Stability Analysis of Continuous PI-like Fuzzy Control Systems based on Vector Norms Approach

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Abstract*—This paper proposes a practical stability analysis of a particular class of continuous PI-like fuzzy control systems based on the convergence of regular vector norms. This approach is based on the comparison, the overvaluing principle and the application of Borne and Gentina criterion. The stability conditions issued from vector norms correspond to a vector Lyapunov function. A comparison system relative to a regular vector norm will be used in order to get the simple arrow form of the state matrix that yields to a suitable use of Borne and Gentina criterion for establishing sufficient conditions for global asymptotic stability.*

Keywords—Continuous PI-like fuzzy control systems, global asymptotic stability, vector norms, Borne and Gentina criterion, Arrow form state matrix.

1 Introduction

Over the last few decades, fuzzy control has received great attention from engineering and science community and has marked rapid growth. However, there isn't a general method for analysis or synthesis of such control strategy. In particular, stability analysis of control systems is an essential step before synthesizing and elaborating the control law.

In this way, many researches have been done on the stability study of fuzzy control systems since their appearance in the middle of the 70's [17].

Indeed, the majority of these studies concern TSK fuzzy systems since consequent fuzzy variables of this type (known as type III) are explicit and many important results have been obtained [13, 18, 23] based on Lyapunov stability theory. Nevertheless, there have been a limited number of stability studies of Mamdani type fuzzy systems (known as type I) due to their complexity, and for this type, fuzzy variables are linguistically understandable in both the premises and consequents. Most of these papers provide a stability analysis of a linear plant controlled by a fuzzy controller and they regard the latter as a nonlinear controller corresponding to a Lur'e system. So, the stability problem of fuzzy control systems comes down to conventional nonlinear stability theory.

Among the methods that have been used, Popov's theorem for time-invariant nonlinearity [16], circle criterion for time-variant nonlinearity [19, 20] and its extended version, the conicity criterion [1].

Other approaches have been used: ones can cite: the hyperstability approach which is equivalent to passivity [9, 10, 24], the input-output stability based on the use of small gain theorem [11, 25] and the Lyapunov function approach [8, 12].

We do remark also that the majority of these papers simplify the study by considering the consequents of the fuzzy system as singletons [22]. So, this type (known as type II) is a special case of both TSK fuzzy systems and those of Mamdani.

In this paper, the stability study of particular class of fuzzy PI controllers of type Mamdani is presented. This study is based on the application of the Borne and Gentina criterion [6] which uses Kotelyanski conditions. In [3], it has been shown that when system state matrix is in arrow form, then Borne and Gentina criterion becomes very simple to apply.

This approach has been used in many previous works [2, 4, 5, 21]. In this way, results proposed in [21] will be used and generalized for stability analysis of continuous Mamdani fuzzy systems in the case of nonlinear processes to be controlled.

The paper is organised as follows. The next section is devoted to the description of the particular class of PI-like fuzzy control system. In section 3, stability conditions of the fuzzy system are established by using Borne and Gentina criterion and vector norms approach. The stability conditions proposed are illustrated with an example presented in section 4. Finally, concluding remarks are drown in section 5.

2 Particular class of PI-like fuzzy controllers

The fuzzy PI control system considered in this study has two inputs the error e and its derivative de and one output the control derivative as shown in Fig. 1, where k_e, k_{de} et k_{du} are scale factors.

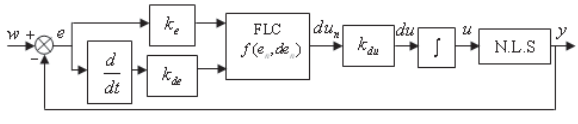

Figure 1: Fuzzy control system.

A particular class of fuzzy PI controllers of type Mamdani is obtained by considering a strong triangular partition of the normalized variables e_n, de_n and du_n presented in Fig. 2.

The rule base considered is an $r \times r$ traditional rule table that is of antidiagonal type such that the Mac Vicar-Whelan one (Table 1).

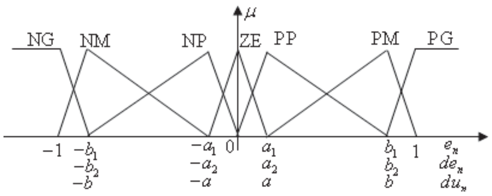

Figure 2: Fuzzy subsets partition.

