Discussions on Interpretability of Fuzzy Systems using Simple Examples

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Abstract*—Two conflicting goals are often involved in the design of fuzzy rule-based systems: Accuracy maximization and interpretability maximization. A number of approaches have been proposed for finding a fuzzy rule-based system with a good accuracy-interpretability tradeoff. Formulation of the accuracy maximization is usually straightforward in each application area of fuzzy rule-based systems such as classification, regression and forecasting. Formulation of the interpretability maximization, however, is not so easy. This is because various aspects of fuzzy rule-based systems are related to their interpretability. Moreover, user's preference should be taken into account when a single fuzzy rule-based system is to be chosen from several alternatives with different accuracy-interpretability tradeoffs. In this paper, we discuss the difficulty in measuring the interpretability of fuzzy rule-based systems using very simple examples. We do not intend to propose any new interpretability measure. Our intention is to help to activate discussions on how to measure the interpretability of fuzzy rule-based systems.*

Keywords—Fuzzy systems, fuzzy rules, accuracy-interpretability tradeoff, multiobjective design of fuzzy systems.

1 Introduction

Handling of the tradeoff between the accuracy maximization and the interpretability maximization has been a hot issue in the design of fuzzy rule-based systems since the mid-1990s [1]-[3]. A number of approaches have already been proposed for improving the accuracy of fuzzy rule-based systems while maintaining their interpretability [1], [2], [4]-[21]. Genetic algorithms have been frequently used in those approaches to search for an accurate and interpretable fuzzy rule-based system. This is because genetic algorithms can perform not only continuous optimization for parameter tuning but also discrete optimization for structure determination. Studies on fuzzy genetics-based machine learning are called genetic fuzzy systems [22]-[24]. In some recent studies [3], [25]-[36], multi-objective genetic algorithms have been used to search for multiple Pareto-optimal fuzzy rule-based systems along the accuracy-interpretability tradeoff surface. Those studies are often referred to as multi-objective genetic fuzzy systems [37]. Recently multi-objective genetic algorithms have also been used for machine learning [38] and data mining [39].

Let us denote a fuzzy rule-based system by S. We can also view S as a set of fuzzy if-then rules. In each application area of fuzzy rule-based systems such as classification, regression and forecasting, the specification of the accuracy of S for the given training data is not difficult (e.g., the number of correctly classified training patterns by S). Let us denote the accuracy measure of S as $Accuracy(S)$. A design problem of fuzzy rule-based systems can be formulated as follows:

$$\text{Maximize } Accuracy(S). \tag{1}$$

Due to the accuracy-interpretability tradeoff relation, the accuracy maximization in (1) often leads to the deterioration in the interpretability of fuzzy rule-based systems. This means that we often obtain from (1) an accurate and complicated fuzzy rule-based system with poor interpretability.

In some application areas, not only the accuracy but also the interpretability is very important. Thus we often want to maximize the accuracy of fuzzy rule-based systems without degrading their interpretability. This maximization problem can be formulated as follows:

$$\text{Maximize } Accuracy(S) \text{ subject to } Interpretability(S) \geq \alpha, \tag{2}$$

where $Interpretability(S)$ is the interpretability measure of the fuzzy rule-based system S and α is the required minimum level of the interpretability.

Of course, we can formulate the maximization problem of the interpretability under the given minimum accuracy level β as follows:

$$\text{Maximize } Interpretability(S) \text{ subject to } Accuracy(S) \geq \beta. \tag{3}$$

One may want to maximize both the accuracy and the interpretability. In this case, a simple approach is to use a scalarizing function $f(\cdot)$ which combines the accuracy and interpretability measures into a single objective function:

$$\text{Maximize } f(Accuracy(S), Interpretability(S)). \tag{4}$$

A well-known scalarizing function is the weighted sum:

$$\text{Maximize } w_1\, Accuracy(S) + w_2\, Interpretability(S), \tag{5}$$

where w_1 and w_2 are non-negative weight values. In addition to the weighted sum in (5), we can use various scalarizing functions developed in the field of multiple criteria decision making (MCDM [40]-[42]).

In general, it is not easy for human users to specify an appropriate scalarizing function for multi-objective problems. Users may want to examine some fuzzy rule-based systems with different accuracy-interpretability tradeoffs (instead of a single best solution with respect to a specific scalarizing function). In this case, the design of fuzzy rule-based systems can be formulated as the following multi-objective problem:

$$\text{Maximize } \{Accuracy(S), Interpretability(S)\}. \tag{6}$$

A large number of Pareto-optimal fuzzy rule-based systems can be obtained by multi-objective genetic algorithms such as NSGA-II [43], SPEA [44] and SPEA2 [45].

