The Fuzzy Induced Generalized OWA Operator and its Application in Business Decision Making

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Abstract—*We present the fuzzy induced generalized OWA (FIGOWA) operator. It is an aggregation operator that uses the main characteristics of the fuzzy OWA (FOWA) operator, the induced OWA (IOWA) operator and the generalized OWA (GOWA) operator. Therefore, it uses uncertain information represented in the form of fuzzy numbers, generalized means and order inducing variables. The main advantage of this operator is that it includes a wide range of mean operators in the same formulation such as the FOWA, the IOWA, the GOWA, the induced GOWA, the fuzzy IOWA, the fuzzy generalized mean, etc. We study some of its main properties. A further generalization by using quasi-arithmetic means is also presented. This operator is called Quasi-FIOWA operator. We also develop an application of the new approach in a strategic decision making problem.*

Keywords— Decision making; OWA operator; Aggregation operators; Fuzzy numbers.

1 Introduction

Different types of aggregation operators are found in the literature for aggregating the information. A very common aggregation method is the ordered weighted averaging (OWA) operator [17]. Since its appearance, the OWA operator has been studied in a wide range of applications [1-3,5,7-8,10-13,15-22]. In [21], Yager and Filev introduced the IOWA operator. It is a generalization of the OWA operator that uses order inducing variables in the reordering of the arguments. In the last years, the IOWA operator has been studied by different authors [5,11-13,16,19,21].

When using the IOWA operator, it is assumed that the available information is exact numbers or crisp values. However, this may not be the real situation found in the decision making problem. Sometimes, the available information is vague or imprecise and it is not possible to analyze it with exact numbers. Then, it is necessary to use another approach to deal with this information such as fuzzy numbers (FN). For these situations, the IOWA is known as fuzzy number induced OWA (FN-IOWA) operator [5].

Recently, [13] have suggested a generalization of the IOWA operator by using generalized means. With this generalization, known as the induced generalized OWA (IGOWA) operator, we are able to include in the same formulation different types of induced aggregation operators such as the IOWA operator, the induced ordered weighted geometric (IOWG) operator and the induced ordered weighted quadratic averaging (IOWQA) operator, among others. Moreover, they also suggested the Quasi-IOWA operator which is a further generalization of the IGOWA operator by using quasi-arithmetic means.

Going a step further, in this paper we present the fuzzy induced generalized OWA operator which generalizes the FN-IOWA by using generalized means. We will call it the fuzzy induced generalized OWA (FIGOWA) operator. Then, we are able to obtain a wide range of fuzzy induced aggregation operators such as the FN-IOWA, the FN-IOWG operator and the FN-IOWQA operator, among others. We study some of the main properties of this generalization and we extend it to a more general formulation by using quasi-arithmetic means. The result is the Quasi-FIOWA operator. We also develop an application of the new approach in a decision making problem about selection of strategies.

This paper is organized as follows. In Section 2, we briefly review some basic concepts such as FN, the FN-IOWA and the IGOWA operator. Section 3 presents the FIGOWA operator and Section 4 studies some of its families. In Section 5 we briefly present the Quasi-FIOWA operator and in Section 6, we develop an application of the new approach in a strategic decision making problem.

2 Preliminaries

2.1 Fuzzy numbers

The FN was first introduced by [4,24]. Since then, it has been studied and applied by a lot of authors such as [6,9-11]. A FN is a fuzzy subset [23] of a universe of discourse that is both convex and normal [9]. Note that the FN may be considered as a generalization of the interval number [14] although it is not strictly the same because the interval numbers may have different meanings.

In the literature, we find a wide range of FNs [6,9]. For example, a trapezoidal FN (TpFN) A of a universe of discourse R can be characterized by a trapezoidal membership function $A = (\underline{a}, \overline{a})$ such that

$$\underline{a}(\alpha) = a_1 + \alpha(a_2 - a_1), \quad \overline{a}(\alpha) = a_4 - \alpha(a_4 - a_3). \tag{1}$$

where $\alpha \in$ [0, 1] and parameterized by (a_1, a_2, a_3, a_4) where $a_1 \le a_2 \le a_3 \le a_4$, are real values. Note that if $a_1 = a_2 = a_3 = a_4$, then, the FN is a crisp value and if $a_2 = a_3$, the FN is

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represented by a triangular FN (TFN). Note that the TFN can be parameterized by (a_1, a_2, a_4).
In the following, we are going to review the FN arithmetic operations as follows. Let A and B be two TFN, where $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$. Then:

1) $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
2) $A - B = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$
3) $A \times k = (k \times a_1, k \times a_2, k \times a_3)$; for $k > 0$.