Table 1 : The Mac Vicar-Whelan rule base.

e_n \ de_n	NG	NM	NP	ZE	PP	PM	PG
NG	NG	NG	NG	NG	NM	NP	ZE
NM	NG	NG	NM	NM	NP	ZE	PP
NP	NG	NM	NP	NP	ZE	PP	PM
ZE	NG	NM	NP	ZE	PP	PM	PG
PP	NM	NP	ZE	PP	PP	PM	PG
PM	NP	ZE	PP	PM	PM	PG	PG
PG	ZE	PP	PM	PG	PG	PG	PG

Let $\sigma(e_n, de_n)$ the surface in the space (e_n, de_n, du_n), verifying the two properties [21]:

$$\begin{cases} \text{i) If } \sigma = 0 \text{ then the input-output characteristic surface } du_n(e_n, de_n) = 0 \\ \text{ii) It exists } k{>}0 \text{ such as } du_n(k\sigma - du_n) \geq 0 \text{ for all } e_n \text{ and } de_n. \end{cases} \quad (1)$$

The first property means that the intersection of the overvaluing surface σ with the plan (e_n, de_n) is a part of the intersection of the characteristic surface $du_n(e_n, de_n)$ with the same plan. The curve $\sigma = 0$ is a straight line when the fuzzy input partition is identical for the inputs and it represents the second bissectrix as shown in Fig. 3.

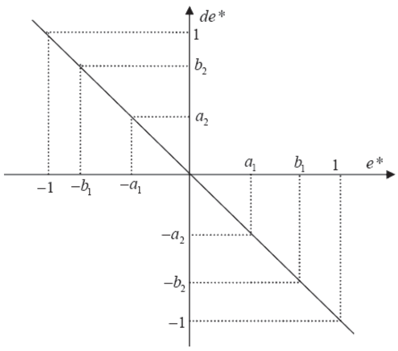

Figure 3: Curve $\sigma = 0$.

According to [21], in the case of an identical partition for the two inputs e_n and de_n (i.e. $a_1 = a_2$ and $b_1 = b_2$) and when using an antidiagonal rule table like the Mac-Vicar Whelan one. This approach is based on the overvaluing and the undervaluing of the fuzzy controller characteristic surface $du_n = (e_n, de_n)$ by two plans crossing the plan (e_n, de_n) in the second bissectrix which is the curve $\sigma = 0$, in this case given by the following equation:

$$e_n + de_n = 0 \quad (3)$$

The slopes of these plans are respectively $k_{\max}$ and $k_{\min}$, those values depend on the geometric parameters of the input variables e_n and de_n $(a_i \text{ and } b_i, i = 1,2)$ and the output variable du_n $(a \text{ and } b)$, on the other hand they depend on the used inference method.

In this way for each point (e_n, de_n) of $[-1,1] \times [-1,1]$ the controller output du_n is such that:

$$k_{\min}(e_n + de_n) \leq du_n \leq k_{\max}(e_n + de_n) .$$

Otherwise $du_n = f(.)(e_n + de_n)$, in such way we can write:

$$k_{\min} \leq f(.) \leq k_{\max} \quad (4)$$

where $f(.)$ is a nonlinear gain.

By programming, we can obtain values of $k_{\max}$ and $k_{\min}$ that allow respectively overvaluing and undervaluing the characteristic surface of the fuzzy controller by PI plans.

In particular, for equidistant fuzzy partition of variables e_n, de_n and du_n $(a_1 = a_2 = a = 0.33 \text{ and } b_1 = b_2 = b = 0.67)$, max-min inference method and centroid defuzzification method, we obtain the action surface for the fuzzy controller given by Fig. 4.

This surface is obtained by distorting the discourse universes of e_n and de_n to 100 points, we obtain:

$$k_{\max} = 1{,}86 \text{ and } k_{\min} = 0{,}47 \quad (5)$$

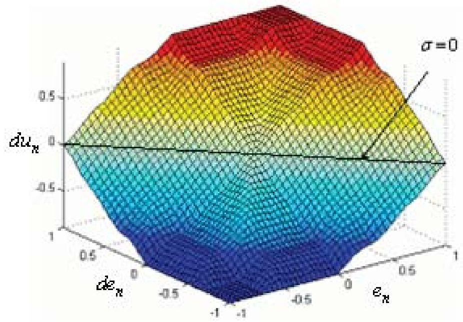

Figure 4: Action surface of the particular Mamdani fuzzy controller.

So the PI fuzzy control system of the Fig. 1 can be set in the following form (Fig. 5):

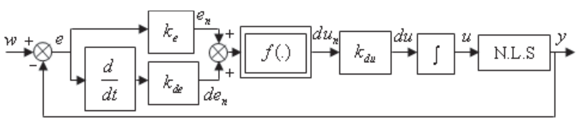

Figure 5: Equivalent fuzzy control system.

