Clustering of Fuzzy Shapes by Integrating Procrustean Metrics and Full Mean Shape Estimation into K-Means Algorithm

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Abstract*—In this paper we propose a generalization of K-means algorithm, which is adapted to integrate Procrustean metrics and full mean shape estimation, with the aim of clustering objects with either multiple or fuzzy contours. First we are concerned with the representation of fuzzy shapes and introduce appropriate shape metrics and descriptors. Next, we discuss Procrustean methods for aligning shapes, finding mutual dissimilarities and estimating shape class centroid. In the case of multiple-contour crisp shapes, we can benefit from the Extended Orthogonal Procrustes method to find mutual distances between shape pairs and from the Generalized Orthogonal Procrustes technique to estimate the Procrustes mean shape of a collection of shapes. On the other hand, dealing with the case of fuzzy shapes needs more advanced Procrustean techniques to consider weighted distances between points placed on $\alpha -$ level contours with different membership degrees. This leads to solve a Weighted Orthogonal Procrustes problem, which typically needs to introduce a weighting matrix of residuals (distances). As an application, we suggest using such methods to cluster ultrasound images of lymph nodes, which typically appear as double-contour shapes.*

Keywords— Clustering of fuzzy shapes, Fuzzy shape metrics and descriptors, Procrustes analysis, Mixing K-means algorithm with Procrustean metrics and mean shape estimation.

1 Shape analysis

Shapes and textures are extremely important features in human as well as machine vision and understanding systems. Shape analysis is concerned with two main classes of algorithms: boundary-based (when only the shape boundary points are used for the description) and region-based (when the whole interior of a shape is used). There are many imaging applications where image analysis can be reduced to the analysis of shapes, in contrast to texture analysis. However, many shape/edge detection techniques use texture information during the segmentation process.

There are several methods for extracting data from shapes, each with their own benefits and weaknesses. These include measurement of lengths and angles, landmark analysis and outline analysis. A landmark is a point of correspondence on each object that matches higher dimensionalities between and within populations. Landmark placement consists of locating a finite number of points on the outline.

More advanced techniques have been designed for semiautomatic and automatic feature extractions. Active contour modeling techniques are commonly used for shape analysis and detection. Some of the techniques for texture feature extraction use gray level co-occurrence matrices, fractal dimension, etc.

Morphometric analysis aims to describe the shape of an object in a way that removes extraneous information and thereby facilitates comparison between different objects. In these terms, a shape is referred to as an invariant to similarity transformations (such as scaling, rotation and translation).

The image fuzzification plays a pivotal role in all image processing systems. Several kinds of image fuzzification can be distinguished:

- histogram-based grey-level fuzzification (e.g. brightness in image enhancement);
- local fuzzification (e.g. edge detection);
- feature fuzzification (scene analysis, object recognition).

2 Representation of fuzzy shapes

2.1 *Crisp shapes*

Crisp shapes represent objects with crisp borders. Furthermore, if a texture is associated with the object, it has to be uniformly represented (e.g. a digitized image, where all pixels are classified as object pixels, or as background pixels).

The coordinates of selected landmarks for a crisp shape can be arranged in a $n \times p$ configuration matrix A, or equivalently on a $np \times 1$ configuration vector $a = vec(A)$.

2.2 *Continuous fuzzy shapes*

This paper primarily focuses on the representation of fuzzy shapes with fuzzy contour, which are commonly obtained through fuzzy segmentation techniques. In particular, we also consider the case of crisp shapes with multiple contours. In the same way as it is convenient to model binary images as crisp objects, it is possible to model grey-level images directly as fuzzy sets. If the grey-level values of an image are scaled to be between 0 and 1, the grey-level of a pixel can be seen as its membership to the set of high-valued (bright) pixels.

Fuzziness of an image representation can arise from various reasons, such as limited acquisition conditions (scanning resolution), but also as intrinsic property of the image, which may have imprecise borders. In such cases, pixels close to the border of the object have assigned to them a fuzzy membership value according to the extent of their belongingness to the object.