Note that other operations could be studied [6,9] but in this paper we will focus on these ones.

2.2 Fuzzy induced OWA operator

The FIOWA (or FN-IOWA) operator was introduced by [5]. It is an aggregation operator that uses uncertain information represented by FNs. It also uses a reordering process different from the values of the arguments. In this case, the reordering step is based on order inducing variables. It can be defined as follows.

Definition 1. *Let Ψ be the set of FN. A FIOWA operator of dimension n is a mapping FIOWA:* $\Psi^n \rightarrow \Psi$ *that has an associated weighting vector W of dimension n such that* $w_j \in [0, 1]$ *and* $\sum_{j=1}^{n} w_j = 1$, *then*:

$$FIOWA(\langle u_1,\tilde{a}_1\rangle, \ldots, \langle u_n,\tilde{a}_n\rangle) = \sum_{j=1}^{n} w_j b_j \qquad (2)$$

where b_j *is the* $\tilde{a}_i$ *value of the FIOWA pair* $\langle u_i, \tilde{a}_i\rangle$ *having the jth largest* u_i, u_i *is the order inducing variable and* $\tilde{a}_i$ *is the argument variable represented in the form FN.*
Note that from a generalized perspective of the reordering step it is possible to distinguish between descending (DFIOWA) and ascending (AFIOWA) orders. Note also that this operator provides a parameterized family of aggregation operators that includes the fuzzy maximum, the fuzzy minimum and the fuzzy average (FA), among others.

2.3 Induced generalized OWA operator

The IGOWA operator was introduced in [13] and it represents a generalization of the IOWA operator by using generalized means. Then, it is possible to include in the same formulation, different types of induced operators such as the IOWA operator or the induced OWG (IOWG) operator. It can be defined as follows.

Definition 2. *An IGOWA operator of dimension n is a mapping IGOWA:* $R^n \rightarrow R$ *that has an associated weighting vector W of dimension n such that* $w_j \in [0, 1]$ *and* $\sum_{j=1}^{n} w_j = 1$, *then*:

$$IGOWA(\langle u_1,a_1\rangle,\ldots,\langle u_n,a_n\rangle) = \left(\sum_{j=1}^{n} w_j b_j^{\lambda} \right)^{1/\lambda} \qquad (3)$$

where b_j *is the* a_i *value of the IGOWA pair* $\langle u_i,a_i\rangle$ *having the jth largest* u_i, u_i *is the order inducing variable,* a_i *is the argument variable and λ is a parameter such that* $\lambda \in (-\infty, \infty)$.
As we can see, if $\lambda = 1$, we get the IOWA operator. If $\lambda = 0$, the IOWG operator and if $\lambda = 2$, the IOWQA operator. Note that it is possible to further generalize the IGOWA operator by using quasi-arithmetic means. The result is the Quasi-IOWA operator.

3 Fuzzy induced generalized OWA operator

The fuzzy induced generalized OWA (FIGOWA) operator is an extension of the GOWA operator that uses uncertain information in the aggregation represented in the form of FNs. The reason for using this operator is that sometimes, the uncertain factors that affect our decisions are not clearly known and in order to assess the problem we need to use FNs. The FN is a very useful technique in decision making because it considers the different uncertain results that could happen in the future. This operator also uses a reordering process based on order inducing variables. It can be defined as follows.

Definition 3. *Let Ψ be the set of FNs. A FIGOWA operator of dimension n is a mapping FIGOWA:* $\Psi^n \rightarrow \Psi$ *that has an associated weighting vector W of dimension n such that* $w_j \in [0, 1]$ *and* $\sum_{j=1}^{n} w_j = 1$, *then*:

$$FIGOWA(\langle u_1,\tilde{a}_1\rangle,\ldots,\langle u_n,\tilde{a}_n\rangle) = \left(\sum_{j=1}^{n} w_j b_j^{\lambda} \right)^{1/\lambda} \qquad (4)$$