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In many cases, the interpretability maximization is handled as the complexity minimization. Thus the above-mentioned formulations in (2)-(6) can be reformulated accordingly. For example, the multi-objective formulation in (6) is rewritten as

Maximize *Accuracy*(*S*) and minimize *Complexity*(*S*), (7)

where *Complexity*(*S*) is a complexity measure.

The main difficulty in the above-mentioned formulations in (2)-(7) for the design of accurate and interpretable fuzzy rule-based systems is the formulation of their interpretability. Whereas the formulation of the accuracy of fuzzy rule-based systems is usually straightforward from their application task such as classification and regression, it is not easy for human users to appropriately formulate the interpretability. This is because various aspects of fuzzy rule-based systems are related to their interpretability [46]-[52]. Moreover, it is not easy for human users to mathematically formulate each of those aspects even when the close relation of each aspect to the interpretability of fuzzy rule-based systems is clear.

In this paper, we explain the difficulty in formulating the interpretability of fuzzy rule-based systems using simple numerical examples. More specifically, we demonstrate the difficulty in comparing different fuzzy rule-based systems with respect to their interpretability even in very simple situations.

2 Interpretability of Fuzzy Partitions

When we use the same type of fuzzy partitions with different granularities (e.g., uniform fuzzy partitions with symmetric triangular membership functions (MFs) in Fig. 1), we can say that the increase in the number of membership functions degrades the interpretability of fuzzy partitions. For example, the fuzzy partition with two membership functions in Fig. 1 (a) is the most interpretable among the four alternatives in Fig. 1. In this case, we can formulate the interpretability of fuzzy partitions by the number of membership functions. That is, the interpretability maximization is realized by minimizing the number of membership functions in fuzzy partitions.

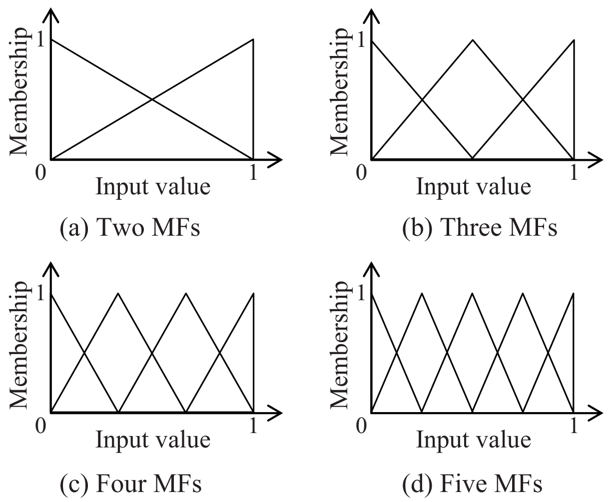

Figure 1: Fuzzy partitions with different granularities.

We can also use the number of membership functions as an interpretability measure (i.e., as a complexity measure to be minimized) when the same fuzzy partition is used for all input variables. For example, we can say that the 3x3 fuzzy grid in Fig. 2 (a) is more interpretable than the 4x4 fuzzy grid in Fig. 2 (b). The comparison, however, becomes difficult when we use different fuzzy partitions for each input variable as shown in the following two examples.

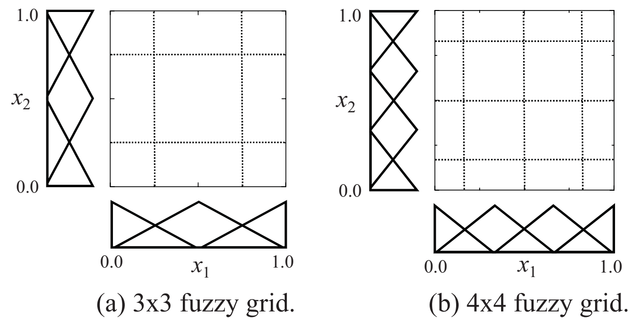

Figure 2: Comparison between the 3x3 and 4x4 fuzzy grids.