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Continuous fuzzy shapes can be described as fuzzy geometric objects. A continuous fuzzy geometric object A in $\Re^p$ is defined as a set of pairs $\{(x, \mu_A(x)) \mid x \in \Re^p\}$ where $\mu_A : \Re^p \to [0,1]$ is the membership function of A in $\Re^n$. It is assumed to have a bounded support.

An alternative representation of fuzzy geometric objects is given by a set of $\alpha-$cuts: $C(A) = \{A_\alpha \mid \alpha \in [0,1]\}$, where $A_\alpha = \{x \in \Re^p \mid \mu_A(x) \geq \alpha\}$ is a crisp object, whose $\alpha-$level contour is obtained for $\mu_A(x) = \alpha$. As a characteristic of fuzzy geometric objects, the membership function is non-increasing away from the interior of the object. For example, in figure 1 is shown a fuzzy disk. Its core is a crisp disk defined by $A_1 = \{x \in \Re^2 \mid (x_1^2 + x_2^2 \leq r_1)\}$ and its contour is the circle defined by $A_1^C = \{x \in \Re^2 \mid (x_1^2 + x_2^2 = r_1)\}$, where r_1 is the length of the corresponding radix. In general, for any $\alpha \in [0,1]$, the $\alpha-$cut is defined by $A_\alpha = \{x \in \Re^2 \mid (x_1^2 + x_2^2 \leq r_\alpha)\}$, and the $\alpha-$level contour is defined by $A_\alpha^C = \{x \in \Re^2 \mid (x_1^2 + x_2^2 = r_\alpha)\}$.

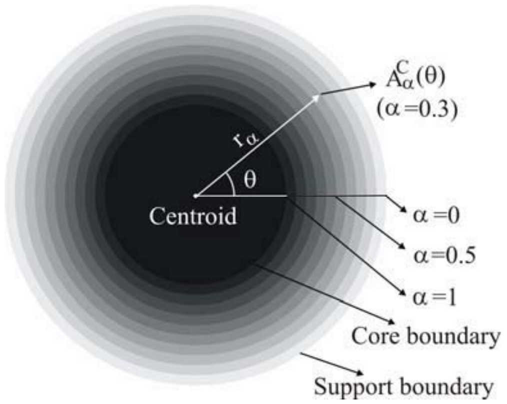

Figure 1: A fuzzy disk: centroid, core, support, α-level contours, radial distance

A shape descriptor based on a one-dimensional functional representation of the two-dimensional shape boundary is called a signature of the shape. The simplest way to generate a signature is to express the radial distance from the centroid to the boundary, as a function of the angle. This is called the centroid distance function. Thus, for crisp objects, the shape signature function corresponds to the Euclidean distance between each boundary point $A(t) = (x(t), y(t))$ and the centroid $A_c = (x_c, y_c)$ of the shape:

$$CD(t) = \sqrt{(x(t) - x_c)^2 + (y(t) - y_c)^2}$$

This shape signature function based on the centroid distance is a convenient choice in the case of star-shaped objects with respect to the centroid (i.e., for each point $y \in A$, the line segment connecting y with the centroid is contained in A).

In the case of a fuzzy object, boundary points are not strictly defined; there is a progressive transition of the membership values from the support outline to the core outline.

The shape signature can be generalized for a continuous fuzzy shape in two possible ways:

- as s radial integral of the membership function:

$$CD_{fuzzy-1}(t) = \int_{A_c}^{A(t)} \mu_A(x(\rho), y(\rho))\, d\rho$$

where $\rho = \rho(t)$ is a parameterization of the straight path between a boundary point and the centroid.

- as an average signature obtained from the $\alpha-$cuts:

$$CD_{fuzzy-2}(t) = \int_0^1 CD_\alpha(t)\, d\alpha$$

where fuzzy star-shaped objects are considered, with all the boundaries of there $\alpha-$cuts jointly indexed by the same parameter t.