where b_j *is the* $\tilde{a}_i$ *value of the FIGOWA pair* $\langle u_i,\tilde{a}_i\rangle$ *having the jth largest* u_i, u_i *is the order inducing variable,* $\tilde{a}_i$ *is the argument variable represented in the form of FN and λ is a parameter such that* $\lambda \in (-\infty, \infty)$.
Note that different types of FNs could be used in the aggregation such as TFNs, TpFNs, L-R FNs, interval-valued FNs, intuitionistic FNs, etc.
As it was explained in [13], when using FN in the OWA operator, we have the additional problem of how to reorder the arguments. In the FIGOWA operator, this is not a problem because the reordering process is developed with order inducing variables and it is independent of the values of the arguments.
From a generalized perspective of the reordering step, it is possible to distinguish between the descending FIGOWA (DFIGOWA) and the ascending FIGOWA (AFIGOWA) operator. The weights of these operators are related by $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the DFIGOWA and w^*_{n-j+1} the jth weight of the AFIGOWA operator.
If B is a vector corresponding to the ordered arguments b_j, we shall call this the ordered argument vector and W^T is the transpose of the weighting vector, then, the FIGOWA operator can be expressed as:

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$$FIGOWA(\langle u_1,\tilde{a}_1\rangle,\ldots,\langle u_n,\tilde{a}_n\rangle) = W^T B \quad (5)$$

Note that if the weighting vector is not normalized, i.e., $W = \sum_{j=1}^{n} w_j \neq 1$, then, the FIGOWA operator can be expressed as:

$$FIGOWA(\langle u_1,\tilde{a}_1\rangle,\ldots,\langle u_n,\tilde{a}_n\rangle) = \frac{1}{W}\sum_{j=1}^{n} w_j b_j \quad (6)$$

The FIGOWA operator is monotonic, commutative, bounded and idempotent.

Theorem 1 (Monotonicity). *Assume f is the FIGOWA operator, if $\tilde{a}_i \geq \tilde{e}_i$, for all $\tilde{a}_i$, then*

$$f(\langle u_1,\tilde{a}_1\rangle,\ldots,\langle u_n,\tilde{a}_n\rangle) \geq f(\langle u_1,\tilde{e}_1\rangle,\ldots,\langle u_n,\tilde{e}_n\rangle) \quad (7)$$

Theorem 2 (Commutativity). *Assume f is the FIGOWA operator, then*

$$f(\langle u_1,\tilde{a}_1\rangle,\ldots,\langle u_n,\tilde{a}_n\rangle) = f(\langle u_1,\tilde{e}_1\rangle,\ldots,\langle u_n,\tilde{e}_n\rangle) \quad (8)$$

where $(\langle u_1,\tilde{e}_1\rangle,\ldots,\langle u_n,\tilde{e}_n\rangle)$ is any permutation of the arguments $(\langle u_1,\tilde{a}_1\rangle,\ldots,\langle u_n,\tilde{a}_n\rangle)$.

Theorem 3 (Boundedness). *Assume f is the FIGOWA operator, then*

$$\min\{\tilde{a}_i\} \leq f(\langle u_1,\tilde{a}_1\rangle,\ldots,\langle u_n,\tilde{a}_n\rangle) \leq \max\{\tilde{a}_i\} \quad (9)$$

Theorem 4 (Idempotency). *Assume f is the FIGOWA operator, if $\tilde{a}_i = \tilde{a}$, for all $\tilde{a}_i$, then*

$$f(\langle u_1,\tilde{a}_1\rangle,\ldots,\langle u_n,\tilde{a}_n\rangle) = \tilde{a} \quad (10)$$

Note that the proofs of Theorems 1 - 4 are omitted because they are trivial.

Another interesting issue when analysing the FIGOWA operator is the problem of ties in the order inducing variables. In order to solve this problem, we recommend to follow the policy explained in [21]. Basically, the idea is to replace each argument of the tied inducing variables by its fuzzy generalized mean. Then, different types of means may be used to replace the arguments depending on the parameter λ.

As it is explained in [21], different kinds of attributes may be used for the order inducing variables of the FIGOWA operator with the only requirement of having a linear ordering.

4 Families of FIGOWA operators

Basically, we can distinguish between two main groups of FIGOWA operators. The first family represents all the families that may be found in the weighting vector W, while the second family represents all the particular cases coming from the parameter λ.

4.1 Analysing the parameter λ

If we analyze different values of the parameter λ, we obtain another group of particular cases such as the FIOWA operator, the fuzzy IOWG (FIOWG), the fuzzy IOWQA (FIOWQA) and the fuzzy induced ordered weighted harmonic averaging (FIOWHA) operator.