3 Proposed stability conditions

3.1 Problem formulatiom

Supposing that the system to be controlled which is nonlinear is represented by the following state matrix given in the Frobenius form such that:

$$\begin{cases} \dot{x} = A(.)x + B(.)u \\ y = C(.)x \end{cases} \quad x \in \mathbb{R}^n \quad (6.a)$$

where:

$$A(.) = \begin{bmatrix} 0 & \dots & 0 & -a_n(.) \\ 1 & \ddots & \vdots & \vdots \\ \vdots & \ddots & 0 & \vdots \\ 0 & \dots & 1 & -a_1(.) \end{bmatrix}, B(.) = \begin{bmatrix} b_n(.) \\ \vdots \\ \vdots \\ b_1(.) \end{bmatrix}, C^T(.) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (6.b)$$

From the diagram given in Fig. 5, we have $du = \dot{u}$, supposing that $v = du = \dot{u}$ and $\xi = u$, which leads to : $\dot{\xi} = v$, the nonlinear system equipped with the integration can be represented by the following state matrix:

ISBN: 978-989-95079-6-8

$$\dot{z} = \begin{bmatrix} A(.) & B(.) \\ 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix} v \quad z \in \mathbb{R}^{n+1} \tag{7}$$

where: $z = \begin{bmatrix} x \\ \xi \end{bmatrix}$ and $\dot{z} = \begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix}$

supposing that: $A'(.) = \begin{bmatrix} A(.) & B(.) \\ 0 & 0 \end{bmatrix}$ and $B'(.) = B' = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$.

So we can write $y = x_n = z_n$ where x_n (respectively z_n) is the n^{th} state variable of state vector x (respectively z), thus: $y = C'(.)z$ where $C'(.) = C' = \begin{bmatrix} 0 & \dots & 1 & 0 \end{bmatrix}$

On the other hand, and by referring to the diagram in Fig. 5 in the autonomous regime, we can write:

$$\begin{aligned} v &= k_{du} f(.)\left[e_n + de_n\right] = -k_{du} f(.)\left[k_e e + k_{de}\dot{e}\right] \\ &= -k_{du} f(.)\left[k_e y + k_{de}\dot{y}\right] \end{aligned} \tag{8}$$

finally: $v = -k_{du} f(.)\left[k_e C'(.)z + k_{de} C'(.)\dot{z}\right]$

then:

$$\begin{aligned} \dot{z} &= A'(.)z + B'(.)v = A'(.)z - B'(.)k_{du} f(.)\left[k_e C'z + k_{de} C'\dot{z}\right] \\ &= A'(.)z - k_{du} f(.)\left[k_e B'C'z + k_{de} B'C'\dot{z}\right] \end{aligned}$$

that leads to:

$$\left[I_{n+1} + k_{du}k_{de} f(.)B'C'\right]\dot{z} = \left[A'(.) - k_{du}k_e f(.)B'C'\right]z \tag{9}$$

supposing now: $\begin{cases} N = I_{n+1} + k_{du}k_{de} f(.)B'C' \\ M = A'(.) - k_{du}k_e f(.)B'C' \end{cases}$

let:

$$N = \begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ 0 & 1 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & 0 & \ddots & 0 \\ 0 & \dots & 0 & k_{du}k_{de} f(.) & 1 \end{bmatrix}, M = \begin{bmatrix} 0 & \dots & 0 & -a_n(.) & b_n(.) \\ 1 & \ddots & & \vdots & \vdots \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ \vdots & & 1 & -a_1(.) & b_1(.) \\ 0 & \dots & 0 & -k_{du}k_e f(.) & 0 \end{bmatrix}$$

where: $\det(N) = 1$ and $N^{-1} = \begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ 0 & 1 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & 0 & \ddots & 0 \\ 0 & \dots & 0 & -k_{du}k_{de} f(.) & 1 \end{bmatrix}$

Finally we obtain the following description of the closed loop system:

$$\dot{z} = A_C(.)z \tag{10}$$

where:

$$\begin{aligned} A_C(.) &= N^{-1}M \\ &= \begin{bmatrix} 0 & \dots & \dots & -a_n(.) & b_n(.) \\ 1 & & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ \vdots & & 1 & -a_1(.) & b_1(.) \\ 0 & \dots & -k_{du}k_{de} f(.) & k_{du} f(.)(k_{de}a_1(.) - k_e) & -k_{du}k_{de} f(.)b_1(.) \end{bmatrix} \end{aligned} \tag{11}$$

For establishing stability conditions for the studied system, we have to make a basic change leading to new representation of the system to get best exploitation.