A path π in $\Re^p$ from a point $x \in \Re^p$ to another point $y \in \Re^p$ is a continuous function $\pi : [0,1] \to \Re^p$, such that $\pi(0) = x$ and $\pi(1) = y$. The length of a path π in A, denoted by $\Pi_A(\pi)$, is the value of the following integration

$$\Pi_A(\pi) = \int_0^1 \mu_A(\pi(t)) \left| \frac{d\pi(t)}{dt} \right| dt$$

where $\Pi_A(\pi)$ is the integral of membership values (in A) along π.

2.3 Discrete fuzzy shapes

Discrete fuzzy objects can arise from the digitization of scanned images. Generally, the gray-level images will be thresholded to calculate geometrical measures. Since the images or their segments have ill-defined or non-crisp boundaries, it is sometimes appropriate to consider them as fuzzy sets. The concept of fuzzy digital geometry has been introduced by Rosenfeld and plays a key role in many image processing applications: "The standard approach to image analysis and recognition begins by segmenting the image into regions and computing various properties of and relationships among these regions. However, the regions are not always 'crisply' defined; it is sometimes more appropriate to regard them as fuzzy subsets of the image... It is not always obvious how to measure geometrical properties of fuzzy sets, but definitions have been given and basic properties established for a variety of such properties and relationships, including connectedness and surroundedness, convexity, area, perimeter and compactness, extent and diameter" ([12]).

The application areas of fuzzy geometry are image representation, enhancement and segmentation. The process of converting the input image into a fuzzy set by indicating, for each pixel, the degree of membership to the object, is referred to as “fuzzy segmentation”. The most straightforward way to perform fuzzy segmentation is to scale grey-levels of an image to be between 0 and 1. Such

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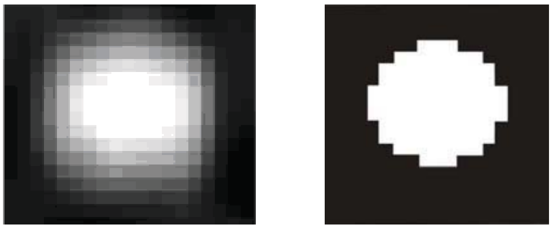
grey levels reflect the area coverage of a pixel by the object, and can be naturally used as membership values. However, in most cases, more advanced segmentation methods are required, especially since it is rarely sufficient to use only the brightness of pixels to calculate fuzzy membership values. For example, fully segmented image can be generated by combining the optimum automatic thresholding procedure with edge detection to produce continuously connected object border.

The object of interest is represented as a discrete spatial fuzzy subset of a grid. It should be noted, however, that the discrete fuzzy objects obtained from the digitization of scanned images (say, using a grey-level scale) are affected by multiple distortions, due to limited representation resolution. Consequently, their properties are significantly different with respect to those of corresponding continuous fuzzy objects. Figure 2 shows a discrete fuzzy disk (a) and, for comparison, its crisp counterpart (b).



(a) discrete fuzzy disk (b) crisp (binary) disk

Figure 2: Thresholding a fuzzy (gray-level) image: pixels with a membership degree below the threshold are lost

The mapping $\mu_A : \Re^p \to [0,1]$ of a continuous fuzzy shape becomes, by discretization, $\mu_A : Z^p \to \left\{0, \frac{1}{k}, \ \ldots, 1\right\}$, where k is the maximal number of grey levels available (e.g., $k =$ 255 for 8-bit pixel representation).

A configuration matrix for a discrete p-dimensional fuzzy shape A can be represented by vertical concatenation of its $\alpha-$level contours into a $nk \times p$ block matrix. Each one of the k sub-matrices defined at each level α collects $n \times p$ landmarks: $\mathrm{A}_\alpha = \left(x_1^A(\alpha) \quad x_2^A(\alpha) \quad \ldots \quad x_p^A(\alpha)\right)$, where $\alpha \in \{\alpha_i \mid 0 = \alpha_1 < \cdots < \alpha_i < \cdots < \alpha_k = 1\}$.