When $\lambda = 1$, the FIGOWA operator becomes the FIOWA operator.

$$FIOWA(\langle u_1,\tilde{a}_1\rangle, \ldots, \langle u_n,\tilde{a}_n\rangle) = \sum_{j=1}^{n} w_j b_j \quad (11)$$

Note that it is possible to study a wide range of families of FIOWA operators by using different weighting vectors in a similar way as it has been explained in Section 4.1. For example, if $w_j = 1/n$, for all $\tilde{a}_i$, we get the FA and if the ordered position of b_j is the same than the position of the values u_i, we get the FOWA operator.

When $\lambda = 0$, we get the FIOWG operator.

$$FIOWG(\langle u_1,\tilde{a}_1\rangle, \ldots, \langle u_n,\tilde{a}_n\rangle) = \prod_{j=1}^{n} b_j^{w_j} \quad (12)$$

Note that in this case we can also study different families of FIOWG operators such as the fuzzy geometric mean or the fuzzy OWG (FOWG) operator. Note also that it is possible to distinguish between descending (DFIOWG) and ascending (AFIOWG) orders.

When $\lambda = -1$, we get the FIOWHA operator.

$$FIOWHA(\langle u_1,\tilde{a}_1\rangle, \ldots, \langle u_n,\tilde{a}_n\rangle) = \frac{1}{\sum_{j=1}^{n} \frac{w_j}{b_j}} \quad (13)$$

From a generalized perspective of the reordering step we find the descending FIOWHA (DFIOWHA) and the ascending FIOWHA (AFIOWHA) operator. Different families of FIOWHA operators are found by using different weighting vectors such as the fuzzy harmonic mean and the fuzzy ordered weighted harmonic averaging (FOWHA) operator.

When $\lambda = 2$, we get the FIOWQA operator.

$$FIOWQA(\langle u_1,\tilde{a}_1\rangle,\ldots,\langle u_n,\tilde{a}_n\rangle) = \left(\sum_{j=1}^{n} w_j b_j^2\right)^{1/2} \quad (14)$$

In this case, we can also study a wide range of families of FIOWQA operators such as the fuzzy quadratic mean and the fuzzy OWQA operator, and distinguish between the DFIOWQA and the AFIOWQA operator.

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4.2 Analysing the weighting vector W

By using a different weighting vector in the FIGOWA operator, we are able to obtain a wide range of aggregation operators. For example, we can obtain the fuzzy maximum, the fuzzy minimum, the FGM, the fuzzy weighted generalized mean (FWGM) and the FGOWA operator.

Remark 1. The fuzzy maximum is obtained if $w_p = 1$ and $w_j = 0$, for all $j \neq p$, and $u_p = \text{Max}\{\tilde{a}_i\}$. The fuzzy minimum is obtained if $w_p = 1$ and $w_j = 0$, for all $j \neq p$, and $u_p = \text{Min}\{\tilde{a}_i\}$. The FGM is found when $w_j = 1/n$, for all $\tilde{a}_i$. The fuzzy weighted generalized mean (FWGM) is obtained if $u_i > u_{i+1}$, for all i, and the FGOWA operator is obtained if the ordered position of u_i is the same than the ordered position of b_j such that b_j is the jth largest of $\tilde{a}_i$.

Remark 2. Other families of FIGOWA operators could be used in the aggregation by using a different manifestation of the weighting vector. For example, we could analyze the step-FIGOWA, the window-FIGOWA, the median-FIGOWA, the olympic-FIGOWA, the centered-FIGOWA, the S-FIGOWA, etc. For more information, see [10-13,17-22].

Remark 3. The step-FIGOWA operator is found when $w_k = 1$ and $w_j = 0$, for all $j \neq k$ and the window-FIGOWA when $w_j = 1/m$ for $k \leq j \leq k + m - 1$ and $w_j = 0$ for $j > k + m$ and $j < k$. Note that k and m must be positive integers such that $k + m - 1 \leq n$.

Remark 4. For the median-FIGOWA, we distinguish between two cases. If n is odd we assign $w_{(n+1)/2} = 1$ and $w_j = 0$ for all others, and this affects the argument $\tilde{a}_i$ with the $[(n + 1)/2]$th largest u_i. If n is even we assign, for example, $w_{n/2} = w_{(n/2)+1} = 0.5$, and this affects the arguments with the $(n/2)$th and $[(n/2) + 1]$th largest u_i.

