second Expert evaluate monotonically. But this is not true for the last Expert, which considers the first criterion more important than the fourth, but assigns to the coalition formed by the two criteria an inferior weight than the minimum of them. While the first Expert exhibits a *disjunctive* behavior (*orness*-type), and the second is *linear*, we can say that the last Expert is characterized by an *exclusive-orness* behavior. In fact, his preference structure follows a rule like:

*The alternative is highly scored if the first or if the fourth criterion is high, but NOT both of them*

The scoring of each alternatives can be obtained aggregating using the (single step) Choquet integral for each Expert. The Table 5 reports the results of the aggregation for the three Experts. For instance, the score of the first alternative calculated for the first Expert, 0.28, is obtained as follows, see (1):

$$\mathbf{x}_1^1 = 0.2 \times 1 + (0.4 - 0.2) \times 0.4 = 0.28 \quad (4)$$

In the subsequent phase, the individual Expert's scores need to be aggregated into a final one. To this purpose, suppose that, after a preliminary briefing, the implicit preference struc-

Table 3: Expert n. 2

| $m^2(2)$ | $m^2(3)$ | $m^2(4)$ |
|---|---|---|
| 0.5 | 0.2 | 0.3 |

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| $m^2(2,3)$ | $m^2(2,4)$ | $m^2(3,4)$ | $m^2(2,3,4)$ |
|---|---|---|---|
| 0.7 | 0.8 | 0.5 | 1 |

Table 4: Expert n. 3

| $m^3(1)$ | $m^3(4)$ | $m^3(1,4)$ |
|---|---|---|
| 0.7 | 0.6 | 0.2 |

ture of the DM about the Expert's confidence/experience be heuristically expressed by the following rules:

Rule # 1: *If the Experts 1 and 3 agree, I have complete confidence about their choice (in this case, the opinion of the second Expert is inessential).*

Rule # 2: *The coalition formed by Experts 1 and 2 is more reliable than the coalition formed by Experts 2 and 3.*

Rule # 3: *Considering each Experts by alone, the first one is the most important, while the second one is the least.*

These rules can be translated in the coalition weights reported in Table 6.

The individual measures can now be aggregated, obtaining the following aggregated alternative scores for the two considered alternatives:

$\sigma_1 = 0.648$, $\sigma_2 = 0.344$

The score $\sigma_1$ is computed as follows:

$$\sigma_1 = 0.2\times 1 + (0.28 - 0.2)\times 0.5 + (0.44 - 0.28)\times 0.3 = 0.648 \quad (5)$$

While for the first and for the second Experts the Shapley and the *extended* Shapley indices coincide (they are monotone), for the third Expert the Shapley index and the *extended* Shapley index are respectively, for the first criterion:

$$p_3 = 0.5 \times \{0.7 + (0.2 - 0.6)\} = 0.15 \quad (6)$$

$$q_3 = 0.5 \times \{0.7 + |0.2 - 0.6|\} = 0.55 \quad (7)$$

Observe that the Shapley index differs from the *extended* Shapley indices. In particular, the extended Shapley index is significatively high (0.55), while the Shapley index is low (0.15). We can conclude that this criterion of the third Expert is *important* even if it is, on average, neither a *benefit*, nor a *cost*.
In a future work, we intend to analyze the properties and the relationships between the Shapley and the *extended* Shapley for the two-step Choquet integral.

Table 5: Alternative scoring of each Expert

| **Alternatives** | Expert n. 1 | Expert n. 2 | Expert n. 3 |
|---|---|---|---|
| **1** | 0.28 | 0.44 | 0.2 |
| **2** | 0.22 | 0.54 | 0.26 |

Table 6: Measure of Expert coalitions

| $u_1$ | $u_2$ | $u_3$ | $u_{1,2}$ | $u_{1,3}$ | $u_{2,3}$ |
|---|---|---|---|---|---|
| 0.2 | 0.3 | 0.7 | 0.5 | 0.7 | 1 |

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