Example 1: Let us consider the 4x4 and 3x5 fuzzy grids in Fig. 3. Both fuzzy grids have eight membership functions in total (i.e., 4+4 = 3+5). Thus they are evaluated as having the same interpretability if they are compared using the number of membership functions. The 3x5 fuzzy grid is, however, viewed as being more interpretable than the 4x4 fuzzy grid if we use the number of fuzzy subspaces as an interpretability measure (i.e., 15 < 16). Since a single fuzzy rule is usually generated for each fuzzy subspace, the 3x5 fuzzy grid in Fig. 3 (b) can be viewed as being more interpretable than the 4x4 fuzzy grid in Fig. 3 (a) if we evaluate the interpretability using the number of fuzzy rules. Some human users, however, may intuitively feel that the 4x4 fuzzy grid with the same fuzzy partition for the two input variables is more interpretable than the 3x5 fuzzy grid with the different fuzzy partitions.

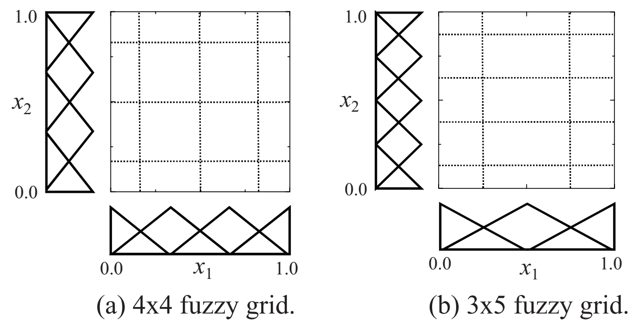

Figure 3: Example 1 with the 4x4 and 3x5 fuzzy grids.

Example 2: Let us consider the 5x5 and 3x8 fuzzy grids in Fig. 4. The 5x5 fuzzy grid in Fig. 4 (a) has less membership functions than the 3x8 fuzzy grid in Fig. 4 (b): 5+5 < 3+8. Thus the 5x5 fuzzy grid is evaluated as more interpretable than the 3x8 fuzzy grid in Fig. 4 (b) if we use the number of membership functions as an interpretability measure. The 5x5 fuzzy grid, however, has more fuzzy subspaces than the 3x8 fuzzy grid: 25 > 24. Thus the 3x8 fuzzy grid is viewed as being more interpretable than the 5x5 fuzzy grid if we use the number of fuzzy subspaces (i.e., the number of fuzzy rules) as an interpretability measure.

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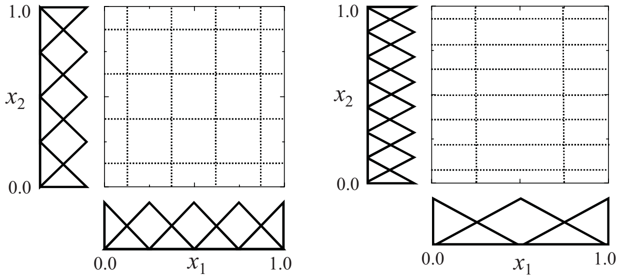

(a) 5x5 fuzzy grid. (b) 3x8 fuzzy grid.

Figure 4: Example 2 with the 5x5 and 3x8 fuzzy grids.

As we have already explained in this section using the two examples, the comparison of different fuzzy grids with respect to their interpretability is not so easy. This means that the choice of an interpretability measure is difficult. If we use the number of fuzzy subspaces in Fig. 4, the 3x8 fuzzy grid is viewed as being more interpretable than the 5x5 fuzzy grid. When we use not only the number of fuzzy subspaces but also the number of membership functions, these two fuzzy grids are viewed as being non-dominated with each other with respect to the interpretability. In this case, the multi-objective formulation in (7) is handled as the three-objective problem:

$$\text{Maximize } Accuracy(S), \text{ and} \\ \text{minimize } \{Complexity_1(S), Complexity_2(S)\}, \quad (8)$$

where $Complexity_1(S)$ and $Complexity_2(S)$ are different complexity measures to be minimized (e.g., the number of fuzzy subspaces and the number of membership functions).

Even if we use these two measures, the 3x5 fuzzy grid is viewed as more interpretable than the 4x4 fuzzy grid. Thus we need another measure if we want to include some bias toward fuzzy grids with the same fuzzy partition for all input variables. For example, the 4x4 fuzzy grid is evaluated as more interpretable than the 3x5 fuzzy grid if the maximum number of membership functions for each input variable is used as an interpretability measure ($\max\{4, 4\} < \max\{3, 5\}$).