Remark 5. The olympic-FIGOWA operator is found if $w_1 = w_n = 0$, and for all others $w_j = 1/(n - 2)$. Note that it is possible to develop a general form of the olympic-POWA by considering that $w_j = 0$ for $j = 1, 2, \ldots, k, n, n - 1, \ldots, n - k + 1$, and for all others $w_{j^*} = 1/(n - 2k)$, where $k < n/2$. Note that if $k = 1$, then this general form becomes the usual olympic-POWA.

Remark 6. A further family is the centered-FIGOWA operator. This type of aggregation operator is symmetric, strongly decaying and inclusive. It is symmetric if $w_j = w_{j+n-1}$. It is strongly decaying when $i < j \leq (n + 1)/2$, then $w_i < w_j$ and when $i > j \geq (n + 1)/2$ then $w_i < w_j$. It is inclusive if $w_j > 0$. Note that it is possible to consider a softening of the second condition by using $w_i \leq w_j$ instead of $w_i < w_j$ which is known as softly decaying centered-FIGOWA operator. Note also the possibility of removing the third condition. Then, we shall refer to this type of aggregation as non-inclusive centered-FIGOWA operator.

Remark 7. A further interesting family is the S-FIGOWA operator. In this case, we can distinguish between three types: the "orlike", the "andlike", and the "generalized" S-FIGOWA operator. The generalized S-FIGOWA operator is obtained when $w_p = (1/n)(1 - (\alpha + \beta)) + \alpha$, with $u_p = \text{Max}\{\tilde{a}_i\}$; $w_q = (1/n)(1 - (\alpha + \beta)) + \beta$, with $u_q = \text{Min}\{\tilde{a}_i\}$; and $w_j = (1/n)(1 - (\alpha + \beta))$ for all $j \neq p,q$ where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. Note that if $\alpha = 0$, we get the andlike S-FIGOWA and if $\beta = 0$, the orlike S-FIGOWA.

Remark 8. Another type is the non-monotonic-FIGOWA operator. It is obtained when at least one of the weights w_j is lower than 0 and $\sum_{j=1}^{n} w_j = 1$. Note that a key aspect of this operator is that it does not always achieve monotonicity.

5 The Quasi-FIOWA operator

The FIGOWA operator may be further generalized by using quasi-arithmetic means. Then, the result is the fuzzy induced ordered weighted quasi-arithmetic averaging operator or Quasi-FIOWA, for short. Note that the Quasi-FIOWA operator is an extension of the Quasi-OWA [1,3,7,11] by using order inducing variables and uncertain information represented with FNs.

Definition 4. *Let Ψ be the set of FNs. A Quasi-FIOWA operator of dimension n is a mapping f: $\Psi^n \rightarrow \Psi$ that has an associated weighting vector W of dimension n such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then*:

$$f(\langle u_1,\tilde{a}_1\rangle,\ldots,\langle u_n,\tilde{a}_n\rangle) = g^{-1}\left(\sum_{j=1}^{n} w_j g(b_j)\right) \qquad (15)$$

where b_j is the $\tilde{a}_i$ value of the Quasi-FIOWA pair $\langle u_i,\tilde{a}_i\rangle$ having the jth largest u_i, u_i is the order inducing variable, $\tilde{a}_i$ is the argument variable represented in the form of FN and $g(b)$ is a strictly continuous monotone function.

As we can see, when $g(b) = b^\lambda$, we get the FIGOWA operator. Note that it is also possible to distinguish between descending (Quasi-DFIOWA) and ascending (Quasi-AFIOWA) orders. Note also that all the properties and particular cases commented in the FIGOWA operator, are also applicable in this case.

6 Application in strategic decision making

In the following, we are going to develop a brief example where we will see the applicability of the new approach. We will focus in a decision making problem about selection of strategies. Note that other business decision making applications could be developed such as financial decision making, human resource selection, etc. Note that the FIGOWA operator may be applied in similar problems than the IOWA and the IGOWA operator.

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Assume a company that operates in North America and Europe is analyzing the general policy for the next year and they consider 5 possible strategies to follow.

1) A_1 = Expand to the Asian market.
2) A_2 = Expand to the South American market.
3) A_3 = Expand to the African market.
4) A_4 = Expand to the 3 continents.
5) A_5 = Do not develop any expansion.

In order to evaluate these strategies, the group of experts considers that the key factor is the economic situation of the next year. Thus, depending on the situation, the expected benefits will be different. The experts have considered 5 possible situations for the next year: S_1 = Very bad, S_2 = Bad, S_3 = Regular, S_4 = Good, S_5 = Very good. The expected results depending on the situation S_i and the alternative A_i are shown in Table 1. Note that the results are TFN.