A similar $nk \times p$-dimensional configuration matrix can be defined for a crisp shape with multiple (say k) contours.

The shape signature can be also generalized for a discrete fuzzy shape in two possible ways:

- using the distance between the boundary points and the centroid:

$$CD_{discrete\_fuzzy-1}(k) = \mu_A(x_c, y_c) + \\ + \sum_{j=1}^{N_k} \delta_k(j) \cdot \mu_A(x_k(j), y_k(j))$$

- as an average signature obtained from the $\alpha-$cuts:

$$CD_{discrete\_fuzzy-2}(k) = \frac{1}{\alpha_{total}} \sum_{\alpha=1}^{\alpha_{total}} CD_{\alpha-resampled}(k)$$

where $CD_{\alpha-resampled}(k)$ is the k th sample of the resampled signature obtained for one $\alpha-$cut.

2.4 Lymph nodes as an example of crisp double-contour shapes

The ultrasound image of a lymph node (see figure 3) is a typical example of a crisp double-contour shape. It appears as an ovoid-shaped masse with an echogenic center, representing the medullary, and a peripheral, hypoechogenic cortical region, interrupted on the hyllum, which give him a reniform shape. Usually, the normal lymph nodes present a thin cortical peripheral zone, while benign inflammatory changes in lymphadenitis may enlarge the node but with preservation of the ovoidal shape and of the ratio cortical/medullary thickness less than 1.0. Malignant metastatic or infiltrated nodes are more apparent than normal ones as they become larger, rounder, and more uniformly hypoechoic by the regularly/irregularly thickening of the cortical zone with progressive restriction of the hyperechogenic medullar area. The Computed Aided Diagnosis develops ultrasound applications, especially for breast imaging, but the complete characterization must include the analysis of the satellite lymph nodes appearance. Because of the large variability of the shape and cortical-to-medullary ratio, computer vision applications are needed to make possible the automatic diagnosis, especially in breast cancer screening.

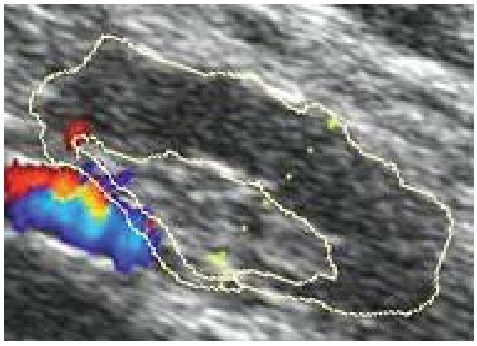


Figure 3: A crisp double-contour shape: the ultrasound image of a lymph node

3 Procrustean shape analysis

A configuration matrix A is not a proper shape descriptor, because it is not pose invariant. For any similarity transformation, i.e. $s \in \Re^+$, $R \in SO(p)$ (the special orthogonal group, i.e. R is ($p \times p$) matrix, s.t. $R'R = I$) and $t \in \Re^p$, the configuration given by $sAR + 1_p t'$ describes the same shape as A, where 1_p is the $p \times 1$ vector $(1\ 1 \ldots 1)'$. To obtain a true shape representation, location, scale and rotational effects need to be filtered out. This is carried out by shape alignment, i.e. by establishing a *coordinate reference*, commonly known as *pose*. A very popular alignment procedure is *Procrustes shape analysis,* which provides a measure, Procrustes distance, that quantifies the dissimilarity of two configurations, and which is invariant with respect to translation, scaling, and rotation.

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Procrustes shape analysis also provides a way to define the average shape, the Procrustes mean shape, which can be viewed as a representative class template.

The *Extended Orthogonal Procrustes* (EOP) problem is a least squares method for fitting a given configuration matrix A to another given matrix B. It is based on the functional model $E = sAR + 1_p t' - B$ and consists of minimizing the Procrustes distance between A and B (i.e. $\|E\|_F^2$), under choice of unknown similarity transformation parameters R, t and s. This leads to solving the problem $\min_{R,t,s} E'E$, subject to the orthogonality restriction $R'R = I$.