3 Interpretability of Fuzzy Rule-Based Systems

In genetic fuzzy systems [22]-[24], almost all aspects of fuzzy rule-based systems can be optimized since genetic algorithms perform continuous, discrete and combinatorial optimization. For example, genetic fuzzy systems can be used for choosing an appropriate type of fuzzy rules (e.g., Takagi-Sugeno, simplified Takagi-Sugeno and Mamdani). In this section, we discuss the interpretability of fuzzy rule-based systems with different fuzzy partitions and different types of fuzzy rules.

Let us consider a simple function approximation problem of a single-input and single-output system $y = f(x)$ in Fig. 5. Our task is to design an accurate and interpretable fuzzy rule-based system from the given input-output data in Fig. 5. For this task, Takagi-Sugeno fuzzy rules are written as

$$\text{Rule } R_i : \text{If } x \text{ is } A_i \text{ then } y = a_i + b_i x\,,\ i = 1, 2, ..., N\,, \quad (9)$$

where i is a rule index, A_i is an antecedent fuzzy set, a_i and b_i are real number coefficients of a consequent linear function of each fuzzy rule, and N is the total number of fuzzy rules.

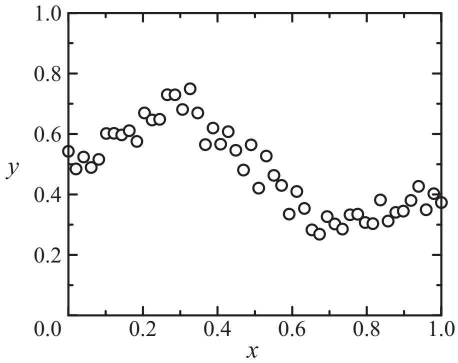

Figure 5: Input-output data in Examples 3 and 4.

When an input value x is presented to the fuzzy rule-based system with the N fuzzy rules in (9), the output value is estimated as follows:

$$y(x) = \frac{\sum_{i=1}^{N} (a_i + b_i x) \cdot \mu_{A_i}(x)}{\sum_{i=1}^{N} \mu_{A_i}(x)}\,, \quad (10)$$

where $y(x)$ is the estimated output value for the input value x, and $\mu_{A_i}(x)$ is the membership value of the antecedent fuzzy set A_i for the input value x.

We can use the same fuzzy reasoning mechanism for the simplified version of Takagi-Sugeno fuzzy rules:

$$\text{Rule } R_i : \text{If } x \text{ is } A_i \text{ then } y \text{ is } h_i\,,\ i = 1, 2, ..., N\,, \quad (11)$$

where h_i is a consequent real number.

Example 3: From the input-output data in Fig. 5, one may think that they can be approximated by a fuzzy rule-based system with three Takagi-Sugeno fuzzy rules. An example of such a fuzzy rule-based system is shown in Fig. 6 where each of the three lines (1), (2) and (3) is the consequent linear function of each of the three fuzzy rules with the trapezoidal antecedent fuzzy sets A_1, A_2 and A_3. The same input-output data can be also approximated by a fuzzy rule-based system with four simplified Takagi-Sugeno fuzzy rules as shown in Fig. 7. Each fuzzy rule in Fig. 7 has a triangular membership function A_i and a consequent real number h_i. The question is which is more interpretable between the two fuzzy rule-based systems in Fig. 6 and Fig. 7.

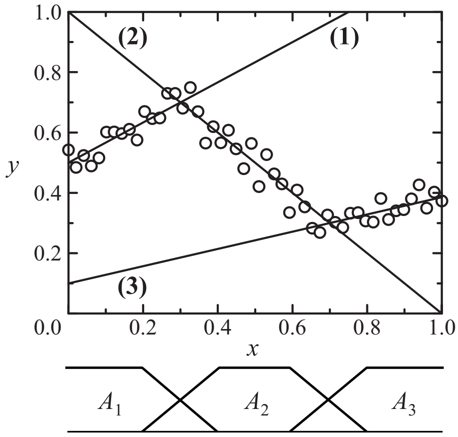

Figure 6: Three Takagi-Sugeno fuzzy rules.

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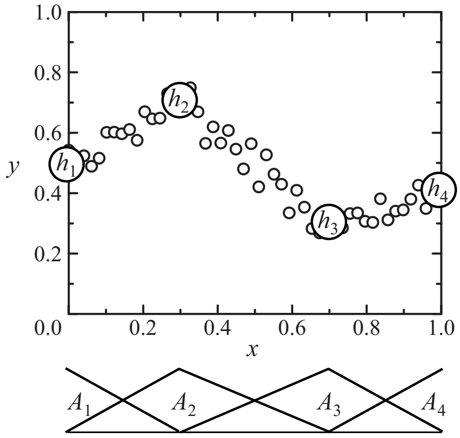

Figure 7: Four simplified Takagi-Sugeno fuzzy rules.