3.2 New state representation of the system

We consider the following passage matrix P allowing passing from the matrix $A_C(.)$ to a matrix $A_C^{'}(.)$, such that:

$$P = \begin{bmatrix} 1 & \alpha_1 & \dots & (\alpha_1)^{n-1} & 0 \\ \vdots & \dots & & \vdots & \vdots \\ 1 & \alpha_{n-1} & \dots & (\alpha_{n-1})^{n-1} & \vdots \\ 0 & \dots & & 1 & 0 \\ 0 & \dots & \dots & 0 & 1 \end{bmatrix} \tag{12}$$

We note: $z' = Pz$ and let $z = P^{-1}z'$.

In this way:

$$\dot{z}' = PA_C(.)P^{-1}z' = A_C^{'}(.)z' \tag{13.a}$$

where: $A_C^{'}(.) = PA_C(.)P^{-1}$

the matrix $A_C^{'}(.)$ is given by:

$$A_C^{'}(.) = \begin{bmatrix} \alpha_1 & 0 & \dots & \delta_1(.) & \upsilon_1(.) \\ \vdots & \ddots & & \vdots & \vdots \\ 0 & \dots & \alpha_{n-1} & \delta_{n-1}(.) & \upsilon_{n-1}(.) \\ \beta_1 & \dots & \beta_{n-1} & \sigma(.) & b_1(.) \\ \lambda_1(.) & \dots & \lambda_{n-1}(.) & \varphi(.) & \psi(.) \end{bmatrix} \tag{13.b}$$

where:

$$\beta_i = \frac{1}{\prod_{\substack{j=1 \\ j \neq i}}^{n} (\alpha_i - \alpha_j)} \quad \forall i = 1,...,n-1 \tag{13.c}$$

$$\begin{cases} \delta_i(.) = -D(.,\alpha_i) & \forall i = 1,...,n-1 \\ D(.,\alpha) = \alpha^n + \sum_{j=1}^{n} a_j(.)\alpha^{n-j} \end{cases} \tag{13.d}$$

$$\begin{cases} \upsilon_i(.) = N(.,\alpha_i) & \forall i = 1,...,n-1 \\ N(.,\alpha) = \sum_{j=1}^{n} b_j(.)\alpha^{n-j} \end{cases} \tag{13.e}$$

$$\sigma(.) = -a_1(.) - \sum_{j=1}^{n-1} \alpha_j \tag{13.f}$$

$$\begin{cases} \lambda_i(.) = -k_{du}k_{de} f(.)\beta_i & \forall i = 1,...,n-1 \\ \varphi(.) = k_{du}k_{de} f(.)\left[a_1(.) + \sum_{j=1}^{n-1} \alpha_j\right] - k_{du}k_e f(.) \\ \psi(.) = -k_{du}k_{de} f(.)b_1(.) \end{cases} \tag{13.g}$$

By observing the matrix $A_C^{'}(.)$, we remark that the nonnull elements are located in the diagonal and in both two last rows and two last columns, this matrix is called in the double arrow form. The nonlinear elements are situated in the last row and the two last columns.

We can also deduce that the Borne and Gentina criterion can not be applied in this case, only the usual stability criteria can be used like Holder norms such that max norm, sum norm and module norm.

In order to use suitably the Borne and Gentina criterion, we can think of the isolation of the nonlinear elements in only one row and one column. In this way, we consider a comparison system relative to the following n regular vector norm:

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$$p(z')=\left[|z'_1|,\ldots,|z'_{n-1}|,\max\left\{|z'_n|,|z'_{n+1}|\right\}\right]^T \quad (14)$$

The vector z' is of order $(n+1)$ and the vector $p(z')$ is of order *n*.