Generalized Orthogonal Procrustes (GOP) *analysis* is a technique that provides least-squares correspondence of more than two model points. The solution of the problem can be thought as the search of the unknown optimal matrix W (also named *consensus matrix*), defined as follows:

$$M + E_i = \hat{A}_i = s_i A_i R_i + 1_p t_i' ; \quad i = 1, \ldots, m$$

$$vec(E_i) \sim N\left(0, \Sigma = \sigma^2 \left(Q_n \otimes Q_p\right)\right)$$

where E_i is the random error matrix in normal distribution, Σ is the covariance matrix, Q_n is the cofactor matrix of the n points, Q_p is the cofactor matrix of the p coordinates of each point, $\otimes$ stands for the Kronecker product, and σ^2 is the variance factor.

Let $C = \sum_{i=1}^{m} \hat{A}_i \big/ m$ be the *geometrical centroid* of the transformed matrices. Therefore, Generalized Orthogonal Procrustes problem can be solved minimizing

$$m \sum_{i=1}^{m} \left\| \hat{A}_i - C \right\|^2 = m \sum_{i=1}^{m} tr \left\{ \left(\hat{A}_i - C\right)' \left(\hat{A}_i - C\right) \right\}$$

Crosilla and Beinat (2002) proved that the shape mean (centroid) C corresponds to the least squares estimation $\hat{M}$ of the true value M: $C = \hat{M} = \sum_{i=1}^{m} \hat{A}_i$.

In the case of *multiple-contour crisp shapes* we can benefit from the *Extended Orthogonal Procrustes* method in order to find mutual distances between shape pairs and from the *Generalized Orthogonal Procrustes* technique in order to estimate the Procrustes mean shape of a collection of shapes. This is illustrated in figures 4 and 5.

On the other hand, dealing with the case of fuzzy shapes needs more advanced Procrustean techniques, which allow us to consider *weighted distances* between points placed on $\alpha -$level contours with different membership degrees. This leads to solve a *Weighted Orthogonal Procrustes* (WOP) problem.

A weighting matrix W of the residual E (defined above) is now introduced and the minimization problem becomes:

$$\min \left\| W \, E \right\|_F^2$$

subject to orthogonality restriction $R'R = I$, $\det(R) = 1$.

Typically, an iterative method is needed to derive a solution to WOP.

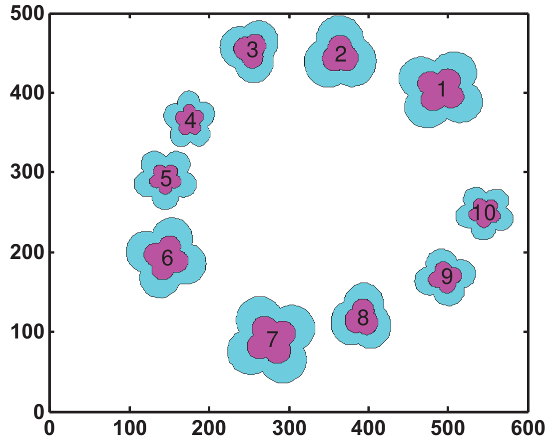

Figure 4: Ten double-contour star-shaped 2D objects with 3, 4 and 5 “lobes”

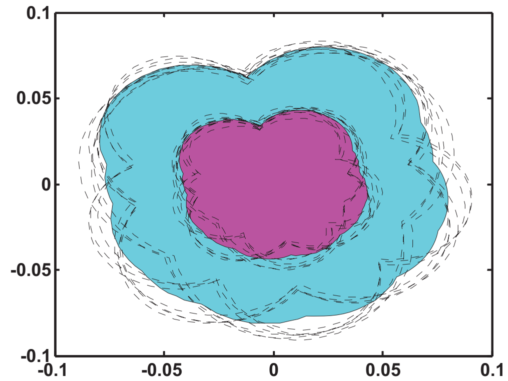

Figure 5: Procrustes mean shape (shape centroid)