In Fig. 8 and Fig. 9, we show the fuzzy reasoning results by these two fuzzy rule-based systems in Fig. 6 and Fig. 7, respectively. We can see that similar results were obtained from the two fuzzy rule-based systems. Since there is no large difference in the approximation accuracy between Fig. 8 and Fig. 9, the interpretability will play an important role in the selection between the two fuzzy models in Fig. 6 and Fig. 7. If we use the number of fuzzy rules as an interpretability measure, the Takagi-Sugeno model in Fig. 6 is viewed as being more interpretable than the simplified Takagi-Sugeno model in Fig. 7. On the other hand, if we use the total number of parameters (i.e., a_i , b_i and h_i) in the consequent part of the fuzzy rules as an interpretability measure, the simplified Takagi-Sugeno model is evaluated as more interpretable than the Takagi-Sugeno model (i.e., 4 < 6).

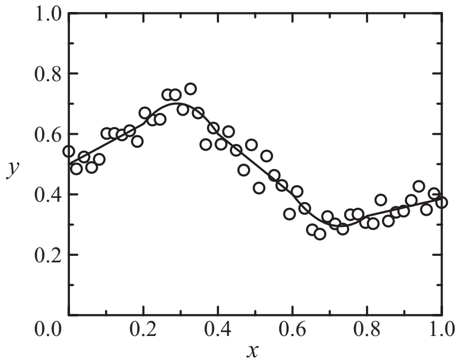

Figure 8: Results by the three Takagi-Sugeno fuzzy rules.

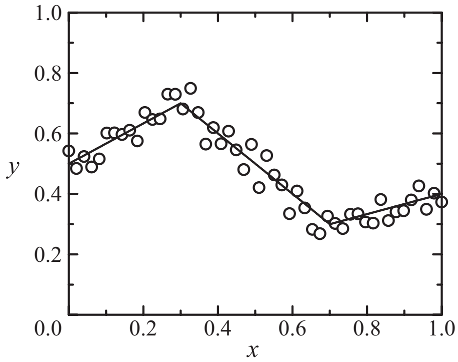

Figure 9: Results by the four simplified fuzzy rules.

Example 4: The given input-output data in Fig. 5 can be also approximated by the two Takagi-Sugeno fuzzy rules in Fig. 10. Whereas data points around $x = 0.5$ are far from the two consequent linear functions in Fig. 10, they can be approximated through the interpolation mechanism of the fuzzy reasoning in Eq. (10). Fig. 11 is the fuzzy reasoning result by the two fuzzy rules in Fig. 10. We can see from Fig. 11 that good approximation was realized by the two fuzzy rules in Fig. 10. Actually, the fuzzy reasoning result in Fig. 11 by the two Takagi-Sugeno fuzzy rules is similar to Fig. 8 and Fig. 9. The question is which is more interpretable between Fig. 6 with the three rules and Fig. 10 with the two rules.

It is clear that Fig. 10 is simpler than Fig. 6 with respect to various aspects of fuzzy rule-based systems (e.g., the number of fuzzy rules, the number of membership functions, and the number of parameters). However, one may think that Fig. 6 is more intuitive than Fig. 10. If we use the local accuracy of each liner function [5] as an interpretability measure, the three Takagi-Sugeno fuzzy rules in Fig. 6 are evaluated as being more interpretable than the two rules in Fig. 10.

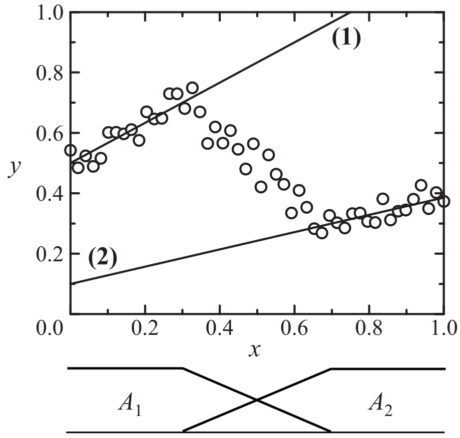

Figure 10: Two Takagi-Sugeno fuzzy rules.