Let $Z=p(z')$, so we define the overvaluing system relative to p such that:

$$\dot{Z}=M_C(.)Z \quad (15.a)$$

The matrix $M_C(.)$ is given by:

$$M_C(.)=\begin{bmatrix}\alpha_1 & 0 & \ldots & \gamma_1(.)\\ 0 & \ddots & & \vdots\\ \vdots & \ldots & \alpha_{n-1} & \gamma_{n-1}(.)\\ \mu_1(.) & & \mu_{n-1}(.) & \mu(.)\end{bmatrix} \quad (15.b)$$

where:

$$\gamma_i(.)=|\delta_i(.)|+|\upsilon_i(.)| \qquad \forall i=1,\ldots,n-1$$
$$=\left|\alpha_i^n+\sum_{j=1}^{n}a_j(.)\alpha_i^{n-j}\right|+\left|\sum_{j=1}^{n}b_j(.)\alpha_i^{n-j}\right| \quad (15.c)$$

$$\mu_i(.)=\max\left\{|\beta_i|,|\lambda_i(.)|\right\}=|\beta_i|\max\left\{1,k_{du}k_{de}f(.)\right\} \quad (15.d)$$

$$\mu(.)=\max\left\{\sigma(.)+|b_1(.)|,|\varphi(.)|+\psi(.)\right\}$$
$$=\max\left\{\begin{array}{l}(-a_1(.)-\sum_{j=1}^{n-1}\alpha_j)+|b_1(.)|,\\ \left|k_{du}k_{de}f(.)\left[a_1(.)+\sum_{j=1}^{n-1}\alpha_j\right]-k_{du}k_ef(.)\right|-k_{du}k_{de}f(.)b_1(.)\end{array}\right\} \quad (15.e)$$

The matrix $M_C(.)$ is in the arrow form. The vector norm p allows passing from the matrix in the double arrow form to a matrix in the simple arrow form by decreasing the order of the system from $(n+1)$ to n order.

To apply the Borne and Gentina criterion to the overvaluing matrices, we substitute the nonlinear elements in the last row $(\mu_i(.))$ by constant elements with the following hypothesis:

$$\mu_i(.)=|\beta_i| \text{ for } k_{du}k_{de}f(.)<1 \quad (16)$$

In these conditions the matrix $M_C(.)$ becomes:

$$M'_C(.)=\begin{bmatrix}\alpha_1 & 0 & \ldots & \gamma_1(.)\\ 0 & \ddots & & \vdots\\ & & \alpha_{n-1} & \gamma_{n-1}(.)\\ |\beta_1| & \ldots & |\beta_{n-1}| & \mu(.)\end{bmatrix} \quad (17)$$

So the nonlinear elements of the matrix $M'_C(.)$ are isolated in the last column.

3.3 Stability conditions

Theorem 1:

If there exist α_i<0 for $i=1,\ldots,n-1$, $\alpha_i\neq\alpha_j$ $\forall i\neq j$ such that $\forall Z\in S$ where S is a neighbourhood domain:

$$i)\,k_{du}k_{de}f(.)<1$$
$$ii)-\mu(.)+\sum_{i=1}^{n-1}\gamma_i(.)\alpha_i^{-1}|\beta_i|>0 \quad (18)$$

then the equilibrium point Z=0 for the continuous system is asymptotically stable.

If $S=\mathbb{R}^n$, the stability is global. ■

Proof:

The matrix $M'_C(.)$ has its off diagonal elements positive and the ones non constant are isolated in the last line.

Let the following comparison system: $\dot{Z}=M'_C(.)Z$

Thus, by referring to results obtained in [3], the conditions of the previous theorem can be deduced from the Kotelyanski conditions [6]. These conditions require having the principal minors with alternating signs (see Appendix), the (α_i) are chosen all negative.

$$\alpha_1<0$$
$$\alpha_1\alpha_2>0$$
$$\vdots$$
$$(-1)^{n-1}\prod_{i=1}^{n-1}\alpha_i>0$$

and $(-1)^n\det(M_C)>0$

The $(n-1)$ first conditions are checked because the α_i are negative, however the last condition yields to:

$$(-1)^n\det\left(M'_C(.)\right)=(-1)^n\begin{vmatrix}\alpha_1 & 0 & \ldots & 0 & \gamma_1(.)\\ 0 & \ddots & \ddots & \vdots & \vdots\\ \vdots & \ddots & \ddots & 0 & \vdots\\ 0 & \ldots & 0 & \alpha_{n-1} & \gamma_{n-1}(.)\\ |\beta_1| & \ldots & \ldots & |\beta_{n-1}| & \mu(.)\end{vmatrix}$$

$$=(-1)^n\left[\mu(.)\prod_{i=1}^{n-1}\alpha_i-\sum_{i=1}^{n-1}\left(|\gamma_i(.)\beta_i|\prod_{\substack{j=1\\ j\neq i}}^{n-1}\alpha_j\right)\right]>0$$

then the theorem is obtained by dividing this condition by $\left((-1)^{n-1}\prod_{i=1}^{n-1}\alpha_i\right)$ such that: $-\mu(.)+\sum_{i=1}^{n-1}\gamma_i(.)\alpha_i^{-1}|\beta_i|>0$. ■

In order to simplify the application of the theorem, we have to exploit the expression of $\mu(.)$. We suppose that:

$$\mu(.)=\left|k_{du}k_{de}f(.)\left[a_1(.)+\sum_{j=1}^{n-1}\alpha_j\right]-k_{du}k_ef(.)\right|-k_{du}k_{de}f(.)b_1(.).$$

Thus, we obtain the following corollary.