Table 1: Available strategies

	S_1	S_2	S_3	S_4	S_5
A_1	(20,30,40)	(60,70,80)	(40,50,60)	(50,60,70)	(50,60,70)
A_2	(30,40,50)	(70,80,90)	(30,40,50)	(30,40,50)	(50,60,70)
A_3	(60,70,80)	(50,60,70)	(40,50,60)	(20,30,40)	(40,50,60)
A_4	(50,60,70)	(30,40,50)	(60,70,80)	(70,80,90)	(10,20,30)
A_5	(40,50,60)	(30,40,50)	(50,60,70)	(60,70,80)	(40,50,60)

In this problem, the experts consider the weighting vector W = (0.1, 0.2, 0.2, 0.2, 0.3). Due to the fact that the attitudinal character is very complex because it involves the opinion of different members of the board of directors, the experts use order inducing variables to express it.

Table 2: Order inducing variables

	S_1	S_2	S_3	S_4	S_5
A_1	7	9	6	5	8
A_2	4	3	6	8	7
A_3	2	8	4	3	6
A_4	5	6	9	2	7
A_5	8	4	3	6	5

With this information, we can aggregate it in order to take a decision. In Table 3 and 4, we show the different results obtained by using different types of FIGOWA operators in the decision process.

Table 3: First aggregation process

	FA	FWA	FOWA
A_1	(44,54,64)	(47,57,67)	(40,50,60)
A_2	(42,52,62)	(44,54,64)	(38,48,58)
A_3	(42,52,62)	(40,50,60)	(38,48,58)
A_4	(44,54,64)	(40,50,60)	(38,48,58)
A_5	(44,54,64)	(44,54,64)	(41,51,61)

Table 4: First aggregation process

	FIOWA	FIOWG	FIOWQA
A_1	(43,53,63)	(40.5,51.1,61.5)	(44.8,54.4,64.2)
A_2	(46,56,66)	(42.8,53.4,63.7)	(49.1,58.6,68.2)
A_3	(43,53,63)	(40.2,50.8,61.2)	(45.2,54.8,64.5)
A_4	(45,55,65)	(36.8,49.1,60.3)	(50.2,59.4,68.7)
A_5	(45,55,65)	(43.7,54.0,64.1)	(46.1,55.9,65.8)

If we establish an ordering of the alternatives, we get the following results shown in Table 5. Note that in this example it is not necessary to establish a criterion for ranking FNs because it is clear which alternative goes first, second and so on, in the ordering process. Note also that "$\succ$" means "preferred to" and "=" means "equal to".

Table 5: Ordering of the investments

	Ordering
FA	A_1=A_4=A_5 $\succ$ A_2=A_3
FWA	A_1 $\succ$ A_2=A_5 $\succ$ A_3=A_4
FOWA	A_5 $\succ$ A_1 $\succ$ A_2=A_3=A_4
FIOWA	A_2 $\succ$ A_4=A_5 $\succ$ A_1=A_3
FIOWG	A_5 $\succ$ A_2 $\succ$ A_1 $\succ$ A_3 $\succ$ A_4
FIOWQA	A_4 $\succ$ A_2 $\succ$ A_5 $\succ$ A_3 $\succ$ A_1

As we can see, depending on the aggregation operator used, the ordering of the investments may be different. Therefore, the decision about which investment select may be also different. For example, the FA gives very similar results between alternatives while the FIOWG and the FIOWQA give more differences between the alternatives.

7 Conclusions

We have presented the FIGOWA operator. It is a generalization of the OWA operator that uses the main characteristics of three well known aggregation operators: the GOWA, the IOWA and the FOWA operator. That is to say, it uses generalized means, order inducing variables and FNs in the aggregation. We have studied some of the main properties of this new aggregation operator. We have further generalized it by using quasi-arithmetic means. Then, we have obtained the Quasi-FIOWA operator. By using this approach, we get a more complete representation of the information because we are using FNs that consider the best and worst result and the possibility that the internal results will occur.

We have also developed an application of the new approach. We have focused in a strategic decision making problem and we have seen that depending on the particular FIGOWA operator used, the results and the decisions may be different.

In future research, we expect to develop further extensions by adding new characteristics [11] in the problem and applying it to other business problems such as financial decision making and human resource management.

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