4 A generalization of k-means algorithm for clustering fuzzy shapes

K-means is a commonly used data clustering for partitioning data points into disjoint groups such that data points belonging to same cluster are similar, while data points belonging to different clusters are dissimilar. The main idea is to define *k* centroids, one for each cluster, and to take each point belonging to a given data set and associate it to the nearest centroid. When no point is pending, an early groupage is done. Next, we need to re-calculate *k* new centroids of the clusters resulting from the previous step. After we have these *k* new centroids, a new binding has to be done between the same data set points and the nearest new centroid. We continue this loop until no more changes are done.

Clustering of objects or images of objects, according to the shapes of their boundaries is of a key importance in computer vision and pattern recognition. This paper was intended to pay attention to this reason by proposing a generalization of K-means algorithm in order to integrate Procrustean metrics and full mean shape estimation, in a way

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making it able of clustering objects with either multiple or fuzzy contours.
We first present the algorithm in pseudo-code, as follows:

- Make initial guesses for the mean shapes v_1, v_2, …, v_k, by choosing the first k shapes from a random permutation.
- While any change still exists in any mean shape
  - Calculate all pair-wise Procrustes distances between shapes using the *Extended Orthogonal Procrustes* algorithm
  - Use the estimated mean shapes to assign the shape samples into clusters
  - For i from 1 to k
    - Replace v_i with the mean shape of all of the samples for cluster i, using the *Generalized Orthogonal Procrustes* algorithm
  - end_for
- end_while

The resulting mean shapes for each one of the 3 clusters are shown below.

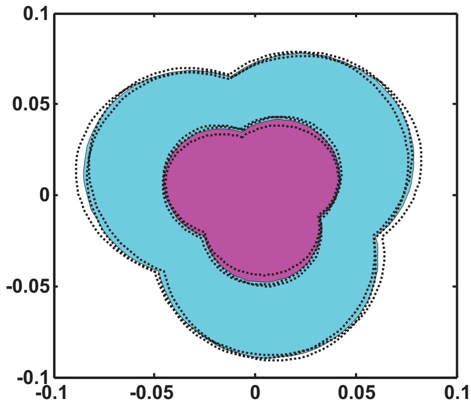

Figure 8: First cluster. Mean shape and three cluster members: {2, 3, 8}

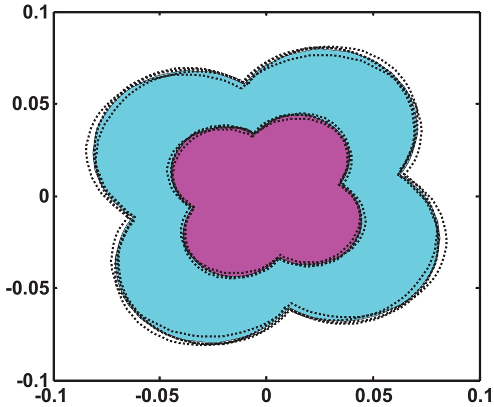

Figure 9: Second cluster. Mean shape and four cluster members: {1, 6, 7, 9}

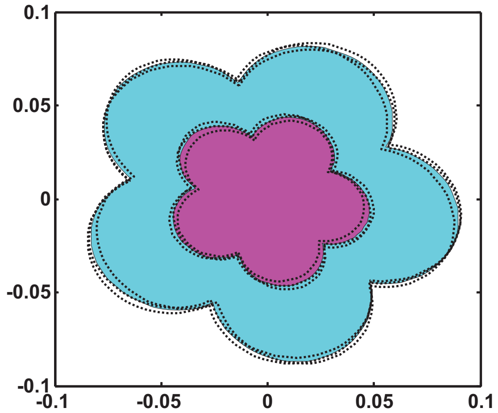

Figure 10: Third cluster. Mean shape and three cluster members: {4, 5, 10}

Our method is thus graphically validated.
As an alternative, one can use a "linkage" method to perform hierarchical shape clustering, i.e. to create a hierarchical tree of clusters starting from the symmetric matrix of Procrustean mutual distances between pairs of shapes. We obtained the same clusters as in the case of using K-means: {6, 7, 9, 1}, {2, 3, 8}, {4, 5, 10}. The dendrogram is shown below.