Corollary 1:

If there exist α_i<0 for $i=1,\ldots,n$, $\alpha_i\neq\alpha_j$ $\forall i\neq j$ such that $\forall Z\in S$ the three following conditions are checked:

$$i)\,k_{du}k_{de}f(.)<1$$
$$ii)\left(-a_1(.)-\sum_{j=1}^{n-1}\alpha_j\right)+|b_1(.)|<\mu(.) \quad (19)$$
$$iii)-\mu(.)+\sum_{i=1}^{n-1}\gamma_i(.)\alpha_i^{-1}|\beta_i|>0$$

then the equilibrium point Z=0 for the system is asymptotically stable.

If $S=\mathbb{R}^n$, the stability is global. ■

Remark 1 :

For $n=1$, the stability condition issued from Theorem 1 is such that: $\mu(.)<0$. ■

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4 Example

For the validation of the results obtained we consider the stabilization of a fuzzy control system, where the controller is of type PI fuzzy controller and the system to be controlled is a speed of DC motor. This motor is supposed with a shunt excitation represented by a transfer function of a first order system which is preceded by a nonlinear element, corresponding to nonlinearity characteristic of the magnetic flux for example (Fig. 6).

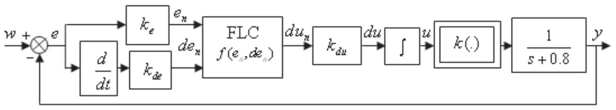

Figure 6: Fuzzy control system for the speed of DC motor with shunt excitation.

The nonlinear gain $k(.)$ is represented by the following allure (Fig. 7):

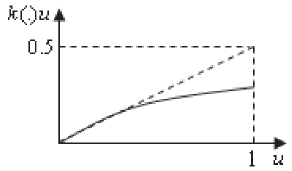

Figure 7: Characteristic of the nonlinear gain.

The system is described by the following state equation:

$$\begin{cases} \dot{x} = A(.)x + B(.)u \\ y = C(.)x \end{cases} \quad (20.a)$$

where:

$$\begin{cases} A(.) = A = -0.8 \\ B(.) = k(.) \\ C(.) = C = 1 \end{cases} \quad (20.b)$$

the matrix $A_C(.)$ is given by :

$$A_C(.) = \begin{bmatrix} -0.8 & k(.) \\ 0.8k_{du}k_{de}f(.) - k_{du}k_{de}f(.) & -k_{du}k_{de}f(.)k(.) \end{bmatrix} \quad (21)$$

By making a basic change to the previous system we obtain:

$$\dot{z}' = A'_C(.)z' \quad (22.a)$$

Such that:

$$\begin{cases} A'_C(.) = PA_C(.)P^{-1} \\ P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2 \end{cases} \quad (22.b)$$

so we can write: $A'_C(.) = A_C(.)$ and:

$$M_C(.) = \max\left\{ \left| k_{du}k_{de}f(.)0.8 - k_{du}k_{de}f(.) \right| - k_{du}k_{de}f(.)k(.), -0.8 + k(.) \right\}$$

the factor scales k_e and k_{de} are chosen such that:

$k_e = k_{de} = 1$.

According to Remark 1, by applying Theorem 1 we get the following condition: $\mu(.) < 0$

which leads to:

$$\begin{cases} -0.8 + k(.) < 0 \\ k_{du}f(.)\left[0.2 - k(.)\right] < 0 \end{cases} \text{ and so: } \begin{cases} 0.2 < k(.) < 0.8 \\ k_{du}f(.) > 0 \end{cases}$$

The area representing the stability domain of the fuzzy system is given by Fig. 8:

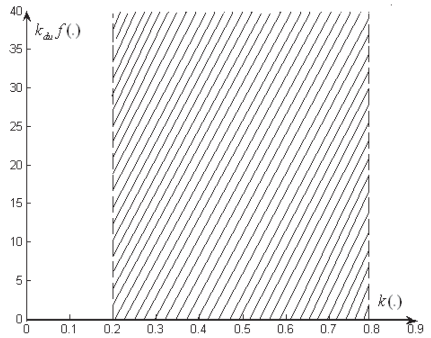

Figure 8: Stability domain of the fuzzy system obtained from Theorem 1.