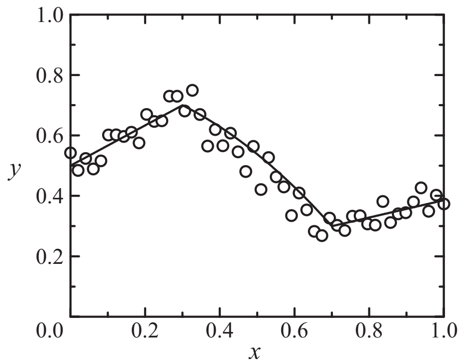

Figure 11: Results by the two Takagi-Sugeno fuzzy rules.

Example 5: We can use different types of fuzzy rules in a single fuzzy rule-based system. We show an example of such a fuzzy rule-based system in Fig. 12 where the second fuzzy rule with the antecedent fuzzy set A_2 has a consequent linear function (line (2)). Each of the other two fuzzy rules with the antecedent fuzzy sets A_1 and A_3 has a consequent real number. As we can expect, good approximation was realized by these three fuzzy rules (due to the page limitation, we can not show

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the fuzzy reasoning result). The given input-output data in Fig. 12 can be also approximated with a similar accuracy by the four simplified Takagi-Sugeno fuzzy rules in Fig. 13. Since the first two fuzzy rules with the antecedent fuzzy sets A_1 and A_2 in Fig. 13 have the same consequent real number, they can be merged into a single rule. The last two fuzzy rules with A_3 and A_4 in Fig. 13 can be also merged into a single rule. As a result, we have a fuzzy rule-based system with the two simplified Takagi-Sugeno fuzzy rules in Fig. 14.

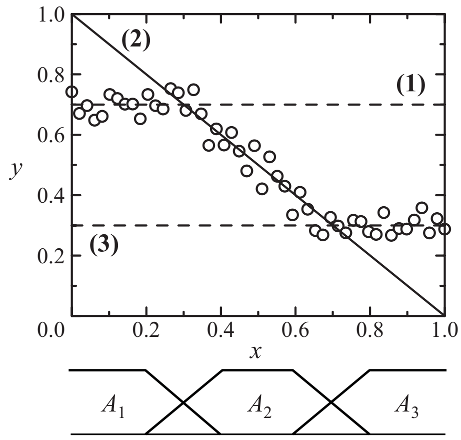

Figure 12: A Takagi-Sugeno rule and two simplified rules.

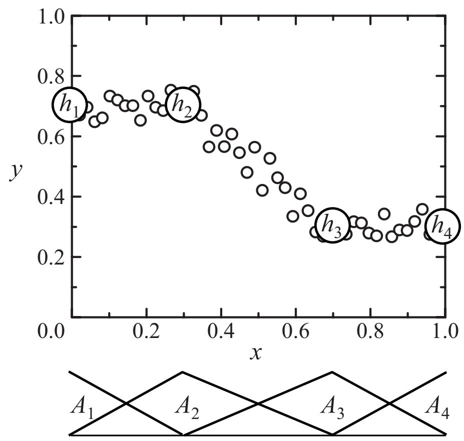

Figure 13: Four simplified Takagi-Sugeno rules.

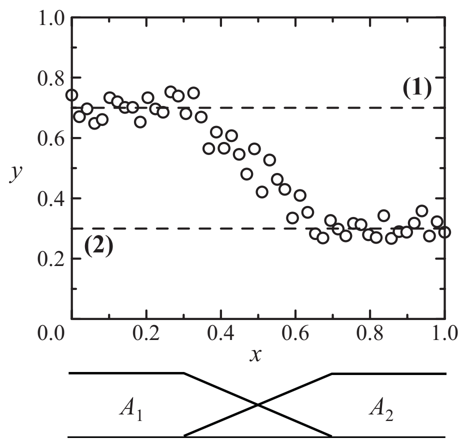

Figure 14: Two simplified Takagi-Sugeno rules.

It is clear that the fuzzy rule-based system with only the two fuzzy rules in Fig. 14 is the simplest one in Figs. 12-14. However, we have no definite answer to the question: Which is the most interpretable in the three models in Figs. 12-14?

4 Conclusions

In this paper, we demonstrated the difficulty in evaluating the interpretability of fuzzy rule-based systems. As shown in this paper, the evaluation of the interpretability is difficult even in very simple situations. Different fuzzy rule-based systems are viewed as being more interpretable according to different interpretability measures. This means that the choice of an appropriate interpretability measure is important in the design of fuzzy rule-based systems. At the same time, such a choice is difficult as shown in this paper. We hope that this paper will activate discussions on the interpretability and help to develop new approaches to fuzzy modelling based on the accuracy-interpretability tradeoff analysis.

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