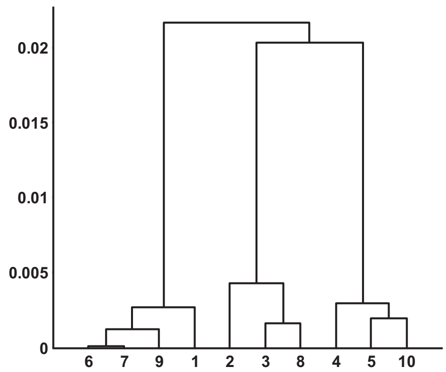

Figure 11: The dendrogram

5 Further remarks

We propose a two stage procedure for landmark placement. In the first stage, landmarks are placed arbitrarily (figure 10).

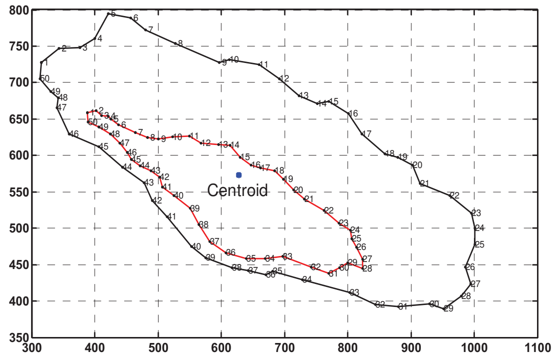

Figure 10: First stage: landmarks are placed arbitrarily

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However, we need a frame of reference to compare and display the differences in shape. Thus, we use Principal Component Analysis (PCA), which uses similarity transformations to produce a standard shape orientation, based on decomposing the overall variation of data. Each axis on a PCA representation of transformed data is an eigenvector of the covariance matrix of shape variables. In this morphospace, the first axis accounts for maximum variation in the sample, with further axes representing further decreasing variations (figure 11).

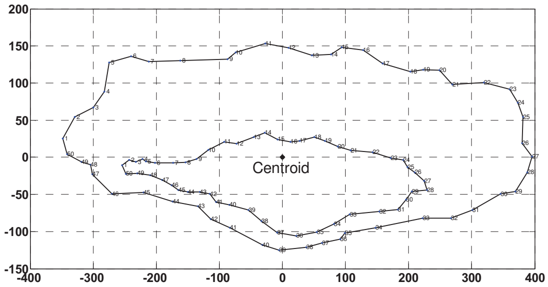

Figure 11: PCA-based alignment along the axes of maximum variation

In the second stage we replace all landmarks based on a standard procedure. The right most object point landmark is taken in the horizontal direction from the centroid. For each pair of $\alpha -$level contours and for each pair of landmarks on these contours, distances are computed starting from the corresponding points, where corresponding points are those located in the same direction from the centroid. It is essential to use an angular landmark placement procedure on the boundaries, to provide appropriate correspondence between the points on the (parts) of the boundaries having different lengths, when the corresponding boundary subparts of different $\alpha -$level contours are matched (figure 12).

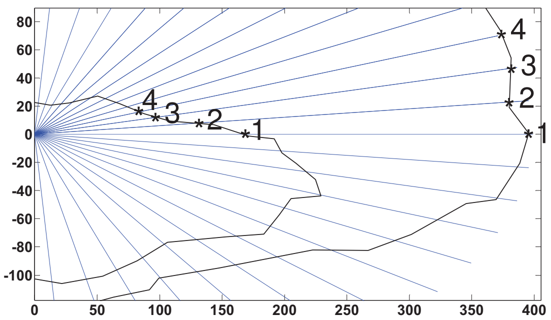

Figure 12: Corresponding points placed on radial directions from the centroid, starting from the right most object point

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