The application of Corollary 1 allows to deduce the following condition: $-0.8 + k(.) < k_{du}f(.)\left[0.2 - k(.)\right] < 0$

then: $k_{du}f(.) < \dfrac{0.8 - k(.)}{k(.) - 0.2}$ and $0.2 < k(.) < 0.8$.

The area representing the stability domain of the fuzzy system is given by Fig. 9:

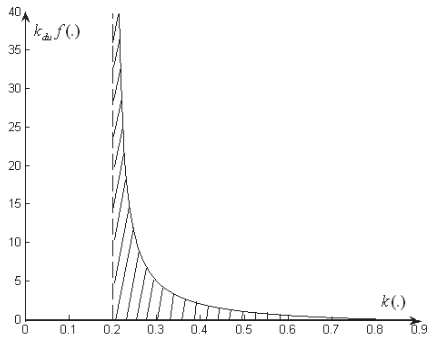

Figure 9: Stability domain of the fuzzy system obtained from Corollary 1.

5 Conclusion

The stability conditions of particular class of PI-like fuzzy control systems were presented in this paper. These conditions were deduced from stability study of overvaluing systems based on vector norms and the application of the Borne and Gentina criterion. After making a basic change on the system, we obtain a state matrix in a double arrow form, to return to the usual arrow form of the matrix and to get matrix with nonlinear elements isolated in only one row or column, we had considered a comparison system relative to a regular vector norm. In this way the Borne and Gentina criterion was used to get sufficient stability conditions. These conditions were applied to nonlinear system given by the control of the speed of DC motor with shunt excitation.

Appendix

Borne-Gentina practical stability criterion [15]

Let consider the nonlinear continuous process described in state space by: $\dot{x} = A(.)x$; $A(.)$ is an $n \times n$ matrix, $A(.) = \left\{ a_{i,j} \right\}$. If the overvaluing matrix $M(A(.))$ has its non constant elements isolated in only one row, the verification of the Kotelyanski condition enables to conclude to the stability of the initial system.

As an example, if the non constant elements are isolated in only one row of $A(.)$, Kotelyanski lemma applied to the

ISBN: 978-989-95079-6-8

overvaluing matrix obtained by the use of the n regular vector norm $p(x)$ with $x = [x_1, x_2, ..., x_n]^T$, such as: $p(x) = [|x_1|, |x_2|, ..., |x_n|]^T$, leads to the following stability conditions of initial system:

$$a_{1,1} < 0, \begin{vmatrix} a_{1,1} & |a_{1,2}| \\ |a_{2,1}| & a_{2,2} \end{vmatrix} > 0, ..., (-1)^n \begin{vmatrix} a_{1,1} & |a_{1,2}| & \cdots & |a_{1,n}| \\ |a_{2,1}| & a_{2,2} & \cdots & |a_{2,n}| \\ \vdots & \vdots & & \vdots \\ |a_{n,1}(.)| & |a_{n,2}(.)| & \cdots & a_{n,n}(.) \end{vmatrix} > 0$$

The Borne-Gentina practical criterion applied to continuous systems generalizes the Kotelyanski lemma for nonlinear systems and defines large classes of systems for which the linear Aizerman conjecture can be applied, either for the initial system or for its comparison system.

References

[1] J. Aracil, A. Garcia-Cerezo, A. Barreiro and A. Ollero , *Fuzzy Control of dynamical systems and stability analysis based on the conicity criterion*, IFSA World Congress, Brussels, Belgium, 1991.

[2] M. Benrejeb, M. Gasmi, *Stability and Stabilization Conditions of TSK Models using Vector Norms*, International Conference on Communication, Computer and Power (ICCCP'01), Muscat, Février 2001.

[3] M. Benrejeb, P. Borne, F. Laurent, *Sur une application de la représentation en flèche à l'analyse des processus*, RAIRO Automatique, 16 (2), pp 133-146, 1982.

[4] M. Benrejeb, A. Sakly, K. Ben Othman and P. Borne, *Choice of conjunctive operator of TSK fuzzy systems and stability domain study*, Math. And Comp. in Simul., Vol 76, pp 410-421, 2008.

[5] M. Benrejeb, D. Soudani, A. Sakly and P. Borne, *New discrete TSK fuzzy systems characterization and stability domain*, Int. Jour. Of Comp. Comm. and Cont., Vol 1, N 4, pp 9-19, 2006.

[6] P. Borne, *Non linear sytem stability*, vector norm approach, System and control Enceclopedia, t.5, 3402-3406, Perganon Press, 1987.

[7] P. Borne, M. Benrejeb, *On the Representation and the Stability Study of Large Scale Systems*, IJCCC, International Journal of Computers, Communications & Control, Vol 3, Supp. Issue: Proceedings of ICCC, pp 55-66, 2008.

[8] F. Bouslama and A. Ishikawa, *Application of limit fuzzy controllers to stability analysis*, Fuzzy Sets and Systems, Vol 49, pp 103-120, 1992.

[9] G. Calcev, *Some remarks on the stability of Mamdani fuzzy control systems*, IEEE Trans. on Fuzzy Systems, Vol 6, Issue 3, pp 436-442, 1998.

[10] G. Calcev, R. Gorez and M.D. Neyer, *Passivity approach of fuzzy control systems*, Automatica, Vol 34, N 3, pp 339-344, 1998.

[11] G. Chen and H. Ying, *Stability analysis of nonlinear fuzzy PI control systems*, Int. Conf. Fuzzy Logic Applict., Houston, TX, 1993.

[12] Y.Y. Chen, *Stability analysis of fuzzy control-A Lyapunov approach*, IEEE Conf. Syst. Man Cybern., Las Vegas, NV, 1987.

[13] H. Feng-Hsiag, H. Jing-Dong, *Stability analysis of fuzzy large-scale systems*, IEEE Trans. on Systems, Man, and Cybernetics, Part B, Vol 32, Issue 1, pp 122-126, 2002.

[14] C.-C. Fuh, P.-C. Tung, *Robust Stability analysis of fuzzy control System*, Fuzzy Sets and Systems, pp 289-298, 1997.

[15] J.C. Gentina, P. Borne, F. Laurent, "Stabilité des systèmes continus non linéaires de grande dimension, " *RAIRO*, Août, J-3, pp 69-77, 1972.

[16] R. Katoh, T. Yamashita and S. Singh, *Stability Analysis of control system having PD type of fuzzy controller*, Fuzzy Sets and Systems, Vol 74, Issue 2, pp 321-334, 1995.

[17] W.M.J. Kickert and E.H. Mamdani, *Analyses of Fuzzy Logic Controllers*, Fuzzy Sets and Systems, Vol.1, pp 29-44, 1978.

[18] J. Ch. Lo, Y. M. Chen, *Stability issues on Takagi-Sugeno fuzzy model-parametric approach*, IEEE Trans. on Fuzzy Systems, Vol 7 Issue 5, pp 597-607, 1999.

[19] K.S. Ray and D.D. Majumder, *Application of Cercle Criteria for Stability Analysis of SISO and MIMO Systems associated with Fuzzy Logic Controllers*, IEEE Trans. Syst. Man & Cyber., Vol 14, Issue 2, pp 345-349, 1984.

[20] A. Sakly, K. Ben Othman and M. Benrejeb, *Mamdani Type Fuzzy Controllers for the Stabilisation of Linear Systems*, WSEAS Transactions on Systems, Issue 4, Volume 2, pp.820-826, 2003.

[21] A. Sakly, B. Zahra and M. Benrejeb, *Stability Study of Mamdani's Fuzzy Controllers applied to Linear Plants*, Studies in Informatics and Control, SIC Journal , vol 17, Issue 4, December 2008.

[22] M. Sugeno , *On stability of fuzzy systems expressed by fuzzy rules with singleton consequences*, IEEE Trans. on Fuzzy Systems, Vol 7, Issue 2, pp 201-223, 1999.

[23] K. Tanaka, M. Sano, *A Robust Stabilization Problem of Fuzzy Control Systems and its Application to Backing up Control of a Track-Trailer*, IEEE Trans. on Fuzzy Systems, Vol 2, Issue 2, pp 119-134, 1994.

[24] L. Wang and R. Langari, *Fuzzy controller design via hyperstability approach*, IEEE Int. Conf. Fuzzy Syst., Orlando, FL, 1994.

[25] H. Ying, *Conditions on general Mamdani fuzzy controllers as nonlinear, variable gain state feedback controllers with stability analysis*, IFSA World Congress and 20th NAFIPS International Conference, 2001.

ISBN: 978-989-95079-6-8